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A Design Method of Discrete-Time Adaptive Control Systems Based on Immersion and Invariance

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Abstract— **Adaptive control systems are designed to achieve desired control performance when plant parameters gains are unknown, or possibly slowly changing. Highly calculation technology is developed, more important discrete-time adaptive control structure is. Because the relative degree condition for strict positive realness (SPR) of the discrete-time error transfer function is different from the condition of the continuous time case, it is hard to prove the stability of the discretized continuous-time adaptive control systems. The main contribution of this paper is the extension of an Immersion and Invariance (I&I)-based adaptive control algorithm from continuous to discrete time.**

The theoretical stability of the proposed discrete time I&I-based adaptive control system is proved. In order to show the effectivaness of the proposed method, numerical simultions are shown.

I. Introduction

There are many cases that a desired control performance cannot be satisfied with a fixed feedback controller (non-adaptive controller) when system parameters are changed according to surrounding environmental situations and/or the time-varing operational condition. For these cases, the design methods of adaptive control system have been proposed (see Table 1). Generally the design method of adaptive control is focused on the design of a parameter adaptive law, i.e. the adjustment parameters of a controller are made to converge on the true value of uncertain parameters for plants (certainly equivalent principle).

While recently the design method of adaptive control is extended to nonlinear adaptive systems, which are not based on this certainly equivalent principle. One of these methods, I&I adaptive control that is nonlinear adaptive control systems based on Immersion and Invariance is proposed.

On the other hand, with the development of computer engineering, the application of the adaptive control in discrete-time becomes more important. Since the realization of adaptive alogrithm is complicated, it would be better to use the digitalcontrollers which can be used easily as computer software. Because the relative degree condition for strict positive realness (SPR) of the discrete-time error transfer function is different from the condition of the continuous time case, it is hard to prove the stability of the discretized continuous-time adaptive control systems. The SPR condition of the error system is important condition of Lyapunov and/or Popov stability theories. Therefore relative degree condition of the error system has to be considered to design the discrete-time adaptive control systems. Table 1 indicates some design methods of continuous-time and discrete-time adaptive control systems in chronological order.

The main contribution of this paper is the extension of an Immersion and Invariance (I&I)-based adaptive control algorithm from continuous to discrete time. The theoretical stability of the proposed discrete time I&I-based adaptive control system is proved. In order to show the effectivaness of the proposed method, numerical simultions are shown.

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II. Continuous-time I&I adaptive control

To begin with, the continuous-time I&I adaptive control is shown.

A. Immersion and Invariance

I&I adaptive control is based on both system immersion and manifold invariance. Consider the nonlinear system:

$$\dot{x} = f(x) + g(x)u, \tag{1}$$

with state $x \in \Re^n$ and control $u \in \Re^m$, with an equilibrium point $x_* \in \Re^n$ to be stabilized. If the nonlinear system satisfies follow fore conditions, the equilibrium point x_* is a (globally) asymptotically stable equilibrium of the closed-loop system

$$\dot{x} = f(x) + g(x)\psi(x, \phi(x)), \tag{2}$$

with mappings $\phi(\cdot) : \Re^n \to \Re^{n-p}$ and $\psi(\cdot,\cdot) : \Re^{n\times(n-p)} \to \Re^m$.

(A1)Target System. Target system is described as

$$\dot{\xi} = \alpha(\xi) \tag{3}$$

with state $\xi \in \Re^p$ mapping $\alpha(\cdot) : \Re^p \to \Re^p$, which has a globally asymptotically stable equilibrium at $\xi_* \in \Re^p$ and $x_* = \pi(\xi_*)$ with mapping $\pi(\cdot) : \Re^p \to \Re^n$.

