

# Dynamic observers for sensor faults detection and diagnosis

A. M. Pertew\*<sup>†</sup>, H. J. Marquez\*<sup>†</sup> and Q. Zhao\*<sup>†</sup>

\* Emails: pertew, marquez, qingzhao @ece.ualberta.ca

<sup>†</sup> Addresses: Department of Electrical and Computer Engineering, University of Alberta, Edmonton, Canada, T6G 2V4

**Abstract**—Most observer-based methods applied in *fault detection and diagnosis (FDD)* schemes use the classical two-degrees of freedom observer structure in which a *constant matrix* is used to stabilize the observer error dynamics while a *post filter* helps to achieve some desired properties for the residual (fault information) signal. In this paper, we consider the use of a more general framework which is the dynamic observer structure in which an observer gain is seen as a filter designed so that the error dynamics has some desirable frequency domain characteristics. This structure offers extra degrees of freedom and we show how this freedom can be used for the sensor faults diagnosis problem with the objective to make the residual converge to the faults vector achieving detection and estimation at the same time. The use of appropriate weightings to transform this problem into a standard  $H_\infty$  optimal control problem is also demonstrated. The introduced strategies are applied to the identified linear state space model of the *PROCON<sup>TM</sup>* level/flow/temperature process training system (manufactured by the Feedback Instrument Limited).

## 1. INTRODUCTION

Motivated by a growing demand for higher reliability in control systems, the *fault diagnosis problem* is gaining increasing consideration world-wide in both theory and application. This problem is defined as the synthesis of a monitoring system to detect faults and specify their location and significance in a control system [1]. Model-based approaches (represented by the structure shown in Fig. 1) have been a useful tool to solve this problem specially for linear time invariant (LTI) systems. The two-stage structure of Fig. 1 was first suggested by Chow and Willsky in 1980 [2] and is now widely accepted by the fault diagnosis community [1]. It consists of the following: (i) A residual generation module that generates a fault indicating signal (residual) using the available input and output information from the monitored system, (ii) A decision making phase where the residuals are examined and a decision rule is applied to determine if a fault has occurred.

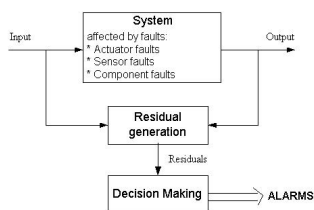


Fig. 1. Structure of model based fault diagnosis system.

Most of the work done in this field is focused on the residual generation problem since the decision making based on well

designed residuals is relatively easy [3], [1].

The observer-based approach, in which an observer plays the role of the residual generation module, is one of the most famous techniques used for residual generation. Many standard observer-based techniques exist in the literature providing different solutions to both the theoretical and practical aspects of the problem (see [3]-[6] for good surveys). The basic idea behind this approach is to estimate the outputs of the system from the measurements by using either *Luenberger* observers in a deterministic framework [7], [8] or *Kalman* filters in a stochastic framework [9], [10]. The weighted output estimation error is then used as the residual in this case.

Different aspects of the fault diagnosis problem have been considered in the literature by using this methodology. Beard used this idea to develop existence conditions for directional residuals (residuals that achieve fault isolation, i.e. determination of the fault location) [7]. Fault isolation has also been considered by using the dedicated observer scheme [11], where a bank of observers is used to differentiate between different fault scenarios. The important problem of robustness to disturbances and modeling uncertainties has also seen much attention. Watanabe and Himmelblau introduced the concept of *Unknown Input Observer* (UIO) for robust sensor fault diagnosis in systems with modeling uncertainty [12]. Their approach was later extended in a series of papers by Wünnenberg and Frank (see [13] and references therein) and also by Patton and Chen (see [14], [1]) to the detection of both sensor and actuator faults. Robustness has also been studied extensively by Patton using the eigen structure assignment approach [15], [8], where the objective is to decouple the residual from unknown disturbances by appropriate design of the observer gain. Optimization techniques have also been widely used in fault detection applications to minimize the disturbance effect and maximize the fault effect when complete decoupling is not possible [16], [1], [4], [5].

In all of these works, the residual generator can be parameterized by the same two-degrees of freedom classical observer structure, in which a *constant observer gain* and a *post filter* help to achieve different specifications of the fault diagnosis problem. In this paper, however, we consider a more general framework, making use of the dynamic observer structure introduced in [17], [18] where an observer gain is seen as a filter designed so that the error dynamics has some desirable frequency domain characteristics. We apply this dynamic structure for the sensor faults

estimation problem where the objective of estimating the faults magnitudes is considered (in addition to detection and isolation). We show that, unlike the classical structure, this objective is achievable by minimizing the faults effect in a narrow frequency band on the observer's state estimation error. Different frequency patterns for the faults are also considered and the use of weightings to model the problem as a standard  $H_\infty$  optimal control problem is illustrated. The introduced techniques are demonstrated through simulations on a model of the *PROCON<sup>TM</sup> level/flow/temperature* process training system. The rest of this paper is organized as follows: section 2 introduces some mathematical background and notations used throughout the paper. In section 3, the problem of diagnosing sensor faults in a narrow frequency band is considered. In section 4, we consider the two cases of low frequency and high frequency ranges, formulating these problems as weighted  $H_\infty$  optimal control problems. Simulation results are presented in section 5 and some conclusions are drawn in section 6.

