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A separation principle for linear switching systems and parametrization of all stabilizing controllers

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***Abstract*—In this paper, we investigate the problem of designing a switching compensator for a plant switching amongst a (finite) family of given configurations (A_i, B_i, C_i). We assume that switching is uncontrolled, namely governed by some arbitrary switching rule, and that the controller has the information of the current configuration i.**

As a first result, we provide necessary and sufficient conditions for the existence of a family of linear compensators, each applied to one of the plant configurations, such that the closed loop plant is stable under arbitrary switching. These conditions are based on a separation principle, precisely, the switching stabilizing control can be achieved by separately designing an observer and an estimated state (dynamic) compensator. These conditions are associated with (non–quadratic) Lyapunov functions. In the quadratic framework, similar conditions can be given in terms of LMIs which provide a switching controller which has the same order of the plant.

As a second result, we furnish a characterization of all the stabilizing switching compensators for such switching plants. We show that, if the necessary and sufficient conditions are satisfied then, given *any arbitrary family* of compensators $\mathcal{K}_i(s)$, each one stabilizing the corresponding LTI plant (A_i, B_i, C_i) for fixed i, there exist suitable realizations for each of these compensators, which assure stability under arbitrary switching.

Index terms– Switching systems, Youla-Kucera parametrization, separation principle, Lyapunov functions.

I. INTRODUCTION

Systems including both logic and continuous variables, the so called hybrid systems, are currently considered a main stream topic as it can be seen from the considerable number of contributions (see for instance [1], [2], [3], [4]). In particular, the so called switching systems, are relevant in many applications and are intensively considered in control theory for two basic reasons.

First, switching is a phenomenon that naturally occurs in several plants that can change suddenly their configuration and an efficient control design must take into account this fact. Basically, determining a single compensator which stabilizes a switching plant can be regarded as a robust design problem and faced with existing techniques [5], [6]. The most efficient techniques are perhaps those based on the Lyapunov approach [7], [8], [9]. In particular, those based on quadratic functions have been successful because of the development of efficient tools based on LMIs [10]. An interesting case is that in which the compensator is informed on–line (not in the design stage) of the plant configuration. This is basically a gain–scheduling problem [11], for which Lyapunov theory has been revealed successful [12], [13], [14], [15].

The second reason of the intense investigation of switching systems is that, even in the case of a single plant, considerable advantages in terms of performances can be achieved by properly switching among compensators. In this case, switching is not imposed by nature, but artificially introduced by the designer. The consequent benefit is well established and indeed switching techniques have been involved in adaptive schemes [16], [17], [18], supervisory control [19], reset design [20] and robust synthesis [21].

In dealing with switching compensators, a fundamental issue is how to guarantee stability. In a recent paper [22] the following essential result has been proved. Given a single linear plant and a family of linear stabilizing compensators, there always exist (possibly non–minimal) realizations for all of them which assures global stability under arbitrary switching. This result is based on a proper formulation of the problem based on the Youla–Kucera parametrization [23], [24] of all stabilizing compensators. The key idea is to show that one can solve the problem, basically, by switching among Youla–Kucera parameters. A key point is that the realization of the Youla–Kucera parameters cannot be arbitrary, but suitably constructed.

The main idea of the present paper is to consider at the same time both the mentioned aspects: controlling a switching linear plant by means of a switching linear controller. We assume that plant switching is homogeneously determined while the compensator commutations are commanded by the plant. Our basic question is the following: given a switching plant, under which conditions there exists a switching compensator which stabilizes the plant under arbitrary switching? This issue was pointed out as an open problem in [22]. Under the assumption that the compensator is granted the instantaneous exact knowledge of the current plant configuration, without delay, we provide the following main results.

- Necessary and sufficient stabilizability conditions are given. These are supported by polyhedral Lyapunov functions and are based on a separation principle. The controller is derived by designing an (extended) observer and a (dynamic) state feedback, although we cannot provide bounds for the compensator order.
- The mentioned conditions are constructive, but computationally demanding. If we strengthen our assumptions to quadratic stabilizability, then the necessary and sufficient conditions are expressed in terms of LMIs. We show that the compensator may have the same order of the plant.

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• Once the necessary and sufficient conditions are assured, we can parametrize the set of all linear switching stabilizing (or quadratically stabilizing) compensators for the switching plant.