(A2)Immersion Condition. For all $\xi \in \Re^p$ and mapping $c(\cdot) : \Re^p \to \Re^m$

$$f(\pi(\xi)) + g(\pi(\xi))c(\pi(\xi)) = \frac{\partial \pi}{\partial \xi}(\xi)\alpha(\xi). \tag{4}$$

(A3)Implicit Manifold. The following set identity holds

$$\begin{aligned}&\{x \in \Re^n | \phi(x) = 0\} \\ &\quad = \{x \in \Re^n |\ x = \pi(\xi)\ forsome\ \xi \in \Re^p\}. \end{aligned} \tag{5}$$

TABLE I
DESIGN METHODS OF ADAPTIVE CONTROL SYSTEMS

	Continuous-TIme	Discrete-Time
Augmented Error	Monopoli(1974)[1]	Monopoli(1977)[5]
Backstepping	Kanellakopoulos(1992)[2]	Yen(1995)[6]
High-order tuners	Morse(1992)[3]	NOT Applicable
I&I	Astolfi (2003)[4]	This Paper

(A4)Manifold Attractivity and Trajectory Boundedness. All trajectories of the system

$$\dot{z} = \frac{\partial \phi}{\partial x}[f(x) + g(x)\psi(x,z)] \quad (6)$$

$$\dot{x} = f(x) + g(x)\psi(x,z) \quad (7)$$

are bounded and satisfy

$$\lim_{t\to\infty} z(t) = 0. \quad (8)$$

B. I&I Adaptive Control Systems

The stabilization of systems of the form (1) is considered under the following assumption.

(A5)Stabilizability. There exists a controller $u = \Phi(x, \theta_*)$, where $\theta_* \in \Re^q$ is unknown, and the system

$$\dot{x} = f_*(x) := f(x) + g(x)\Phi(x, \theta_*) \quad (9)$$

has a globally asymptotically stable equilibrium at $x = x_*$.

(A6)Linearly parameterized plant. The vector field $f(x)$ can be written in the form

$$f(x) = f_0(x) + f_1(x)\theta_* \quad (10)$$

for some known functions $f_0(x) : \Re^n \to \Re^n$ and $f_1(x) : \Re^n \to \Re^{n\times q}$.

The system (1) is said to be adaptively I&I stabilizable if there exist an adaptive controller

$$\begin{cases} u = \Psi(x, \hat{\theta} + \beta_1(x)) \\ \dot{\hat{\theta}} = \beta_2(x, \hat{\theta}), \end{cases} \quad (11)$$

with extended states x,$\hat{\theta}$, and controls β_1,β_2, and the system (1) is I&I stabilizable with target system $\dot{\xi} = f_*(\xi)$ using this controller.

The system (1) can be written in the error equation by (11).

$$\dot{x} = f_*(x) + g(x)[\Psi(x, \hat{\theta} + \beta_1(x)) - \Psi(x, \theta_*)] \quad (12)$$

Then the parameter error that is added to the classical certainty equivalent control a new term is chosen as follow.

$$z \equiv \hat{\theta} - \theta_* + \beta_1(x) \quad (13)$$

The differential of parameter error is calculated

$$\begin{aligned} \dot{z} &= \beta_2(x, \hat{\theta}) + \frac{\partial \beta_1(x)}{\partial x}\dot{x} \\ &= \beta_2(x, \hat{\theta}) + \frac{\partial \beta_1(x)}{\partial x}[f_0(x) + f_1(x)\theta_* + g(x)u] \end{aligned} \quad (14)$$

using by (10). Parameter update law is choosen

$$\beta_2(x, \hat{\theta}) = -\frac{\partial \beta_1(x)}{\partial x} (f_0(x) + f_1(x)[\hat{\theta} + \beta_1(x)] + g(x)u). \quad (15)$$

Therefore the error equation of this system is indicated

$$\begin{cases} \dot{x} = f_*(x) + g(x)[\Psi(x, z + \theta_*) - \Psi(x, \theta_*)] \\ \dot{z} = -\frac{\partial \beta_1(x)}{\partial x} f_1(x) z \end{cases} \quad (16)$$

Finally if the control β_1 is chosen adequately, the system is adaptively I&I stable.

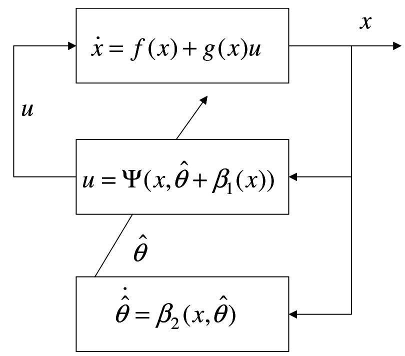

Fig. 1. Continuous-time I&I adaptive control systems

Therefore the condition (A4) is satisfied. Similarly to the nonlinear stabilization problem of using immersion and invariance, the conditions (A1)-(A3) are verified. First, (A1) is automatically satisfied from (3). Second, for the immersion condition (A2) mappings $\pi(\xi)$ and $c(\pi(\xi))$ are looked for, with