## 2. PRELIMINARIES AND NOTATION

The linear fault detection and diagnosis (FDD) problem considers the general class of LTI-MIMO systems affected by faults that can be modeled as follows:

$$\dot{x}(t) = Ax(t) + Bu(t) + R_1 f(t) \quad (1)$$

$$y(t) = Cx(t) + Du(t) + R_2 f(t) \quad (2)$$

where  $x(t) \in \mathbb{R}^n$ ,  $u(t) \in \mathbb{R}^m$ ,  $y(t) \in \mathbb{R}^p$  and  $f(t) \in \mathbb{R}^s$ , and where the matrices  $A, B, C, D, R_1$  and  $R_2$  are known matrices of appropriate dimensions. Here  $f(t)$  is the fault vector, and can represent the different types of system faults shown in Fig. 1 (i.e, sensor, actuator and component faults).

As mentioned in section 1, the most famous technique used for residual generation is the observer-based approach that uses the following *Luenberger* observer structure:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - \hat{y}(t)) \quad (3)$$

$$\hat{y}(t) = C\hat{x}(t) + Du(t) \quad (4)$$

in addition to a transfer matrix (weighting)  $Q(s)$  to generate the residual  $r$  as:

$$r = Q(s)(y - \hat{y}); \quad r(t) \in \mathbb{R}^q \quad (5)$$

The residual obtained from (5) is therefore the weighted output estimation error of the observer, and the residual generator (3)-(5) has two degrees of freedom, namely, the constant observer gain  $L$  and the post filter  $Q(s)$ . This freedom can be used to achieve different specifications of the FDD problem. The following definitions are widely accepted by the FDD community and are related to the different tasks of a residual generator [1]:

*Definition 1:* (Fault detection) The residual generator achieves fault detection if the following condition is satisfied:

$$\begin{aligned} r_i(t) &= 0 ; \quad \text{for } i = 1, \dots, q ; \quad \forall t \\ \iff f_i(t) &= 0 ; \quad \text{for } i = 1, \dots, s ; \quad \forall t \end{aligned}$$

*Definition 2:* (Fault isolation) The residual generator achieves fault isolation if the residual has the same dimension as  $f(t)$  (i.e,  $q = s$ ) and if the following condition is satisfied:  $(r_i(t) = 0 ; \quad \forall t \iff f_i(t) = 0 ; \quad \forall t) ; \quad \text{for } i = 1, \dots, s$

*Definition 3:* (Fault identification) The residual generator achieves fault identification if the residual has the same dimension as  $f(t)$  and if the following condition is satisfied:

$$(r_i(t) = f_i(t) ; \quad \forall t) ; \quad \text{for } i = 1, \dots, s$$

According to the previous definitions, in fault detection a binary decision could be made either that a fault occurred or not, while in fault isolation the location of the fault is determined and in fault identification the size of the fault is estimated. The relative importance of the three tasks is subjective and depends on the application, however it is important to note that fault identification implies isolation and that fault isolation implies detection (but not the opposite).

For the sensor faults diagnosis problem (which is our focus in this paper), the system (1)-(2) is the special case where  $R_1 = 0$ ,  $R_2 = I_p$  (the identity matrix of order  $p$ ) and  $f(t) = f_s(t) \in \mathbb{R}^p$ . The effect of using the classical residual generator in (3)-(5) is clear by noting that the observer error dynamics is given from (6)-(7) (where  $e = x - \hat{x}$ ,  $\tilde{y} = y - \hat{y}$ ).

$$\dot{e}(t) = (A - LC)e(t) - Lf_s(t) \quad (6)$$

$$\tilde{y}(t) = Ce(t) + f_s(t) \quad (7)$$

The fault vector  $f_s$  has direct effect on the output estimation error  $\tilde{y}$ , and hence on the residual. Therefore *sensor fault detection* according to definition 1 is achievable by this structure [1]. *Sensor fault isolation* can also be achieved by using the dedicated observer scheme, where a bank of observers (3)-(4) is used to differentiate between different faults. However, for this approach, the number of sensor faults need to be known a priori, and also some restrictive observability conditions need to be satisfied [11]. In this paper we consider the multiple sensor faults identification problem using a novel approach. Our methodology is based on the extension of the *Luenberger* structure in (3)-(4) to a more general dynamic framework. We tackle the case when the sensor faults  $f_s$  are in a narrow frequency band by showing that the sensor fault identification problem is equivalent to an output zeroing problem which is solvable only with a dynamic observer. We further consider the cases of low and high frequency ranges showing that the problem can be modeled as a weighted  $H_\infty$  optimal control problem. The extra design freedom offered by the dynamic formulation is used to solve the proposed problems.