The results have several implications as well as applications. For instance, the complete parametrization is given in a form which is suitable for optimal design, since the closed loop map is shown to be an affine function of the Youla–Kucera parameter, the natural extension of the standard linear time–invariant theory. We will investigate on what we call the paradox of the "zero transfer functions compensator". Given a system which is stable in any fixed configuration, but switching unstable, under the given conditions, we can assure switching stability by means of a switching compensator with the (surprising) property of having zero transfer function for each fixed configuration. The explanation of this paradox is quite intriguing. Precisely the switching compensator must be *properly realized* in such a way that *its observable and reachable subsystems interact* only during switching. We propose a "switching manager" control as an application of this paradox.

The paper is organized as follows. After the formulation of the problem in Sections II, the main results are all stated in Section III without proofs which are essentially based on previous results on non–quadratic Lyapunov functions (see [25], [26], [27] and [9] for a survey), on generalized observers [28], [29] and duality properties between observer and state feedback design [15]. In the quadratic stabilization case the results are based on standard LMI techniques [10]. The parametrization of all stabilizing compensators is achieved by generalizing ideas described in [5] (see also [6]). The implications are described in section IV and we finally discuss the results in section VI. The present article is the conference version of [30] and, for space reasons, the proofs and most of the examples in the original paper have been omitted. We refer the interested reader to [30] for the full version.

II. DEFINITIONS AND PROBLEM STATEMENT

Consider the time–varying system

$$\begin{array}{rcl} \delta x(t) & = & A_i x(t) + B_i u(t) \\ y(t) & = & C_i x(t) \end{array} \qquad (1)$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^p$. δ represents the derivative in the continuous–time case and the one–step shift operator $\delta x(t) = x(t+1)$ in the discrete–time case. We assume that the plant matrices can switch arbitrarily, precisely that $i = i(t) \in I = \{1, 2, \ldots, r\}$ and that for each i the plant $(A_i,\ B_i,\ C_i)$ is stabilizable. For the simple notations, we have dropped the time t from the index i with the understanding that $(A_i, B_i, C_i) = (A_{i(t)}, B_{i(t)}, C_{i(t)})$. For this system, we consider the class of linear switching controllers

$$\begin{array}{rcl} \delta z(t) & = & F_i z(t) + G_i y(t) \\ u(t) & = & H_i z(t) + K_i y(t) \end{array} \qquad (2)$$

where, again, $i = i(t) \in I$, and $(F_i, G_i, H_i, K_i) = (F_{i(t)}, G_{i(t)}, H_{i(t)}, K_{i(t)})$. In the following, it will be assumed that **(a)** the number of switching instants is finite (although it may be arbitrarily large, in the continuous–time case) on every finite interval and **(b)** the switching time is zero for both plant and compensator and there is no delay in the communication between the plant and the controller, which knows exactly the current $y(t)$ and configuration $i(t)$. We stress that the finite-switching assumption is not actually a restriction (it can be easily dropped and avoid the well–posedness issue), whereas the no-delay assumption might be a restriction in practice, though fairly acceptable in most plants.

The closed loop system matrix achieved from (1) and (2) becomes

$$A_i^{cl} = \left[\begin{array}{cc} A_i + B_i K_i C_i & B_i H_i \\ G_i C_i & F_i \end{array} \right] \qquad (3)$$

For this system (or any arbitrary switching system) we adopt the following stability definitions.

Definition 2.1: The system governed by matrices A_i^{cl}, $i(t) \in I$ is: **(a) Hurwitz (Schur) stable**, if for every fixed value i, its eigenvalues have negative real parts (respectively modulus less than one), **(b) switching stable**, if it is asymptotically stable for any switching signal $i(t) \in I$, and **(c) quadratically stable**, if these matrices share a common quadratic Lyapunov function.

In the sequel, when we will talk about "stability", we will always refer to "switching stability". It is well established that the three definitions are not equivalent, precisely **(c)⇒(b)⇒(a)** [8] (we remind that we assumed zero dwell time). In a Lyapunov framework, switching stability is equivalent to the existence of a Lyapunov function which is a polyhedral norm (see [26], [27], [25]). We will use this fact later.

The next two problems are addressed in this paper.