$$\begin{aligned} x &= \pi_1(\xi) \\ \hat{\theta} &= \pi_2(\xi) \\ \beta_1(x) &= c_1(\pi_1(\xi)) \\ \beta_2(x, \hat{\theta}) &= c_2(\pi(\xi)) \end{aligned}$$

where, for notational convenience we have introduced the partitions, that solve the regulator equation (4)

$$\begin{aligned} \frac{\partial \pi_1}{\partial \xi} &= f(\pi_1(\xi)) + g(\pi_1(\xi))\Psi[\pi_1(\xi), \pi_2(\xi) + c_1(\pi_1(\xi))] \\ \frac{\partial \pi_2}{\partial \xi} &= c_2(\pi(\xi)) \end{aligned}$$

For any function $c_1(\pi(\xi))$ a solution of these equations is clearly given by

$$\begin{aligned} \pi_1(\xi) &= \xi \\ \pi_2(\xi) &= \theta_* - c_1(\pi_1(\xi)) \end{aligned}$$

while $c_2(\pi(\xi))$ is defined by the last identity. This varifies condition (A2) and, selecting $\beta_1(\xi) = c_1(\pi(\xi))$, we get the implicit manifold condition (A3) as

$$\phi(x, \hat{\theta}) = \hat{\theta} - \theta_* + \beta_1(x) = 0 \tag{17}$$

Now, replacing the control law in (11) we get $\dot{x} = f(x) + g(x)\Psi(x, \hat{\theta} + \beta_1(x)))$. Writing this equation in terms of the off-the manifold coordinates $z = \hat{\theta} - \theta_* + \beta_1(x)$ and adding and subtracting $\Psi(x, \theta_*)$ yield the error equation (16).

C. Stability Analysis

Consider (16) and the following three assumption is satisfied.

(A7)The controller satisfies the Lipschiz condition

$$|\Psi(x, z + \theta_*) - \Psi(x, \theta_*)| \leq L|z| \tag{18}$$

for all $z \in \Re^q$ and for some value $L \in \Re > 0$.

(A8)There exist a function $\beta_1 : \Re^n \to \Re^q$ such that

$$\frac{\partial \beta_1(x)}{\partial x} f_1(x) + (\frac{\partial \beta_1(x)}{\partial x} f_1(x))^T > 0 \tag{19}$$

(A9)There exist a Lyapunov function of the target system $V : \Re^n \to \Re > 0$ and V satisfies

$$\frac{\partial V(x)}{\partial x} f_*(x) \leq 0 \tag{20}$$

$$\limsup_{\|x\| \to \infty} \frac{\|\frac{\partial V(x)}{\partial x} f_*(x)\|}{\|\frac{\partial V(x)}{\partial x} g(x)\|^2} \leq K < \infty \tag{21}$$

To establish the stability of the error system (16) consider a Lyapunov function candidate for (16) of the form $W(x, z) = V(x) + \frac{\rho}{2}\|z\|^2$. Using Young's inequality and assumption (A7), the derivative of $W(x, z)$ is calculated as follow.

$$\begin{aligned} \dot{W}(x, z) &= \tfrac{\partial V}{\partial x}\dot{x} + \rho z^T \dot{z} \\ &\leq \tfrac{\partial V}{\partial x} f_*(x) + \tfrac{1}{2\lambda^2}\|\tfrac{\partial V}{\partial x} g(x)\|^2 \\ &\quad + z^T(\tfrac{\lambda^2}{2} M^2 I - \rho \tfrac{\beta_1(x)}{\partial x} f_1(x))z \end{aligned} \tag{22}$$

As a result $\dot{W}(x, z) < 0$ is established from (A8) and (A9) and selecting ρ sufficiently large.

