Multirate MVC Design and Control Performance Assessment: a Data Driven Subspace Approach*

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Abstract

This paper discusses minimum variance control (MVC) design and control performance assessment based on the MVC-benchmark for multirate systems. In particular, a dual-rate system with a fast control updating rate and a slow output sampling rate is considered. The lifting technique is used as an alternative way to derive a subspace equation, which is used to design the multirate MVC law, and to estimate the multirate MVC-benchmark variance. The multirate optimal controller design is data-driven since it does not involve solving Diophantine or Riccati equations, and it does not even involve identification of a parametric model. The presented MVC-benchmark variance estimation algorithm is model-free because no prior knowledge, namely, transfer function matrices, Markov parameters or interactor matrices, are needed. The proposed methods are illustrated through a simulation example.

1. Introduction

In many applications of electrical, mechanical and chemical engineering, control signals and output measurements need to be sampled at different rates, leading to multirate systems. The research on multirate systems can be traced back to 1957, when the concept of lifting was developed by Kranc from the switch decomposition technique [1]. From then on, the lifting technique has been a most powerful tool used for multirate systems design and analysis. Much of the recent work on multirate systems has been done on the LQG/LQR design [2], the $\mathscr{H}_2$ design [3, 4, 5, 6], the $\mathscr{H}_\infty$ design [3, 7], and model identification and validation [8, 9]. Instead of treating general multirate systems, in this paper we consider a multirate system where the sampling frequency of the controller output is M (a positive integer) times that of the plant output. This setup captures most of the fundamental features of multirate systems while maintains some clarity in the exposition. There exist other cases of dual-rate systems, e.g., those systems where the controller input is fast sampled and the controller output is slowly sampled. However, the former one is commonly used in industry [10].

The first objective of this work is to explore a dual-rate minimum variance control (MVC) law, when only the system input/output data is available. For a known system model, the principle of MVC can be obtained by solving either a Diophantine equation [11] or two Riccati equations [3] since the MVC problem can be reformulated into an $\mathscr{H}_2$ optimal control problem. Also, numerical solutions can also be found by solving a group of linear matrix inequalities [12, 13]. Unlike these foregoing methods which require parametric models, we will explore a method based on certain results from the area of subspace system identification [14, 15]. Subspace identification methods, which were developed in the late 80's and early 90's, allow the identification of a system state space model directly from the data [16, 17, 18]. The idea of designing predictive controllers using subspace system identification techniques has been around for a few years. For instance, there are the model-free LQG design [19] and subspace predictive control design [20]. In addition, the extended state space model is used to obtain predictive controllers [21]. A predictive control law, with all the important predictive control features, is investigated via subspace approach in [22]. All these aforementioned designs are for single-rate systems. In this paper, we will consider the MVC design for dual-rate systems.

The second objective of this work is to assess dual-rate control-loop performance using input/output data. Due to the lack of published work on multi-

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rate control performance assessment, we will only review the important research which has been done on single-rate control performance assessment. For linear systems, it is known that the MVC is the best possible control in the sense that no controllers can provide a lower closed-loop variance [23]. Many papers [24, 25, 26, 27, 28] have shown that MVC is a useful benchmark to assess control-loop performance. A comprehensive overview of research up to 1998 on control performance assessment using minimum variance principles can be found in [29]. To estimate the MVC-benchmark variance by traditional performance assessment algorithms, the time delay (SISO case) [24, 25] or the interactor matrix (MIMO case) [28, 30] or the first few Markov parameters [31] must be known a priori. Recently, a framework based on subspace matrices is studied to estimate the MVC-benchmark variance for multivariate feedback control systems [32]. No prior knowledge is needed anymore in this algorithm. Estimating the dual-rate MVC-benchmark variance is an extension of the work in [32] to dual-rate systems, but we use a different subspace algorithm.