The following definitions and notations will be used throughout the paper:

*Definition 4:* ( $\mathcal{L}_2$  space) The space  $\mathcal{L}_2$  consists of all Lebesgue measurable functions  $u : \mathbb{R}^+ \rightarrow \mathbb{R}^q$ , having a finite  $\mathcal{L}_2$  norm  $\|u\|_{\mathcal{L}_2}$ , where  $\|u\|_{\mathcal{L}_2} \triangleq \sqrt{\int_0^\infty \|u(t)\|^2 dt}$ , with  $\|u(t)\|$  as the *Euclidean* norm of the vector  $u(t)$ .

For a system  $H : \mathcal{L}_2 \rightarrow \mathcal{L}_2$ , we will represent by  $\gamma(H)$  the  $\mathcal{L}_2$  gain of  $H$  defined by  $\gamma(H) = \frac{\|Hu\|_{\mathcal{L}_2}}{\|u\|_{\mathcal{L}_2}}$ . It is well known that, for a linear system  $H : \mathcal{L}_2 \rightarrow \mathcal{L}_2$  (with a transfer matrix

$\hat{H}(s)$ ,  $\gamma(H)$  is equivalent to the H-infinity norm of  $\hat{H}(s)$  defined as follows:

$$\gamma(H) \equiv \|\hat{H}(s)\|_\infty \triangleq \sup_{\omega \in \mathbb{R}} \sigma_{\max}(\hat{H}(j\omega))$$

where  $\sigma_{\max}$  represents the maximum singular value of  $\hat{H}(j\omega)$ . The matrices  $I_n$ ,  $0_n$  and  $0_{nm}$  represent the identity matrix of order  $n$ , the zero square matrix of order  $n$  and the zero  $n$  by  $m$  matrix respectively.  $Diag_r(a)$  represents the diagonal square matrix of order  $r$  with  $[a \ a \ \cdots \ a]_{1 \times r}$  as its diagonal vector, while  $diag(a_1, a_2, \dots, a_r)$  represents the diagonal square matrix of order  $r$  with  $[a_1 \ a_2 \ \cdots \ a_r]$  as its diagonal vector. The symbol  $\hat{T}_{yu}$  represents the transfer matrix from input  $u$  to output  $y$ . The partitioned matrix  $G = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$  (when used as an operator from  $u$  to  $y$ , i.e.,  $y = Gu$ ) represents the state space representation ( $\dot{\xi} = A\xi + Bu$ ,  $y = C\xi + Du$ ), and in that case the transfer matrix is  $\hat{G}(s) = C(sI - A)^{-1}B + D$ . We will also make use of the following property on the rank of  $\hat{G}(s)$  [19]:

$$\text{rank} \begin{bmatrix} A - sI & B \\ C & D \end{bmatrix} = n + \text{rank}(\hat{G}(s)) \quad (8)$$

if  $s$  is not an eigenvalue of  $A$  and where  $n$  is the dimension of the matrix  $A$ .

### 3. NARROW FREQUENCY BAND SENSOR FAULTS DIAGNOSIS

In almost all observer-based FDD designs, maximizing the faults effect on the observer's estimation error is considered as an optimal objective. However, for the sensor faults case (as shown in (6)-(7)) the opposite is true. By minimizing  $e$ , the output estimation error  $\hat{y}$  converges to  $f_s$  which guarantees fault identification in this case. In this section, we consider the solution of this design problem (when  $f_s$  is in a narrow frequency band around a nominal frequency  $\omega_o$ ) by using a dynamic observer structure, showing that the problem is not tractable for the static gain structure in (3)-(4).

#### 3.1. Dynamic generalization of the classical observer structure

Throughout this paper, following the approach in [17], [18], we will make use of dynamical observers of the form:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + \eta(t) \quad (9)$$

$$\hat{y}(t) = C\hat{x}(t) + Du(t) \quad (10)$$

where  $\eta(t)$  is obtained by applying a dynamical compensator on the output estimation error ( $y - \hat{y}$ ). In other words  $\eta(t)$  is given from

$$\dot{\xi} = A_L\xi + B_L(y - \hat{y}) \quad (11)$$

$$\eta = C_L\xi + D_L(y - \hat{y}). \quad (12)$$

We will also write

$$K = \begin{bmatrix} A_L & B_L \\ C_L & D_L \end{bmatrix} \quad (13)$$

to represent the compensator in (11)-(12). It is straightforward to see that this observer structure reduces to the usual observer in (3)-(4) in the special case where the gain  $K$  is the constant gain given by  $K = \begin{bmatrix} 0_n & 0_{np} \\ 0_n & L \end{bmatrix}$ . The additional dynamics provided by this observer brings additional degrees of freedom in the design, something that will be exploited in the minimization of the sensor faults effect.