III. Discrete-time I&I adaptive control

Consider discrete-time I&I adaptive control systems. When just discretize the continuous-time I&I control systems, the desired control performance is not satisfied. For example, consider a scalar system

$$\dot{x} = \theta_* + u \tag{23}$$

where θ_* is unknown. The system is stabilized by continuous-time I&I adaptive control systems with target system

$$\dot{\xi} = -\xi + \sin(\frac{\Pi}{10}t) + \sin(\frac{\Pi}{100}t) \tag{24}$$

and unknown $\theta*$ is $\theta_* = 2, (0 \leq t \leq 30)$ and $\theta_* = 4, (30 \leq t)$. In the case that the unknown parameter of a system is changed, the continuous-time I&I adaptive control satisfy the desired output (Fig.2). To make discrete-time I&I adaptive control systems, when just discretize the continuous-time one, the discrete-time compensator can not satisfy the desired output as following figures (Fig.3).

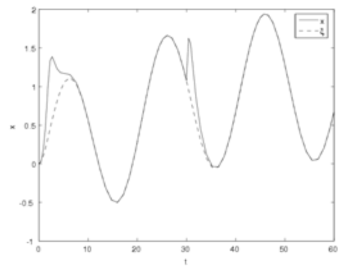

Fig. 2. Output $x(t)$ (adaptive I&I)

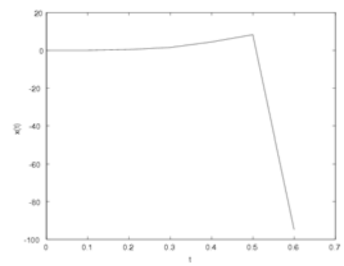

Fig. 3. Output $x(t)$ (just discretize)

Therefore discrete-time adaptive control system must be designed for original method and we propose a design method of discrete-time I&I adaptive control systems based on immersion and invariance.

A. Immersion and Invariance

Discrete-time immersion and invariance is now proposed. Consider the nonlinear system

$$x_{k+1} = f(x_k) + g(x_k)u_k \tag{25}$$

with state $x \in \Re^n$ and control $u \in \Re^m$, with an equilibrium point $x_* \in \Re^n$ to be stabilized. If the nonlinear system satisfies follow four conditions, the equilibrium point x_* is a (globally) asymptotically stable equilibrium of the closed-loop system

$$x_{k+1} = f(x_k) + g(x_k)\psi(x_k, \phi(x_k)) \tag{26}$$

with mappings $\phi(\cdot) : \Re^n \to \Re^{n-p}$ and $\psi(\cdot, \cdot) : \Re^{n \times (n-p)} \to \Re^m$.

(B1)Target System. Target system

$$\xi_{k+1} = \alpha(\xi_k) \tag{27}$$

with state $\xi \in \Re^p$ mapping $\alpha(\cdot) : \Re^p \to \Re^p$, has a globally asymptotically stable equilibrium at $\xi_* \in \Re^p$ and $x_* = \pi(\xi_*)$ with mapping $\pi(\cdot) : \Re^p \to \Re^n$.

(B2)Immersion Condition. For all $\xi \in \Re^p$ and mapping $c(\cdot) : \Re^p \to \Re^m$

$$f(\pi(\xi_k)) + g(\pi(\xi_k))c(\pi(\xi_k)) = \pi(\alpha(\xi_k)) \tag{28}$$

(B3)Implicit Manifold. The following set identity holds

$$\begin{aligned} \{x_k \in \Re^n | \phi(x_k) = 0\} \\ = \{x_k \in \Re^n | \; x_k = \pi(\xi_k)\} \end{aligned} \tag{29}$$

(B4)Manifold Attractivity and Trajectory Boundedness. All trajectories of the system

$$z_k = \phi(x_k) \tag{30}$$
$$x_{k+1} = f(x_k) + g(x_k)\psi(x_k, z_k) \tag{31}$$

are bounded and satisfy

$$\lim_{k\to\infty} z_k = 0 \tag{32}$$

B. I&I Adaptive Control Systems

The stabilization of systems of the form (25) is considered under the following assumption.

(B5)Stabilizability. There exists a controller $u = \Psi(x_k, \theta_*)$, where $\theta_* \in \Re^q$ is unknown, and the system

$$x_{k+1} =: f_*(x_k) = f(x_k) + g(x_k)\Psi(x_k, \theta_*) \tag{33}$$

has a globally asymptotically stable equilibrium at $x = x_*$.

(B6)Linearly parameterized plant. The vector field $f(x_k)$ can be written in the form

$$f(x_k) = f_0(x_k) + f_1(x_k)\theta_* \tag{34}$$

for some known functions $f_0(x_k) : \Re^n \to \Re^n$ and $f_1(x_k) : \Re^n \to \Re^{n\times q}$.