2. System description and problem statement

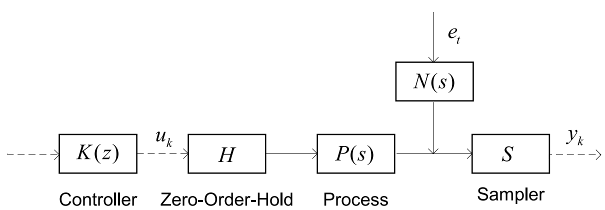

Figure 1. Block diagram of a sampled-data system

Consider a discrete-time, time-invariant, linear state space model of the form

$$x_{k+1} = Ax_k + Bu_k + Ee_k \quad (1)$$
$$y_k = Cx_k + Du_k + Fe_k \quad (2)$$

where k is a discrete-time instance, $x \in R^n$ is the state vector, $y \in R^m$ is the system output, $u \in R^r$ is the system input, and $e_k \in R^m$ is a standard white noise with zero mean and identity covariance matrix. A, B, C, D, E, F are system matrices with appropriate dimensions. We assume that (C, A) is observable, (A, B) is controllable, and u and e are independent signals. The system is shown in Figure 1, where H and S denote a zero-order-hold and an output sampler with interval T, respectively. Thus the discrete-time state space model of the system SPH is (A, B, C, D), and the discretized state space model of the disturbance SN is (A, E, C, F). In certain industrial applications [10], it is common that the control updating rate is faster than the output sampling rate by a certain factor. Thus we assume that the input variable is sampled with a fast sampling rate $T_{\text{fast}} = T$ and the output variable is sampled with a slow sampling rate $T_{\text{slow}} = MT_{\text{fast}}$. The issues investigated in this paper are to design the dual-rate MVC law and to estimate the MVC-benchmark variance directly from the input/output data. The data sets are

$$\begin{cases} u_k, \ \forall k = k_0, k_0+1, \cdots, k_0+N_u-1, \\ y_k, \ k = jM + k_0, \ \forall j = 0, 1, \cdots, N_y - 1, \end{cases}$$

where $k_0 \geq 1$ is the first discrete-time instance when y_k is available. The lengths of input and output data satisfy $N_u = k_0 + (N_y - 1)M$, which means that the last time instance when u_k is available equals to the last time instance when y_k is available.

3. A dual-rate subspace equation with lifting approach

The two objectives of this work are both based on a dual-rate input-output subspace equation. This subspace equation, also named as extended state space model, is derived by Di Ruscio in the deterministic and stochastic subspace system identification and realization (DSR) algorithm [15, 33]. Here, we will derive it for dual-rate systems with the lifting technique, which makes the derivation simple, concise and consistent with our subsequent performance assessment work.

3.1. Definitions

For the sake of presentation, the following definitions are adopted in this paper. For a sequence of data

$$s_t \in R^{n_r \times n_c}, \ \forall t = 0, 1, \cdots, k, k+1, \cdots$$

with n_r being the number of the rows and n_c the number of the columns in s_t, given positive integers k, J, L and K, the extended signal sequence $s_{k|L} \in R^{Ln_r \times n_c}$ is defined as

$$s_{k|L} \overset{\text{def}}{=} \begin{bmatrix} s_k \\ s_{k+1} \\ \vdots \\ s_{k+L-1} \end{bmatrix}, \quad (3)$$

and the Hankel matrix $S_{k|L} \in R^{Ln_r \times Kn_c}$ is defined as

$$S_{k|L} \overset{\text{def}}{=} \begin{bmatrix} s_k & s_{k+1} & \cdots & s_{k+K-1} \\ s_{k+1} & s_{k+2} & \cdots & s_{k+K} \\ \vdots & \vdots & \ddots & \vdots \\ s_{k+L-1} & s_{k+L} & \cdots & s_{k+K+L-2} \end{bmatrix}, \quad (4)$$

where k is the starting index ($k = k_0 + JM$); J is the past horizon to define the instrumental variable matrix; L, the number of n_r-block rows in $s_{k|L}$ and $S_{k|L}$, is defined as the prediction horizon. It is chosen such that $L \geq L_{\min}$, where the minimal number of L is defined by [21]

$$L_{\min} \overset{\text{def}}{=} \begin{cases} n - \text{rank}(C) + 1 \text{ when rank}\,(C) < n, \\ 1 \text{ when rank}\,(C) \geq n. \end{cases}$$

K is the number of n_c-block columns in $S_{k|L}$. In this paper the past and prediction horizons are chosen as $J = L$, and the number of block columns in $S_{k|L}$ is chosen as $K = N_y - L - J$ [33].