First, note that the observer error dynamics in (6) is now given by ( $\dot{e} = Ae - \eta$ ) which can also be represented by the following so-called standard form:

$$\dot{z} = [A]z + \begin{bmatrix} 0_{np} & -I_n \end{bmatrix} \begin{bmatrix} \omega \\ \nu \end{bmatrix} \quad (14)$$

$$\begin{bmatrix} \zeta \\ \varphi \end{bmatrix} = \begin{bmatrix} I_n \\ C \end{bmatrix} z + \begin{bmatrix} 0_{np} & 0_n \\ I_p & 0_{pn} \end{bmatrix} \begin{bmatrix} \omega \\ \nu \end{bmatrix} \quad (15)$$

where

$$\begin{aligned} \omega &= f_s \\ \nu &= \eta = K(y - \hat{y}) \\ \zeta &= e = x - \hat{x} \\ \varphi &= y - \hat{y} \end{aligned} \quad (16)$$

which can also be represented by Fig. 2 where the plant  $G$  has the state space representation in (17) with the matrices in (14)-(15) and where the controller  $K$  is given in (13).

$$\hat{G}(s) = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \quad (17)$$

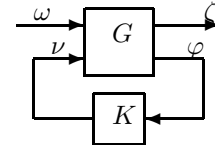


Fig. 2. Standard setup.

Therefore, all possible observer gains for the observer in (9)-(13) can be parameterized by the set of all stabilizing controllers for the setup in Fig. 2. This is a standard result in control theory [19] and, for the observer problem considered in this paper, it can be represented by the following theorem (as a special case of Theorem 11.4 in [19]):

*Theorem 1:* Let  $F$  and  $L$  be such that  $A + LC$  and  $A - F$  are stable; then all possible observer gains  $K$  for the observer (9)-(13) can be parameterized as the transfer matrix from  $\varphi$  to  $\nu$  in Fig. 3 with any  $\hat{Q}(s) \in RH_\infty$ .

$$\hat{J}(s) = \begin{bmatrix} A - F + LC & -L & -I_n \\ F & 0_{np} & I_n \\ -C & I_p & 0_{pn} \end{bmatrix}$$

Fig. 3. Parametrization of all observer gains.

### 3.2. State and sensor faults estimation

As mentioned earlier, our objective is to minimize (in some sense) the effect of sensor faults (in a narrow frequency band around a nominal frequency  $\omega_o$ ) on the state estimation error in order to achieve sensor faults estimation. Towards that goal, we will denote  $\mathcal{G}$  as the set of all scalar continuous functions  $g(\omega)$  which are symmetric around  $\omega_o$ , and  $F_s(j\omega)$  as the fourier transform of  $f_s(t)$ . We will then define an optimal observer gain in  $\mathcal{L}_2$  sense as follows:

**Definition 5:** (Optimal observer gain) An observer gain is said to be optimal with respect to the nominal frequency  $\omega_o$  if the following property is satisfied for the estimation error  $e(t)$  resulting from the sensor faults vector  $f_s(t)$ :  
 $\forall \epsilon > 0$  and  $\forall g(\omega) \in \mathcal{G}$ ,  $\exists \Delta\omega > 0$  such that  $F_s(j\omega)$  in (18)  $\implies \|e\|_{\mathcal{L}_2} \leq \epsilon$ .

$$|F_s(j\omega)| = \begin{cases} g(\omega); & |\omega - \omega_o| < \Delta\omega \\ 0; & \text{otherwise} \end{cases} \quad (18)$$

Equation (18) means that the frequency pattern for  $f_s(t)$  is confined to the region  $[\omega_o - \Delta\omega, \omega_o + \Delta\omega]$ . It is easy to see that if  $\hat{T}_{ef_s}(s)$  is the transfer matrix from  $f_s$  to  $e$ , then an optimal observer gain is one that satisfies  $\hat{T}_{ef_s}(j\omega_o) = 0$ . The following lemma shows that a static observer gain can never be an optimal observer gain.

**Lemma 1:** A static observer gain (such as the constant matrix  $L$  in (3)-(4)) can never be an optimal observer gain according to Definition 5.

*Proof:* The proof follows by noting that the transfer matrix from  $f_s$  to  $e$  (as seen in (6)) is  $\hat{T}_{ef_s}(s) = \left[ \begin{array}{c|c} A-LC & -L \\ \hline I_n & 0_{np} \end{array} \right]$ . And since the gain  $L$  is chosen to stabilize  $(A-LC)$ , then  $(\forall \omega_o) j\omega_o$  is not an eigenvalue of  $(A-LC)$ . Therefore, by using (8), we have  $\text{rank}(\hat{T}_{ef_s}(j\omega_o)) = \text{rank} \left[ \begin{array}{c|c} A-LC-j\omega_o I_n & -L \\ \hline I_n & 0_{np} \end{array} \right] - n$ .  
 But  $\text{rank} \left[ \begin{array}{c|c} A-LC-j\omega_o I_n & -L \\ \hline I_n & 0_{np} \end{array} \right] = \text{rank} \left[ \begin{array}{cc} L & 0_n \\ 0_{np} & I_n \end{array} \right] = n + \text{rank}(L)$ . Therefore,  $\text{rank}(\hat{T}_{ef_s}(j\omega_o)) \neq 0$  unless  $L = 0$ .