Problem 1: Given the switching plant represented by (1), does there exist a family of matrices (F_i, G_i, H_i, K_i), $i \in I$ such that the system governed by (3) is switching stable?

Once the previous problem has received a "yes" answer, the next question is in order.

Problem 2: Given a set of transfer functions $\mathcal{K}_i(s)$ assuring that the ith closed loop system is Hurwitz (respectively Schur), namely stable for fixed i, does there exist realizations for the $\mathcal{K}_i(s)$ such that the system is switching stable?

In the next section we come up with a necessary and sufficient condition for Problem 1 and with an "always yes" reply to the question of Problem 2.

III. MAIN RESULTS

A. Necessary and sufficient stabilizability conditions

To state our results we need a technical definition. Given a square matrix P, $\|P\|_l$, $l = 1,\ \infty$, denotes the standard induced matrix norms with respect to $\|\cdot\|_l$ norm for vectors.

Definition 3.1: The square matrix M is of class $\mathcal{H}_1$ if there exists $\tau > 0$ such that $\|I + \tau M\|_1 < 1$. It is of class $\mathcal{H}_\infty$ if there exists $\tau > 0$ such that $\|I + \tau M\|_\infty < 1$.

The above classes, introduced to state the continuous–time conditions, are associated to existing algorithms based on the Euler approximating systems (see [9]). The following holds:

Theorem 3.1: The following two statements are equivalent for continuous–time (resp. discrete–time) systems.

i) There exists a linear switching compensator (2) for the switching plant (1) which assures switching stability to the closed loop system.

ii) There exist $\mu\times\mu$ matrices $P_i \in \mathcal{H}_1$ and $\nu\times\nu$ matrices $Q_i \in \mathcal{H}_\infty$, (respectively matrices $\|P_i\|_1 < 1$, and $\|Q_i\|_\infty < 1$), $m\times\mu$ matrices U_i, $p\times\nu$ matrices L_i , $n\times\mu$ matrix X, and $\nu\times n$ matrix R , of full row rank and full column rank, respectively, such that

$$A_iX + B_iU_i = XP_i \tag{4}$$

$$RA_i + L_iC_i = Q_iR \tag{5}$$

Corollary 3.1: If the necessary and sufficient conditions are satisfied, then a stabilizing compensator is given by

$$\begin{aligned} \delta w(t) &= Q_iw(t) - L_iy(t) + RB_iu(t) & (6)\\ \hat{x}(t) &= Mw(t) & (7)\\ \delta z(t) &= F_iz(t) + G_i\hat{x}(t) & (8)\\ u(t) &= H_iz(t) + K_i\hat{x}(t) + v(t) & (9)\end{aligned}$$

where $v(t) = 0$ (the reason of introducing this dummy signal will become clear later). The matrix M is any left inverse of R ($MR = I$), while (F_i, G_i, H_i, K_i) can be computed as

$$\begin{bmatrix} K_i & H_i \\ G_i & F_i \end{bmatrix} = \begin{bmatrix} U_i \\ V_i \end{bmatrix} \begin{bmatrix} X \\ Z \end{bmatrix}^{-1} \tag{10}$$

where Z is any complement of X which makes the square matrix invertible and $V_i = ZP_i$. The compensator is of order $\nu+\mu-n$.

Remark 3.1: The compensator has a separation structure. Indeed it can be shown that $\hat{x}(t)$ estimates asymptotically $x(t)$ since $\|Rx-w\|_\infty \to 0$ and the dynamic compensator having z as state variable is a dynamic state feedback stabilizing compensator. The state feedback condition (4) was previously given in [31].

Unfortunately, the computation of the solution of (4) and (5) may be non trivial. Indeed, (4)–(5) are bilinear and therefore they cannot be easily solved for fixed dimensions ν and μ of R and X (which is equivalent to fix the compensator complexity). However they can be solved by means of known iterative procedures to determine polyhedral Lyapunov functions [32], [33], [9] although there is no upper bound for the order of the compensator, which depends on the system data.