3.2. Lifting and input-output subspace equation

The dual-rate controller in Figure 1 is time-varying due to the presence of the fast-rate hold and the slow-rate sampler. To avoid dealing with the time-varying system directly, let us introduce the lifting technique. The slow sampled output signal is $\{ y_{k_0}, \ \cdots \ y_k, \ y_{k+M}, \ \cdots \}$, and the fast sampled input signal is $\{ u_{k_0}, \ \cdots \ u_k, \ u_{k+1}, \ \cdots \}$. By the definition in equation (3), the control signals can be stacked as vectored-valued sequences as $\underline{u} = \{u_{k_0|M}, \cdots, u_{k|M}, u_{k+M|M}, \cdots\}$. After lifting, both the signal dimension and the underlying period are increased by a factor of M. The lifted system G_M from $\underline{u}$ to y is single-rate and linear time-invariant [34]. From the fast-rate plant model (A, B, C, D), where D can be set to a zero matrix without loss of generality, G_M can be written as [3]:

$$x_{k+M} = A^M x_k + \underline{B} u_{k|M}, \quad (5)$$

$$y_k = C x_k + D_M u_{k|M} \quad (6)$$

where $\underline{B} = \begin{bmatrix} A^{M-1}B & \cdots & AB & B \end{bmatrix} \in R^{n \times Mr}$ is the lifted external input matrix, and $D_M = \begin{bmatrix} 0 & \cdots & 0 \end{bmatrix} \in R^{m \times Mr}$ is the lifted direct control input-to-output matrix. Again, by lifting y_k and $u_{k|M}$ with the user chosen prediction horizon L, equation (6) can be written as

$$y_{k|L} = O_L x_k + H_L u_{k|LM}, \quad (7)$$

where O_L, the dual-rate extended observability matrix for the pair $(C,\ A^M)$, is defined as

$$O_L \overset{\text{def}}{=} \begin{bmatrix} C \\ CA^M \\ \vdots \\ CA^{M(L-1)} \end{bmatrix} \in R^{Lm \times n},$$

and the lower block triangular Toeplitz matrix $H_L \in R^{Lm \times LMr}$ is defined as

$$H_L \overset{\text{def}}{=} \begin{bmatrix} D_M & 0 & \cdots & 0 \\ C\underline{B} & D_M & \cdots & 0 \\ CA^M\underline{B} & C\underline{B} & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ CA^{(L-2)M}\underline{B} & CA^{(L-3)M}\underline{B} & \cdots & D_M \end{bmatrix}. \quad (8)$$

Then it can be obtained from (5, 7) that [14]

$$\begin{aligned} y_{k+M|L} &= O_L x_{k+M} + H_L u_{k+M|LM} \\ &= O_L(A^M x_k + \underline{B} u_{k|M}) + H_L u_{k+M|LM} \\ &= \tilde{M} y_{k|L} + \tilde{N} u_{k|(L+1)M} \end{aligned} \quad (9)$$

where $\tilde{M}$ and $\tilde{N}$ are defined as

$$\tilde{M} \overset{\text{def}}{=} O_L A^M (O_L^T O_L)^{-1} O_L^T,$$

$$\tilde{N} \overset{\text{def}}{=} \begin{bmatrix} O_L\underline{B} & H_L \end{bmatrix} - \tilde{M} \begin{bmatrix} H_L & 0_{Lm \times Mr} \end{bmatrix}.$$

The subspace model in (9) is disturbance free. The disturbance is in the slow sampling rate and the corresponding discrete-time state space model is as (A^M, E_s, C, F_s), where the subscript s denotes the slow sampling rate. This model can also be obtained from the fast-rate noise model (A, E, C, F) by lifting and inner-outer factorization [13]. So by taking into account the disturbance, via similar derivation as in (9), it can be seen that

$$y_{k+M|L} = \tilde{M} y_{k|L} + \tilde{N} u_{k|(L+1)M} + \tilde{T} e_{k|L+1} \quad (10)$$

where $e_{k|L+1}$ is defined in (3), $\tilde{T} \overset{\text{def}}{=} \begin{bmatrix} O_L E_s & H_L^s \end{bmatrix} - \tilde{M} \begin{bmatrix} H_L^s & 0_{Lm \times m} \end{bmatrix}$, and $H_L^s \in R^{Lm \times Lm}$ is defined as

$$H_L^s \overset{\text{def}}{=} \begin{bmatrix} F_s & 0 & \cdots & 0 \\ CE_s & F_s & \cdots & 0 \\ CA^M E_s & CE_s & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ CA^{(L-2)M}E_s & CA^{(L-3)M}E_s & \cdots & F_s \end{bmatrix}. \quad (11)$$