The system (25) is said to be adaptively I&I stabilizable if there exist an adaptive controller

$$\begin{cases} u_k = \Psi(x_k, \hat{\theta}_k + \beta_1(x_k)) \\ \hat{\theta}_{k+1} = \hat{\theta}_k + \beta_2(x_k, \hat{\theta}_{k+1}) \end{cases} \tag{35}$$

with extended states x_k,$\hat{\theta}_k$, and controls β_1,β_2, and the system (25) is I&I stabilizable with target system $\xi_{k+1} = f_*(\xi_k)$ using this compensator. Then we can choose the parameter update law $\hat{\theta}_{k+1} - \hat{\theta}_k = \beta_2(x_k, \hat{\theta}_{k+1})$ considering the discrete-time adaptive control problem of relative degree.

The system (25) can be written in the error equation by (35).

$$x_{k+1} = f_*(x_k) \tag{36}$$
$$+g(x_k)[\Psi(x_k, \hat{\theta}_k + \beta_1(x_k)) - \Psi(x_k, \theta_*)]. \tag{37}$$

Then the parameter error that is added to the classical certainty equivalent control a new term is chosen as follow.

$$z_k \equiv \hat{\theta}_k - \theta_* + \beta_1(x_k) \tag{38}$$

Consider the attractivity of the manifold we calculate z_{k+1} as follow.

$$\begin{aligned} z_{k+1} &= \hat{\theta}_{k+1} - \theta_* + \beta_1(x_{k+1}) \\ &= \hat{\theta}_{k+1} - \hat{\theta}_k + \hat{\theta}_k - \theta_* \\ &\quad +\beta_1(x_k) + \beta_1(x_{k+1}) - \beta_1(x_k) \\ &= \beta_2(x_k, \hat{\theta}_{k+1}) + z_k + \beta_1(x_{k+1}) - \beta_1(x_k) \end{aligned}$$

Then we choose the control β_1

$$\beta_1(x_k) = K(x_{k-1})x_k \tag{39}$$

with $K(x_k) : \Re^n \to \Re^{(q\times n)}$, z_{k+1} becomes as follows:

$$\begin{aligned} z_{k+1} &= \beta_2(x_k, \hat{\theta}_{k+1}) + z_k + \beta_1(x_{k+1}) - \beta_1(x_k) \\ &= \beta_2(x_k, \hat{\theta}_{k+1}) + z_k + K(x_k)x_{k+1} - K(x_{k-1})x_k \\ &= \beta_2(x_k, \hat{\theta}_{k+1}) + z_k - K(x_{k-1})x_k \\ &\quad +K(x_k)(f_0(x_k) + f_1(x_k)\theta_* + g(x_k)u_k) \end{aligned} \tag{40}$$

Parameter update law should be designed with paying attention that it includes the function of $\hat{\theta}_{k+1}$, because the relative degree of the error system is zero.

$$\begin{aligned} \beta_2(x_k, \hat{\theta}_{k+1}) &= -K(x_k)(f_0(x_k) + f_1(x_k)(\hat{\theta}_{k+1} \\ &\quad +K(x_k)x_{k+1}) + g(x_k)u_k) \\ &\quad +K(x_{k-1})x_k \end{aligned} \tag{41}$$

At this time, z_{k+1} becomes as follow.

$$\begin{aligned} z_{k+1} &= z_k - K(x_k)f_1(x_k)(\hat{\theta}_{k+1} - \theta_* + K(x_k)x_{k+1}) \\ &= z_k - K(x_k)f_1(x_k)z_{k+1} \end{aligned} \tag{42}$$

Finally if the control β_1 is chosen adequately, the system is adaptively I&I stable and the error equation of this system is

$$\begin{cases} x_{k+1} = f_*(x_k) + g(x_k)[\Psi(x_k, z_k + \theta_*) - \Psi(x_k, \theta_*)] \\ z_{k+1} = (I + K(x_k)f_1(x_k))^{-1} z_k \end{cases} \tag{43}$$

The parameter update law (41) cannot be implemented, because both sides of this equation has $\hat{\theta}_{k+1}$. Then the parameter update law is used as following equation.