This implies that no gain  $L$  can satisfy  $\hat{T}_{ef_s}(j\omega_o) = 0$ , and therefore a static observer gain can never be an optimal gain according to Definition 5.  $\triangle$

We now consider the case of the dynamic observer introduced in (9)-(13). As a result of the gain parametrization presented in theorem 1, the transfer matrix from  $f_s$  to  $e$ , achievable by an internally stabilizing gain  $K$ , is equal to the Linear Fractional Transformation (LFT) between  $T$  and  $Q$  as follows [19]:

$$\hat{T}_{ef_s}(s) \equiv LFT(T, Q) = \hat{T}_{11}(s) + \hat{T}_{12}(s)\hat{Q}(s)\hat{T}_{22}(s) \quad (19)$$

where  $\hat{Q}(s) \in RH_\infty$  and where  $T$  is given from

$$\left[ \begin{array}{cc} T_{11} & T_{12} \\ T_{21} & T_{22} \end{array} \right] = \left[ \begin{array}{cc|cc} A-F & F & 0_{np} & -I_n \\ 0_n & A+LC & L & 0_n \\ \hline I_n & 0_n & 0_{np} & 0_n \\ 0_{pn} & C & I_p & 0_{pn} \end{array} \right] \quad (20)$$

We will denote  $\hat{T}_{11}(s)$ ,  $\hat{T}_{12}(s)$  and  $\hat{T}_{21}(s)$  by  $\hat{T}_1(s)$ ,  $\hat{T}_2(s)$  and  $\hat{T}_3(s)$  respectively. The following lemma presents a result on the invertibility of the transfer matrices  $\hat{T}_2(s)$  and  $\hat{T}_3(s)$  at a frequency  $\omega_o$  (i.e., at  $s = j\omega_o$ ).

**Lemma 2:** The  $(n \times n)$  and  $(p \times p)$  matrices  $\hat{T}_2(j\omega_o)$  and  $\hat{T}_3(j\omega_o)$  are invertible if  $j\omega_o$  is not an eigenvalue of  $A$ .

*Proof:* By (20),  $\hat{T}_2(s) = \left[ \begin{array}{c|c} A-F & F \\ \hline 0_n & A+LC \\ I_n & 0_n \end{array} \middle| \begin{array}{c} -I_n \\ 0_n \end{array} \right]$   
 $= \left[ \begin{array}{c|c} A-F & -I_n \\ \hline I_n & 0_n \end{array} \right]$ . Similarly,  $\hat{T}_3(s) = \left[ \begin{array}{c|c} A+LC & L \\ \hline C & I_p \end{array} \right]$ .  
 Therefore, using the rank property in (8):

$$\begin{aligned} \text{i) } \text{rank}(\hat{T}_2(j\omega_o)) &= \text{rank} \left[ \begin{array}{c|c} A-F-j\omega_o I_n & -I_n \\ \hline I_n & 0_n \end{array} \right] - n \\ \text{ii) } \text{rank}(\hat{T}_3(j\omega_o)) &= \text{rank} \left[ \begin{array}{c|c} A+LC-j\omega_o I_n & L \\ \hline C & I_p \end{array} \right] - n \end{aligned}$$

But  $\text{rank} \left[ \begin{array}{c|c} A-F-j\omega_o I_n & -I_n \\ \hline I_n & 0_n \end{array} \right] = 2n$ ,  $\forall \omega_o$ . Also,  
 $\text{rank} \left[ \begin{array}{c|c} A+LC-j\omega_o I_n & L \\ \hline C & I_p \end{array} \right] = \text{rank} \left[ \begin{array}{cc} A-j\omega_o I_n & L \\ 0_{pn} & I_p \end{array} \right]$   
 $= n + p$ ; if  $j\omega_o$  is not an eigenvalue of  $A$ . Therefore,  $\text{rank}(\hat{T}_2(j\omega_o)) = n$  and  $\text{rank}(\hat{T}_3(j\omega_o)) = p$  (full ranks) if  $j\omega_o$  is not eigenvalue of  $A$ , and the proof is completed.  $\triangle$

Based on the results in lemma 2, it can be proven that, for  $\hat{T}_{ef_s}(s)$  in (19),  $\exists$  a transfer matrix  $\hat{Q}(s) \in RH_\infty$  that satisfies  $\hat{T}_{ef_s}(j\omega_o) = 0$  (proof is omitted). Therefore, for the dynamic observer in (9)-(13), an optimal gain (in the sense of Definition 5) can be found (unlike the static case). This shows the advantage of using the dynamic observer.