From (10), it can be observed that

$$Y_{k+M|L} = \tilde{M} Y_{k|L} + \tilde{N} U_{k|(L+1)M} + \tilde{T} E_{k|L+1} \quad (12)$$

where the data matrices $Y_{k+M|L}$, $Y_{k|L}$, $U_{k|(L+1)M}$ and $E_{k|L+1}$ are defined as in (4). Equations (10) and (12) are equivalent, and they both are input-output equations for dual-rate systems.

3.3. Determination of subspace matrices

Since the subspace matrices $\tilde{M}$, $\tilde{N}$ and $\tilde{T}$ in (10, 12) will play important roles in the later dual-rate MVC controller design and MVC based performance assessment, in this part we will show how to determine them directly from the open-loop input/output data. The estimation steps come from the DSR algorithm [15, 33]. It is important to mention that estimating the subspace matrices is not equivalent to estimating system matrices of the lifted system, since the system parameters A^M, $\underline{B}$, C, D_M, E_s and F_s are never explicitly calculated. This is the reason we refer our methods as data-driven or model free. The sketch to determine the subspace matrices mainly comprises a QR decomposition [15, 33]. After structuring the data matrix in the left-hand side of (13) from the open-loop input/output data set, the QR decomposition of the data matrix can be computed as

$$\frac{1}{\sqrt{K}}\begin{bmatrix} U_{k|(L+1)M} \\ W \\ Y_{k|L} \\ Y_{k+M|L} \end{bmatrix} = RQ \tag{13}$$
$$= \begin{bmatrix} R_{11} & 0 & 0 & 0 \\ R_{21} & R_{22} & 0 & 0 \\ R_{31} & R_{32} & R_{33} & 0 \\ R_{41} & R_{42} & R_{43} & R_{44} \end{bmatrix}\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{bmatrix},$$

where $W = \begin{bmatrix} Y_{k_0|J}^T & U_{k_0|JM}^T \end{bmatrix}^T$ denotes the past information. The QR decomposition can be treated as a data compression step, i.e., the data matrix can be compressed to a usually much smaller lower triangular matrix R which contains the relevant information of the system. By certain derivations [15], the subspace matrices can be obtained as follows:

$$\tilde{M} = R_{42}R_{32}^{\dagger}$$
$$\tilde{N} = (R_{41} - \tilde{M}R_{31})R_{11}^T(R_{11}R_{11}^T)^{-1}$$
$$\tilde{T} = R_{43} - \tilde{M}R_{33}$$

where the superscript † denotes the Moore-Penrose pseudo inverse.

4. Data-driven MVC law design

4.1. Control objective and prediction model

The control objective for the system in Figure 1 can be expressed as

$$J_{\text{mvc}} \overset{\text{def}}{=} \min \; E[y_k^T y_k] = \min \lim_{N\to\infty}\frac{1}{N}\sum_{k=1}^{N} y_k^T y_k \tag{14}$$

It can be seen that the subspace input-output equations (10) and (12) contain inherent relationship between the past and the future control/output signals. In the next part a prediction model is derived to present this relationship explicitly.