B. The quadratic stabilization case

If we strengthen our assumption, invoking quadratic stabilizability, the following holds

Theorem 3.2: The following two statements are equivalent in the continuous–time case.

i) There exists a linear switching compensator (2) for the switching plant (1) assuring switching quadratic stability.

ii) There exist positive definite symmetric $n\times n$ matrices P and Q, and $m\times n$ matrices U_i and $n\times p$ matrices Y_i such that

$$PA_i^T + A_iP + B_iU_i + U_i^TB_i^T < 0 \tag{11}$$

$$A_i^TQ + QA_i + Y_iC_i + C_i^TY_i^T < 0 \tag{12}$$

In the discrete–time case the LMIs are different, precisely, i) is equivalent to the next statement.

iii) There exist symmetric positive–definite $n\times n$ matrices P and Q, and $m\times n$ matrices U_i and $n\times p$ matrices Y_i such that

$$\begin{bmatrix} P & (A_iP + B_iU_i)^T \\ A_iP + B_iU_i & P \end{bmatrix} > 0 \tag{13}$$

$$\begin{bmatrix} Q & (QA_i + Y_iC_i)^T \\ QA_i + Y_iC_i & Q \end{bmatrix} > 0 \tag{14}$$

Corollary 3.2: If the necessary and sufficient conditions are satisfied, then a stabilizing compensator is given by the standard observer + feedback compensator

$$\begin{aligned} \delta\hat{x}(t) &= (A_i + L_iC_i + B_iJ_i)\hat{x}(t) - L_iy(t) + B_iv(t) \\ u(t) &= J_i\hat{x}(t) + v(t) \end{aligned} \tag{15}$$

with $v(t) = 0$ (again, this signal will be used later), and

$$J_i = U_iP^{-1} \quad \text{and} \quad L_i = Q^{-1}Y_i$$

where P and Q are the matrices defined in (11) and (12).

Remark 3.2: Also this compensator has an observer–based structure. It is of order n, and thus of fixed complexity. This shows that, for switching systems, quadratic stabilizability is equivalent to quadratic stabilizability with a compensator of at most the same order of the plant.

Note that (11)–(12) and (13)–(14) are LMIs, thus easily solvable. We stress that this kind of conditions are known in the LMI literature for both state feedback and observer design [12], [13], [34]. They have been proposed for instance for LPV systems [12] (see also [10]). In [12] when the LMIs are stated (Th. 4.3) it is assumed that B and C are certain matrices. This is a critical assumption in the LPV case but not an issue in the switching case. The conditions based on LMIs and quadratic functions lead to efficient algorithms but they are conservative. Indeed, there are switching stable systems which do not admit quadratic Lyapunov functions. Less conservative results can be achieved if one considers synthesis results based on parameter–dependent Lyapunov functions [14], [35], [36].

C. The set of all stabilizing compensators

In this section, we consider the problem of parametrizing all the switching compensators which can be associated with a switching plant. An efficient parametrization setup is achieved by means of an observer–based pre-compensator and an input injection [5] (see also [6]). We adapt such a structure (which can be derived if the provided stabilizability conditions are satisfied) to switching plants. Once the pre-compensator is determined, the free parameter is a proper stable transfer function which must be properly realized, in agreement with the results presented in [22] for the case of a single plant. Henceforth, we will always assume stabilizability conditions (quadratic stabilizability) are satisfied. The main result of this section is simply stated as follows.

Theorem 3.3: Assume that the necessary and sufficient conditions for switching stabilizability of Theorem 3.1 (switching quadratic stabilizability of Theorem 3.2) are satisfied. Then, given any arbitrary family of transfer functions $\mathcal{K}_i(s)$, $i = 1,\ldots,r$ each stabilizing the i-th plant, there exists a suitable realization for each of them such that the closed loop system is switching stable (switching quadratically stable).

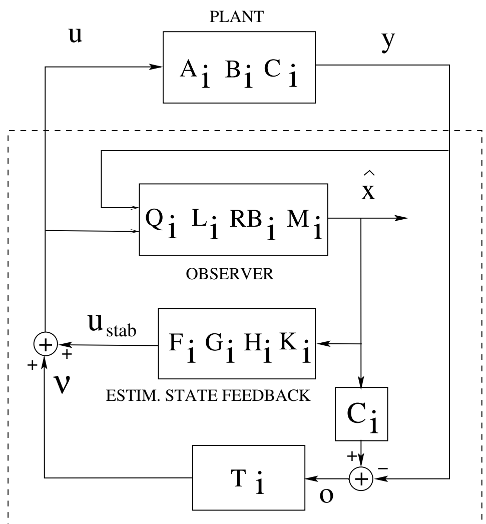

Figure 1. The observer–based compensator structure

The realization of such compensator $\mathcal{K}_i(s)$ is derived from the previously introduced regulators and is illustrated in Fig.1. More precisely, consider the observer–based compensator (6)-(9) or (15) and, instead of assuming $v \equiv 0$, take