To summarize, based on the previous results, we will define an optimal residual generator as follows:

**Definition 6:** (Optim. residual for narrow frequency band) An observer of the form (9)-(13) along with  $r = y - \hat{y}$  is an optimal residual generator for the sensor faults identification problem (with faults in a narrow frequency band around  $\omega_o$ ) if the dynamic gain  $K$  is chosen as the Linear Fractional Transformation  $LFT(J, Q)$  in Fig. 3 where  $\hat{Q}(s) \in RH_\infty$  solves the problem  $\hat{T}_{ef_s}(j\omega_o) = 0$  for the transfer matrix  $\hat{T}_{ef_s}(s)$  in (19).

#### Remarks

- According to the previous definition, an optimal residual generator guarantees sensor faults estimation and at the same time state estimation (with minimum energy for the estimation errors). An advantage of having *state* estimation in presence of sensor faults is the possibility to use the observer in fault tolerant output feedback control (i.e., if a reconfiguration control action is involved).
- From the special cases of interest is the case of sensor bias, where the previous approach can be used to get an *exact* estimation of all sensor biases at the same time. A sufficient condition is that the matrix  $A$  has no eigenvalues at the origin.

#### 4. $H_\infty$ SENSOR FAULTS DIAGNOSIS

In this section, we consider two different cases: the low frequency range and the high frequency range. For the first

case, we assume the system to be affected by sensor faults of low frequencies determined by a cutoff frequency  $\omega_l$ , i.e. the frequency pattern for  $f_s(t)$  is confined to the region  $[0, \omega_l]$ . On the other hand, in the high frequency case, we assume these faults to have very high frequencies above a minimum frequency  $\omega_h$ , i.e. the frequencies are confined to the region  $[\omega_h, \infty)$ . As mentioned in section 3, by using the dynamic observer in (9)-(13), the error dynamics (due to general sensor faults) can be represented by Fig. 2 where the plant  $G$  has the state space representation shown in (17) with the matrices defined in (14)-(15) and where the controller  $K$  is given in (13). Therefore, the two previous problems can be solved by adding weightings to the standard setup in Fig. 2 that emphasize the frequency range under consideration, and by solving these problems as weighted  $H_\infty$  control problems.

However, before introducing weightings, it is important to note that the standard form in (14)-(15) does not satisfy all of the regularity assumptions in the  $H_\infty$  framework (Notice that  $D_{12}^T D_{12}$  is singular), and hence observer synthesis can not be carried out directly using the standard  $H_\infty$  solution. Fortunately, regularization can be done by extending the external output  $\zeta$  in Fig. 2 to include the “scaled” vector  $\beta\nu$ ; with  $\beta > 0$ . It can be seen that the standard form in (14)-(15) has now the following form:

$$\dot{z} = [A] z + [0_{np} \quad -I_n] \begin{bmatrix} \omega \\ \nu \end{bmatrix} \quad (21)$$

$$\begin{bmatrix} e \\ \beta\nu \\ \varphi \end{bmatrix} = \begin{bmatrix} I_n \\ 0_n \\ C \end{bmatrix} z + \begin{bmatrix} 0_{np} & 0_n \\ 0_{np} & \beta I_n \\ I_p & 0_{pn} \end{bmatrix} \begin{bmatrix} \omega \\ \nu \end{bmatrix} \quad (22)$$

which can also be represented by the standard setup shown in Fig. 2 with the same variables in (16), except for redefining the matrices of  $\hat{G}(s)$  in (17) and defining  $\zeta$  as:  $\zeta \triangleq [e \quad \beta\nu]^T$ . All the regularity assumptions summarized below, [19], are now satisfied iff  $A$  has no eigenvalues on the imaginary axis:

- 1)  $(A, B_2)$  stabilizable: satisfied for any matrix  $A$ .  
 $(C_2, A)$  detectable: satisfied, since  $(A, C)$  is detectable.
- 2)  $D_{21} D_{21}^T = I_p$ , which is nonsingular.  
 $D_{12}^T D_{12} = \beta^2 I_n$ , which is nonsingular.
- 3)  $\text{rank} \begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix} = 2n = \text{full column rank } \forall \omega$ .  
 $\text{rank} \begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix} = n + p = \text{full row rank}$ ; if  $j\omega$  is not an eigenvalue of  $A$ .
- 4)  $D_{22} = 0$ .

The following lemma demonstrates a certain equivalence relationships between the standard form in (14)-(15) and the regularized one in (21)-(22) (proof is omitted)

**Lemma 3:** Let  $R_1$  be the setup in Fig. 2 associated with (14)-(15),  $R_2$  be the one associated with (21)-(22) and consider a stabilizing controller  $K$  for both setups. Then  $\|\hat{R}_1\|_\infty < \gamma$  if and only if  $\exists \beta > 0$  such that  $\|\hat{R}_2\|_\infty < \gamma$ .