Proposition 1: The prediction model for the future outputs is

$$\begin{aligned}\hat{y}_{k+M|L} =& P_y y_{k-(L-1)M|L} + P_{L-1}u_{k-(L-1)M|(L-1)M} \\ &+ F_{L+1}u_{k|(L+1)M},\end{aligned} \tag{15}$$

and the terms are given by

$$\begin{aligned} P_y &= \tilde{M}^L, \\ P_{L-1} &= [\; p_1 \quad p_2 \quad \cdots \quad p_{L-1} \;], \\ F_{L+1} &= [\; f_1 \quad f_2 \quad \cdots \quad f_{L+1} \;], \end{aligned} \tag{16}$$

where

$$p_i = \sum_{j=1}^{i} H_{j+L-i,j} \;\; \forall i = 1,\cdots,L-1, \tag{17}$$

$$f_i = \sum_{j=1}^{L-i+2} H_{j,j+i-1} \;\; \forall i = 2,\cdots,L+1, \;\; f_1 = \sum_{j=1}^{L} H_{j,j}. \tag{18}$$

$H_{i,j}$ denotes the sub-block of the Hankel Matrix H, which is structured by the subspace matrices $\tilde{M}$ and $\tilde{N}$:

$$H = \begin{bmatrix} \tilde{N} \\ \tilde{M}\tilde{N} \\ \vdots \\ \tilde{M}^{L-1}\tilde{N} \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} & \cdots & H_{1,L+1} \\ H_{21} & H_{22} & \cdots & H_{2,L+1} \\ \vdots & \vdots & \ddots & \vdots \\ H_{L1} & H_{L2} & \cdots & H_{L,L+1} \end{bmatrix}. \tag{19}$$

Proof. Omitted due to the limit of space. The prediction model for the single-rate case can be found in [21]. ■

4.2. Computing MVC control law

Proposition 2: Considering the prediction model in (15), the L-step ahead optimal control variables, which minimize the control objective

$$J_{\text{mvc}} = \min \; y_{k+M|L}^T y_{k+M|L}, \tag{20}$$

can be computed as

$$\begin{aligned}u_{k|(L+1)M} =& -(F_{L+1}^T F_{L+1})^{-1}F_{L+1}^T(\tilde{M}y_{k-(L-1)M|L} \\ &+ P_{L-1}u_{k-(L-1)M|(L-1)M}).\end{aligned} \tag{21}$$

The equivalence of (20) and (14) was shown in [19] when the past horizon is infinity.

Proof. By inserting (15) into (20) and setting

$\partial J_{\text{mvc}}/\partial u_{k|(L+1)M} = 0$, the optimal control variables can be obtained. ■

After the optimal control sequence $u_{k|(L+1)M}$ is known, the first row block of $u_{k|(L+1)M}$, $u_{k|M}$, is implemented as the receding horizon MVC law.

5. Estimation of the MVC-benchmark variance directly from input/output data

In [32], it presents the extended output $y_{k|L}$ of the process (in Figure 1) as

$$y_{k|L} = O_L x_t + H_L u_{k|L} + H_L^s e_{k|L},$$

and the output variance under MVC can be written as

$$\begin{aligned} J_{\text{mvc}} &\stackrel{\text{def}}{=} \min\ E[y_k^T y_k] = \text{trace}(\min\ E[y_k y_k^T]) \\ &= \text{trace}\ (I - H_L H_L^\dagger) H_{L,1}^s H_{L,1}^{sT} (I - H_L H_L^\dagger)^T \end{aligned} \tag{22}$$

where H_L is defined as in (8) except for $M = 1$; H_L^s is defined as in (11), and $H_{L,1}^s$ denotes the first block column of H_L^s. Equation (22) shows that the J_{mvc} can be estimated if H_L and $H_{L,1}^s$ are known. In the following part, we will see how to estimate these two matrices for dual-rate systems. From (15, 16) it can be seen that

$$\begin{aligned} \hat{y}_{k|L} =& \tilde{M}^L y_{k-LM|L} + P_{L-1} u_{k-LM|(L-1)M} \\ &+ F_{L+1} u_{k-M|(L+1)M} \\ =& L_w W_p + H_L u_{k|LM}, \end{aligned}$$

where $W_p = \begin{bmatrix} y_{k-LM|L}^T & u_{k-LM|LM}^T \end{bmatrix}^T$ denotes the past information and L_w is the linear operator of W_p. The estimation of H_L, $\hat{H}_L$, can be determined as