$$v(s) = T_i(s)(\hat{y}(s) - y(s))$$

where $\hat{y}(t) = C_i\hat{x}(t)$ is the estimated output, precisely

$$u(s) = u_{stab}(s) + v(s) = u_{stab}(s) + T_i(s)(C_i\hat{x}(s) - y(s)) \quad (16)$$

In other words, u_{stab} is derived by means of the feedback (6)–(9) (or (15)), and $T_i(s)$ is a stable transfer function (the Youla–Kucera parameter [23], [24]). Note that the structure in Fig. 1 is valid for both type of observer–based compensators (indeed, (6)–(7) parametrize all type of observers for fixed i [29], [28]) including (15) as special case with $Q_i = A_i + L_iC_i$, $R = I$ and $M = I$.

The transfer function $T_i(s)$ can be selected in such a way that the resulting compensator transfer function is the desired one $\mathcal{K}_i(s)$. The only problem with $T_i(s)$ is its implementation, which cannot be arbitrary. In this case it is sufficient to exploit the idea of [22] and realize $T_i(s)$ as $T_i(s) = H_i^{(T)}\left(sI - F_i^{(T)}\right)^{-1} G_i^{(T)} + K_i^{(T)}$ in such a way that the family $F_i^{(T)}$ is switching stable. This results in switching stability and transfer function matching for any i. The procedure for the control synthesis is the following

Procedure 3.1: Given $\mathcal{K}_i(s)$ $i = 1,\ldots,r$ each stabilizing (A_i, B_i, C_i) perform the following operations.

1. Check the necessary and sufficient conditions and if the system passes the test synthesize any stabilizing control of the form (6)–(9) (or (15)).

2. Select the free stable parameter $T_i(s)$ in such a way that the ith compensator has transfer function $\mathcal{K}_i(s)$. This is always possible according to Lemma 10.2 in [5].

3. Select a Hurwitz (Schur) realization for each $T_i(s)$. Make all these realizations of the same order, possibly adding dummy non reachable and non–observable asymptotically stable dynamics: $T_i(s) = \hat{H}_i^{(T)}\left(sI - \hat{F}_i^{(T)}\right)^{-1}\hat{G}_i^{(T)} + \hat{K}_i^{(T)}$

4. Find a switching–stable realization $\left(F_i^{(T)}, G_i^{(T)}, H_i^{(T)}, K_i^{(T)}\right)$ for $T_i(s)$ as follows. Take the (arbitrary) previous realizations $T_i(s):\ (\hat{F}_i^{(T)}, \hat{G}_i^{(T)}, \hat{H}_i^{(T)}, \hat{K}_i^{(T)})$ and apply a transformation to each of them in such a way that all the $F_i^{(T)}$ share $\|\cdot\|_2^2$, the square of the Euclidean norm, as a Lyapunov function. This can be done by solving the Lyapunov equations $\hat{F}_i^{(T)^T}\Pi_i + \Pi_i\hat{F}_i^{(T)} = -I$ and denoting by Ω_i the positive square root of Π_i, say such that $\Pi_i = \Omega_i^T\Omega_i$. Apply the transformation [22] (an alternative is to use proper reset maps) $F_i^{(T)} = \Omega_i\hat{F}_i^{(T)}\Omega_i^{-1}$, $G_i^{(T)} = \Omega_i\hat{G}_i^{(T)}$, $H_i^{(T)} = \hat{H}_i^{(T)}\Omega_i^{-1}$, $K_i^{(T)} = \hat{K}_i^{(T)}$ (in the discrete–time case we have to use the equivalent Lyapunov equations).

5. Realize the compensator as in Fig. 1.

The next corollary formalizes the fact that, if we are seeking a single compensator transfer function for all plants, our parametrization works as well.