#### 4.1. The low frequency range case

We now consider sensor faults of low frequencies determined by a cutoff frequency  $\omega_l$ . The SISO weighting

$\hat{w}_l(s) = \frac{as+b}{s}$ , [19], emphasizes this low frequency range with “ $b$ ” selected as  $\omega_l$  and “ $a$ ” as an arbitrary small number. Therefore, with a diagonal transfer matrix  $\hat{W}(s)$  that consists of these SISO weightings, the observer problem in Fig. 2 can be modified to the weighted version in Fig. 4.

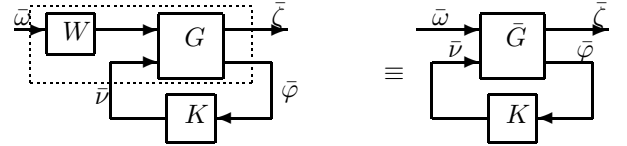


Fig. 4. Weighted standard setup.

It can be seen that the augmented plant  $\bar{G}$  (consisting of the weighting  $W$  cascaded with  $G$  in (21)-(22)) is given by:

$$\hat{G}(s) = \begin{bmatrix} \bar{A} & \bar{B}_1 & \bar{B}_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & \bar{D}_{21} & \bar{D}_{22} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} A_w & 0_{pn} \\ 0_{np} & A \end{bmatrix} & \begin{bmatrix} I_p \\ 0_{np} \end{bmatrix} & \begin{bmatrix} 0_{pn} \\ -I_n \end{bmatrix} \\ \begin{bmatrix} 0_{np} & I_n \\ 0_{np} & 0_n \\ C_w & C \end{bmatrix} & \begin{bmatrix} 0_{np} \\ 0_{np} \\ D_w \end{bmatrix} & \begin{bmatrix} 0_n \\ \beta I_n \\ 0_{pn} \end{bmatrix} \end{bmatrix} \quad (23)$$

where  $A_w = 0_p$ ,  $C_w = \text{diag}_p(b)$  and  $D_w = \text{diag}_p(a)$ .

However, this standard form violates assumptions 1 and 3 of the regularity assumptions summarized earlier; since  $(\bar{A}, \bar{B}_2)$  is not stabilizable and the rank conditions are not satisfied at  $\omega = 0$ . Therefore, we introduce the modified weighting  $\hat{w}_{lmod}(s) = \frac{as+b}{s+c}$ ; with arbitrary small positive “ $c$ ”. It is easy to see that, with this modification, the augmented plant  $\bar{G}$  is the same as (23) except for  $A_w$  which is now given by the stable matrix  $\text{diag}_p(-c)$  and  $C_w$  given by  $\text{diag}_p(b - ac)$ . Similar to the non weighted case, all the regularity assumptions are satisfied iff  $A$  has no eigenvalues on the imaginary axis. To this end, we define the regular  $H_\infty$  problem associated with the low frequency range as follows:

**Definition 7:** (Low frequency  $H_\infty$ ) Given  $\beta > 0$ , find  $\mathcal{S}$ , the set of admissible controllers  $K$  satisfying  $\|\hat{T}_{\zeta\bar{w}}\|_\infty < \gamma$  for the setup in Fig. 4 where  $\bar{G}$  has the state space representation (23) with  $A_w = \text{diag}_p(-c)$ ,  $C_w = \text{diag}_p(b - ac)$  and  $D_w = \text{diag}_p(a)$ .

Based on the previous results, we now present the main result of this section in the form of the following definition for an optimal residual generator in  $\mathcal{L}_2$  sense:

**Definition 8:** (Optimal residual for low frequencies) An observer of the form (9)-(13) along with  $r = y - \hat{y}$  is an optimal residual generator for the sensor faults identification problem (with faults of low frequencies below the cutoff frequency  $\omega_l$ ) if the dynamic gain  $K \in \mathcal{S}^*$  (the set of controllers solving the  $H_\infty$  optimal control problem in Definition 7 with the minimum possible  $\gamma$ ).

#### Comments

- A residual generator that is optimal in the sense of Definition 8 can be found by using an iterative binary search algorithm over the constant  $\beta$  (in order to achieve the minimum possible  $\gamma$  for the problem in Definition 7

which has  $\beta$  as one of its parameters). Existing software packages can be used to solve the regular  $H_\infty$  problem in Definition 7 for a given  $\beta$ .

- The constants  $a$  and  $c$  should be selected as arbitrary small positive numbers, while  $b$  must approximately be equal to  $\omega_l$  (the cutoff frequency). Different weightings could also be used for the different sensor channels. In this case  $A_w = \text{diag}(-c_1, \dots, -c_p)$ ,  $C_w = \text{diag}(b_1 - a_1 c_1, \dots, b_p - a_p c_p)$  and  $D_w = \text{diag}(a_1, \dots, a_p)$ .