$$\begin{aligned} \hat{\theta}_{k+1} &= (I + K(x_k)f_1(x_k))^{-1}(\hat{\theta}_k - K(x_k)(f_0(x_k) \\ &\quad +f_1(x_k)K(x_k)x_{k+1} + g(x_k)u_k) + K(x_{k-1})x_k) \end{aligned} \tag{44}$$

Therefore the condition (B4) is satisfied. Similarly to the nonlinear stabilization problem of using immersion and invariance, we have to verify the condition (B1)-(B3). First, (B1) is automatically satisfied from the target system. Second, for the immersion condition (B2) we are looking for mappings $\pi(\xi)$ and $c(\pi(\xi))$, with

$$\begin{aligned} &x_k = \pi_1(\xi_k) \\ &\hat{\theta}_k = \pi_2(\xi_k) \\ &\beta_1(x_k) = c_1(\pi_1(\xi_k)) \\ &\beta_2(x_k, \hat{\theta}_{k+1}) = c_2(\pi_1(\xi_k), \pi_2(\xi_{k+1})) \end{aligned}$$

where, for notational convenience we have introduced the partitions, that solve the regulator equation (28)

$$\begin{aligned} \pi_1(\xi_{k+1}) &= f(\pi_1(\xi_k)) + g(\pi_1(\xi_k)) \\ &\quad \times\Psi[\pi_1(\xi_k), \pi_2(\xi_k) + c_1(\pi_1(\xi_k))] \\ \pi_1(\xi_{k+1}) &= c_2(\pi_1(\xi_k), \pi_2(\xi_{k+1})) + \pi_2(\xi_k) \end{aligned}$$

For any function $c_1(\pi(\xi))$ a solution of these equations is clearly given by

$$\begin{aligned} \pi_1(\xi_k) &= \xi_k \\ \pi_2(\xi_k) &= \theta_* - c_1(\pi_1(\xi_k)) \end{aligned}$$

while $c_2(\pi(\xi))$ is defined by the last identity. This verifies condition (B2) and, selecting $\beta_1(\xi) = c_1(\pi(\xi))$, we get the implicit manifold condition (B3) as

$$\phi(x_k, \hat{\theta}_k) =: \hat{\theta}_k - \theta_{*k} + \beta_1(x_k) = 0 \tag{45}$$

Now, replacing the control law in (35) we get $x_{k+1} = f(x_k) + g(x_k \Psi(x_k, \hat{\theta}_k + \beta_1(x_k)))$. Writing this equation in terms of the off-the manifold coordinates $z_k = \hat{\theta}_k - \theta_* + \beta_1(x_k)$ and adding and subtracting $\Psi(x_k, \theta_*)$ yield the error equation (43).

C. Stability Analysis

Consider (43) and the following three assumptions are satisfied.

(B7)The controller satisfies the Lipschiz condition

$$|\Psi(x_k, z_k + \theta_*) - \Psi(x_k, \theta_*)| \leq L|z_k| \tag{46}$$

for all $z \in \Re^q$ and for some value $L \in \Re > 0$.

(B8)There exist a function $K(x_k) : \Re^n \rightarrow \Re^{(q \times n)}$ such that

$$K(x_k)f_1(x_k) + [K(x_k)f_1(x_k)]^T > 0 \tag{47}$$

(B9)There exist a Lyapunov function of the target system $V : \Re^n \rightarrow \Re > 0$ and V satisfies

$$\begin{aligned} V(x_{k+1}) - V(x_k) &= (\frac{\delta V}{\delta x_1} \dots \frac{\delta V}{\delta x_n})(x_{k+1} - x_k) \\ &= D_V(f_*(x_k) - x_k) \leq 0 \\ D_V &=: (\frac{\delta V}{\delta x_1} \dots \frac{\delta V}{\delta x_n}) \end{aligned} \tag{48}$$

$$\limsup_{\|x_k\| \rightarrow \infty} \frac{\|D_V(f_*(x_k) - x_k)\|}{\|D_V g(x)\|^2} \leq K < \infty \tag{49}$$

To establish the stability of the error system (43) consider a Lyapunov function candidate for (43) of the form $W(x_k, z_k) = V(x_k) + \rho \|z_k\|^2$. Using Young's inequality and assumption (A7), the difference of $W(x_k, z_k)$ is calculated as follow.