Corollary 3.3: Assume that the stabilizability conditions are satisfied. Then a single compensator $C(s)$ stabilizes the plant (under switching) if and only if it can be represented as in Fig. 1 with a proper T_i (suitably realized). Moreover, if there exists $C(s)$ such that all the closed loop systems are Hurwitz (Schur) stable, then there exist proper realizations for $C(s)$ such that the overall system is switching stable.

Remark 3.3: Clearly, we have no guarantee that a single realization of a compensator which assures Hurwitz (Schur) stability preserves stability also under switching. This property becomes true under suitable and, in general, different realizations of such a compensator.

IV. Implications of the Results

The proposed scheme can be successfully adopted to decouple the solutions of the following problems: **(a)** achieving optimality of any fixed configuration i and **(b)** assuring stability under switching.

A. Switching systems and optimization

Consider the problem of optimizing a set of transfer functions

$$\begin{array}{rcllll} \delta x(t) & = & A_ix(t) & +B_iu(t) & +B_i^{\omega}\omega(t) & \\ y(t) & = & C_ix(t) & & +D_i^{y,\omega}\omega(t) & \quad (17) \\ \xi(t) & = & E_ix(t) & +D_i^{\xi,u}u(t) & +D_i^{\xi,\omega}\omega(t) & \end{array}$$

according to arbitrary criteria. The following holds true.

Proposition 4.1: If the necessary and sufficient conditions are satisfied, we can derive an optimal control law for each plant of the family and such that switching stability is assured. Once we have computed the observer–based pre–compensator the i-th input output map is of the form

$$\xi(s) = [M_i^{\xi,\omega}(s) + M_i^{\xi,v}(s)T_i(s)M_i^{o,\omega}(s)]\omega(s)$$

where $M_i^{\xi,\omega}(s)$, $M_i^{\xi,v}(s)$ and $M_i^{o,\omega}(s)$ are the ω–to–ξ, v–to–ξ and ω–to–o transfer functions for the i-th configuration. This is an important feature, since it allows for the well–known Wiener-Hopf design [24] for the ith transfer function (which is affine with respect to T_i).

Remark 4.1: The procedure can be extended to achieve "contractive performances" precisely to assure the discrete–time condition $\|x(t)\| \leq \gamma_1 \|x(0)\|\ \lambda^t$ or the continuous–time condition $\|x(t)\| \leq \gamma_2 \|x(0)\|\ e^{\beta t}$. The reader is referred to [30] for details.

B. The zero transfer functions paradox

Assume that we are given a plant composed by a (finite) family of switching systems which are Hurwitz (Schur), but not switching stable. According to the developed theory, if this plant is switching stabilizable, then we can apply a switching compensator such that the system is switching stable, but, at the same time, for any fixed i the compensator transfer function is $\mathcal{K}_i(s) \equiv 0$. A potential application of this property is what we call the switching manager, a device which leaves the plant uncontrolled as long as it remains on a fixed configuration (for instance because optimal compensators are already applied for each i) and activates the control only under switching. As an example, consider the dynamic system with two vertices

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & 1 \\ -2+\gamma & -0.01 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \\ y &= \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{aligned}$$

where $\gamma \in \{-1,\ 1\}$. Denote by A_1 and A_2 the values of the state matrix for $\gamma = -1$ and 1, respectively. Though each A_i is clearly stable, the time-varying system governed by the switch rule $\gamma(t) = sign[x_1(t)x_2(t)]$ (x_i represents the state component) is unstable (see [8] for details). By means of the next observer and state feedback gains

$$L_1 = L_2 = \begin{bmatrix} -10 & 0 \end{bmatrix}^T, \qquad J_1 = J_2 = \begin{bmatrix} 0 & -10 \end{bmatrix}$$

switching stability is assured and hence, by realizing the switching regulator as

$$\begin{aligned} \dot{\hat{x}} &= A_i\hat{x} + L_i(C_i\hat{x} - y) + B_i u \\ u &= J_i\hat{x} + v \\ \dot{z} &= \Omega_i A_i \Omega_i^{-1} z + \Omega_i L_i (C_i\hat{x} - y) \\ v &= -J_i \Omega_i^{-1} z \end{aligned} \tag{18}$$

with

$$\Omega_1 = \begin{bmatrix} 14.142 & 0.0075 \\ 0.0075 & 8.165 \end{bmatrix}, \quad \text{and} \quad \Omega_2 = \begin{bmatrix} 10.001 & 0.0245 \\ 0.025 & 10.00 \end{bmatrix}$$

we obtain a zero-transfer function regulator for each i and switching stability.