$$\hat{H}_L = [\ h_1 \quad h_2 \quad \cdots \quad h_L\ ] \tag{23}$$

where

$$h_i = \sum_{j=1}^{L-i+1} \hat{H}_{j,j+i},\ \forall i = 1, \cdots L, \tag{24}$$

and $\hat{H}$ is the estimation of the Hankel Matrix defined in (19). $\hat{H}_{L,1}^s$ can be obtained as

$$\hat{H}_{L,1}^s = \sum_{j=1}^{L} \Lambda_{j,j+1} \tag{25}$$

where Λ is defined as

$$\Lambda = \begin{bmatrix} \tilde{T} \\ \tilde{M}\tilde{T} \\ \vdots \\ \tilde{M}^{L-1}\tilde{T} \end{bmatrix} = \begin{bmatrix} \Lambda_{11} & \Lambda_{21} & \cdots & \Lambda_{1,L+1} \\ \Lambda_{21} & \Lambda_{22} & \cdots & \Lambda_{2,L+1} \\ \vdots & \vdots & \ddots & \vdots \\ \Lambda_{L1} & \Lambda_{L1} & \cdots & \Lambda_{L,L+1} \end{bmatrix}. \tag{26}$$

Table 1. Results of the simulation example

Output data set length	6000
Theoretical minimum variance	2.4838
Simulated minimum variance	2.5150
Estimated minimum variance	2.4633

The derivation of (23-25) is omitted. Therefore, the estimated MVC-benchmark variance can be computed as

$$\hat{J}_{\text{mvc}} = \text{trace}\ (I - \hat{H}_L \hat{H}_L^\dagger) \hat{H}_{L,1}^s \hat{H}_{L,1}^{sT} (I - \hat{H}_L \hat{H}_L^\dagger)^T. \tag{27}$$

Notice that $H_{L,1}^s$ is preferably estimated from a set of representative close-loop routine operating data [32]. Then the close-loop performance index can be computed as the ratio between the estimated MVC-benchmark variance and the current output variance.

6. A simulation example

In order to illustrate the developed algorithms, a numerical example is shown in this section . The open-loop system model is taken from [15] except that one time delay is added to the process. The process and disturbance models are

$$P = z^{-1}\frac{0.5}{1 - 0.9z^{-1}},\ N = \frac{1 - 0.3z^{-1}}{1 - 0.9z^{-1}}$$

with the sampling period $T_{\text{fast}} = 1\,\text{sec}$. The system output signal is slow sampled with the sampling period $T_{\text{slow}} = 2\,\text{sec}$. The lifted system model ($M = 2$) and the slow-rate disturbance model are

$$\begin{aligned} \underline{P} &= z^{-1} \left[\ \tfrac{0.45}{1-0.81z^{-1}} \quad \tfrac{0.5}{1-0.81z^{-1}} \ \right], \\ N_s &= \frac{1.576 + 0.1713z^{-1}}{1 - 0.81z^{-1}}. \end{aligned}$$

The theoretical minimum variance is calculated as $J_{\text{mvc}} = 2.4838$. From the system input/output data, where the input is a white noise of unit variance, the dual-rate MVC law is computed as $K(z) = \begin{bmatrix} 0.7710 & 0.9145 \end{bmatrix}^T$ when $L = J = 1$. The simulated close-loop variance under $K(z)$ and the estimated MVC-benchmark variance are listed in Table 1. It can be seen that the error between the simulated minimum variance and the theoretical minimum variance is small, and the difference between the estimated MVC-benchmark variance and the theoretical value is negligible. Thus, this numerical example validates both the data-driven dual-rate MVC design algorithm and the control performance assessment method.

7. Conclusion and future work

In this paper, the data-driven minimum variance control (MVC) problem and the MVC-benchmark variance estimation were discussed for a dual-rate system. A subspace input-output equation is derived by the lifting technique to obtain a prediction model. The dual-rate optimal controller design is data-driven since it only requires a set of input/output open-loop experimental data and the process model is not needed anymore. The presented MVC-benchmark variance estimation algorithm requires a set of open-loop experimental data plus a set of close-loop routine operating data. These two proposed methods were illustrated through a numerical example. Based on the work we have done so far, it will be of interest to extend current results to the case where the control objective takes both the output variable and the input variable into account, i.e., to design multirate predictive controllers via data-driven approach.

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