$$\begin{aligned} \triangle W(x_k, z_k) &= W(x_{k+1}, z_{k+1}) - W(x_k, z_k) \\ &= V(x_{k+1}) - V(x_k) + \rho(|z_{k+1}|^2 - |z_k|^2) \\ &\leq D_V(f_*(x_k) - x_k) + \frac{1}{2\alpha}|D_V g(x_k)|^2 \\ &\quad + \frac{\alpha}{2}|\Psi(x_k, z_k + \theta *) - \Psi(x_k, \theta *)|^2 \\ &\quad - 2\rho(K(x_k)f_1(x_k)z_{k+1})^T z_{k+1} \\ &\quad - \rho(K(x_k)f_1(x_k)z_{k+1})^T(K(x_k)f_1(x_k)z_{k+1}) \\ &\leq D_V(f_*(x_k) - x_k) + \frac{1}{2\alpha}|D_V g(x_k)|^2 \\ &\quad + \frac{\alpha}{2}L^2|z_k|^2 - 2\rho(K(x_k)f_1(x_k)z_{k+1})^T z_{k+1} \\ &\quad - \rho(K(x_k)f_1(x_k)z_{k+1})^T(K(x_k)f_1(x_k)z_{k+1}) \\ &= D_V(f_*(x_k) - x_k) + \frac{1}{2\alpha}|D_V g(x_k)|^2 \\ &\quad - z_{k+1}^T(I + (K(x_k)f_1(x_k))^T) \\ &\quad \times (\frac{\rho}{2}K(x_k)f_1(x_k) - \frac{\alpha}{2}L^2 I)z_{k+1} \\ &\quad - (\frac{\rho}{2} - \frac{\alpha}{2}L^2)z_{k+1}^T(I + K(x_k)f_1(x_k))^T \\ &\quad \times K(x_k)f_1(x_k)z_{k+1}^T \\ &\quad - \rho z_{k+1}(Kf_1)^T z_{k+1} \end{aligned} \tag{50}$$

As a result, $\triangle W(x, z) < 0$ is established from (B8) and (B9) and selecting ρ sufficiently large.

D. Numerical Simulations

It is said that I&I system has a certain level of robustness, and we compare discrete-time adaptive I&I which is proposed to discrete-time adaptive backstepping system [6]. Consider a continuous-time system

$$\ddot{x}(t) = -\theta_* \sin(x(t)) - (x(t) - x_*) - \dot{x}(t) + u(t) + w \tag{51}$$

where w is step disturbance. We stabilize this continuous-time system at $x_* = 1$ using proposed discrete-time adaptive I&I systems. Then target system choosed

$$x_{k+1} = \frac{3}{4}x_k + \frac{1}{4} \tag{52}$$

Therefore the compensator is calculated as follow.

$$\begin{cases} u_k = (\hat{\theta}_k - \frac{x_k}{T_s}\sin(x_{k-1}))\sin x_k + x_k - x_* \\ \qquad + \frac{1}{T_s}(x_k - x_{k-1}) + \frac{1}{T_s^2}\left(\frac{1}{4} + x_{k-1} - \frac{5}{4}x_k\right) \\ \hat{\theta}_{k+1} = \frac{1}{1 + T_s \sin^2(x_k)}\Big(\hat{\theta}_k - \frac{x_k}{T_s}\sin(x_{k-1}) + \frac{1}{T_s}\sin(x_k) \\ \quad \times\Big(-T_s(x_k - x_{k-1}) + 2x_k - x_{k-1} - T_s^2(x_k - x_*) \\ \quad + T_s x_{k+1}\sin^2(x_k) + T_s^2 u_k)\Big)\Big) \end{cases}$$

The parameter update law cannot be available, because both sides of the equation has $\hat{\theta}_{k+1}$. Then the parameter update law is used as following equation.

$$\begin{aligned} \hat{\theta}_{k+1} &= (I + K(x_k)f_1(x_k))^{-1}(\hat{\theta}_k - K(x_k)(f_0(x_k) \\ &\quad + f_1(x_k)K(x_k)x_{k+1} + g(x_k)u_k) + K(x_{k-1})x_k) \end{aligned} \tag{53}$$

Unknown parameter θ_* and disturbance term w are given in as follow.