V. EXAMPLE

Consider a very simple (academic) system with two vertices (including input and measurements noises)

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} x(t) + B_i u(t) + w_x(t) \\ y(t) &= C_i x(t) + w_y(t) \end{aligned}$$

where the input and output matrices are

$$C_2 = B_1^T = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad C_1 = -B_2^T = \begin{bmatrix} 0 & 1 \end{bmatrix}^T$$

For this model, with specific cost indexes and noise covariances, we computed two optimal LQG regulators $\mathcal{K}_i(s)$

$$\begin{aligned} \dot{q} &= (A + B_i J_i + L_i C_i)\, q - L_i y \\ u &= J_i q \end{aligned}, \quad i = 1,2$$

with

$$\begin{aligned} J_1 &= \begin{bmatrix} -3167.6 & -13655.7 \end{bmatrix} & J_2 &= \begin{bmatrix} 20491.1 & 10003.1 \end{bmatrix} \\ L_1 &= \begin{bmatrix} -27.64 & -13.50 \end{bmatrix}^T & L_2 &= \begin{bmatrix} -13.50 & -27.64 \end{bmatrix}^T \end{aligned}$$

Such regulators are such that, if used alternatively with sampling time $T = 0.1$, result in an unstable switching behaviour (it is sufficient to check that $exp[A_1^{cl}T]exp[A_2^{cl}T]$ is unstable).

Since the above switching system passes the switching stabilizability test (and the quadratic stability test, which can be checked via the package CVX [37]), it was possible to find proper realizations for the determined optimal controllers, $\mathcal{K}_1$ and $\mathcal{K}_2$. Fig. 2 depicts the time evolution of the system state and the switching signal with the switching stable realizations of the optimal controllers.

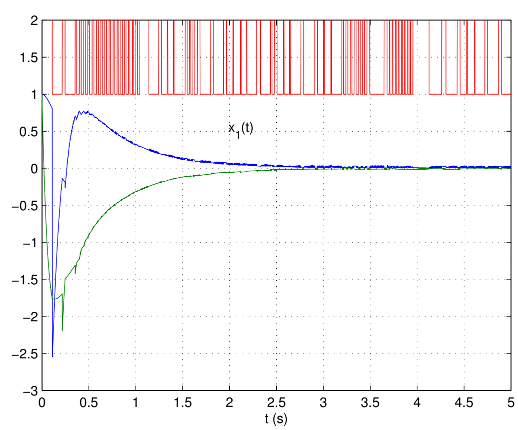

Figure 2. The state and switching signal evolution

VI. DISCUSSION AND CONCLUSIONS

In this paper, necessary and sufficient stabilizability conditions for the existence of a stabilizing and quadratically stabilizing switching linear compensators for a switching plant were reported. If these conditions are satisfied, no matter how we associate a family of compensators with a family of plants, then there exist realizations for which the closed loop system is switching stable. We have shown how to derive these realizations. The results have several implications such as the "zero transfer functions paradox", with its application to the switching manager, the optimal Wiener-Hopf synthesis and contractive design. We conjecture that important connections can be found with recent interesting results proposed in [38] concerning l_∞ performance optimization.

The stabilizability conditions suffer of the well known insurmountable problem (which is, even for autonomous systems, NP-hard [39]) of the complexity of the required algorithms and compensator. By resorting to quadratic stabilizability, efficient LMI algorithms are involved and the compensator has an a–priori fixed complexity. The results are suitable for several extensions and investigations and we believe that there are several interesting issues to be investigated such as the case of delayed information on the current configuration i or the case in which the controller has to identify the current system configuration. For the latter problem the observer–based structure of the proposed compensator seems to be promising [40], [34]. Another intersting issue is the switched case, namely the discrete variable is a control signal (see for instance [41]), for which the presented results could be successfully applied to enforce stability jointly with another scheme which controls switching to optimize performances. Another issue worth investigating is the extension to LPV systems rather than switching systems. We conjecture that as long as B and C are constant and known matrices, results along these lines can be established.

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