Backstepping

$$\theta_* = 2(0 \leq t \leq 80), \theta_* = 2(80 \leq t)$$
$$w = 0(0 \leq t \leq 160), w = 2(160 \leq t)$$

I&I based Adaptive Control

$$\theta_* = 2(0 \leq t \leq 40), \theta_* = 2(40 \leq t)$$
$$w = 0(0 \leq t \leq 80), w = 2(80 \leq t)$$

Then simulation results are given in Figs.4 and 5(Backstepping) and Figs.6-9(I&I).

We indicate effectivity of proposed method by simulating two discrete-time adaptive control, adaptive backstepping and adaptive I&I control. Both adaptive backstepping control and Adaptive I&I control satisfy the desired output when system unknown parameter is changed. When disturbance term is added to input, adaptive backstepping control can not satisfy the desired output. While adaptive I&I control systems which is proposed method can satisfy the desired output. Therefore it is said that adaptive I&I control has a certain robustness. The robustness of proposed method is an excellent point compared to design methods of previous adaptive control systems.

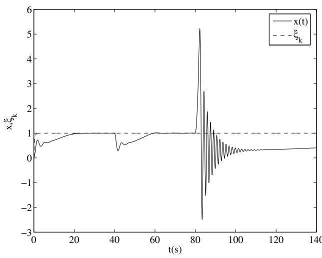

Fig. 4. Output $x(t)$ (Backstepping)

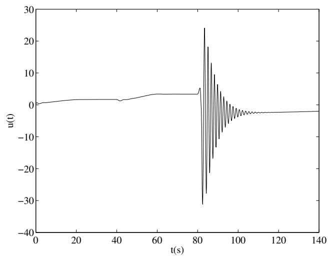

Fig. 5. Input $u(t)$ (Backstepping)

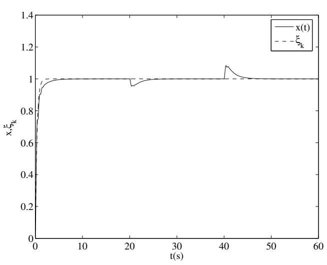

Fig. 6. Output $x(t)$ (I&I)

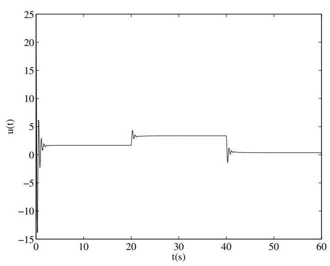

Fig. 7. Input $x(t)$ (I&I)

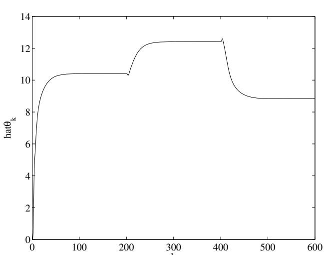

Fig. 8. $\hat{\theta}_k$ (I&I)

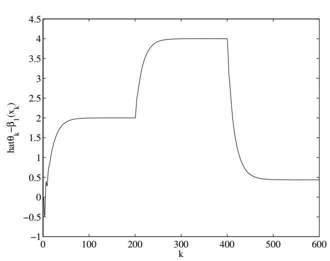

Fig. 9. $\hat{\theta}_k + \beta_1(x_k)$ (I&I)

IV. CONCLUSIONS

In this paper, we propose a design method of discrete-time adaptive control systems based on immersion and invariance.

With the development of computer engineering, the application of the adaptive control in discrete-time becomes important. Due to the construction of adaptive controller is complicated, it would be better to realize the controllers in digital algorithm, which can be used as computer software. However, there are many cased that the discrete-time adaptive control systems are not stable when just discretize the continuous-time adaptive control systems. Because relative degree condition of strict positive real (SPR) condition in discrete-time is different from continuous-time the condition. The notion of SPR condition is important condition of Lyapunov and Popov stability theories. Therefore relative degree condition has to be considered to design the discrete-time adaptive control systems, and discrete-time adaptive control systems must be designed for original method.

We propose a design method to prove the stability of adaptive control systems by Lyapunov stability theory and the adaptive law is designed considering the discrete-time adaptive control problem of relative degree condition. Therefore the stability of the proposed discrete-time adaptive control system is proved and simulations show the value of this system. A previous adaptive control system does not have robustness, while adaptive I&I control has certain robustness. The robustness of proposed method is an excellent point compared to design methods of previous adaptive control systems.

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