#### 4.2. The high frequency range case

The SISO weighting  $\hat{w}_{hmod}(s) = \frac{s+(a \times b)}{\epsilon s + b}$ , [19], could be selected to emphasize the high frequency range  $[w_h, \infty)$  with “ $b$ ” selected as  $w_h$  and, “ $a$ ” and “ $\epsilon$ ”  $> 0$  as arbitrary small numbers. Similar to the low frequency range, a regular  $H_\infty$  problem related to this case can be defined. Also, an optimal residual generator can be defined in a similar way to Definition 8. (details are omitted due to similarity).

### 5. SIMULATION RESULTS

The PROCON Level/Flow/Temperature Process Control System (shown in Fig. 5) includes two rigs which can be either controlled independently or by connecting them together to achieve simultaneous level and temperature control.

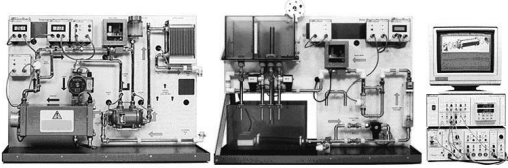


Fig. 5. The Level/Flow/Temperature Process Control System.

In these simulation experiments, we consider the configuration obtained by connecting the two modules in cascade as shown in Fig. 6. In this case, there are two water circuits, namely, the hot water circuit and the cold water circuit. The water of both circuits flows into a heat exchanger where the heat energy can be transferred from the hot water flow into the cold water flow. The hot water temperature is controlled manually by the on-off switch of the heater, while the flow rates of both circuits can be controlled through the two servo valves connected to the computer. A level sensor is used to measure the level of the cold water in the main upper tank, while the temperature (at exactly one position) can be measured through the transmitter. It is important to note that there are 5 available positions for temperature measurement:  $T_1$  ( $T_2$ ) for the hot water input flow to (output flow from) the heat exchanger,  $T_3$  ( $T_4$ ) for the cold water input (output) flow, and  $T_5$  for the cold water output flow from the cooling radiator. In this experiment, our objective is to control the water level and the temperature of the hot water circuit by controlling the flow rates of the valves. According to this configuration, the process has two inputs (the cold water and hot water servo valves) and two outputs (the level of the water in the upper tank and the temperature  $T_2$ ). The

inputs will be denoted  $u_1$  and  $u_2$  respectively and they both have the same operating range of 0 to 4 litres/min. The operating ranges for the outputs  $y_1$  and  $y_2$  are (0, 14 cm) and (0, 100 Celsius) respectively. The heater set point (i.e,  $T_1$ ) is chosen as 80 Celsius, while the cold water in the reservoir is at the room’s temperature (i.e,  $T_3 \approx 23$  Celsius).

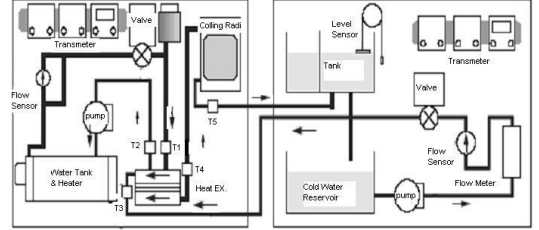


Fig. 6. Structure of the connected rigs.

Using the first principle physical laws, a model of the process can be developed. However, this model is highly nonlinear and many of its parameters are unknown. Therefore, identification experiments are conducted, and based on the operating point ( $u_1 = 2.8$  litres/min,  $u_2 = 0.8$  litres/min,  $y_1 = 6.35$  cm and  $y_2 = 35$  Celsius) a 5<sup>th</sup> order state space model of the form ( $\dot{x} = Ax + Bu$ ;  $y = Cx + Du$ ) is identified, where  $u = [u_1 \ u_2]^T$  and  $y = [y_1 \ y_2]^T$  (see Appendix I for the system matrices). This model is used to demonstrate the proposed observer-based sensor faults identification schemes. Towards that goal, the system is controlled to stabilize the output at  $y_1 = 8$  cm and  $y_2 = 40$  Celsius as seen in Fig. 7 .

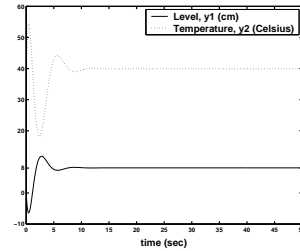


Fig. 7. Actual system outputs for the controlled process.

**Case study 1:** In this case, the system is assumed to be affected by sensor biases, where both measured outputs are affected by piecewise constant faults. This is the special case where  $\omega_o = 0$  for the problem in section 3.2, and since the matrix  $A$  in Appendix I has no eigenvalues on the origin, an optimal observer gain that can estimate both sensor biases can be designed. This optimal gain  $K$ , in our case, is the LFT in Fig. 3 with  $\hat{Q}(s)$  as follows:

$$\hat{Q}(s) = \hat{Q}(0) = \begin{bmatrix} 104.9570 & -116.5842 \\ -75.2350 & -356.1628 \\ -6.8637 & 783.5496 \\ -74.8358 & -694.0620 \\ 36.3072 & 112.4389 \end{bmatrix} \quad (24)$$

Using this observer gain with the observer IC as  $[0 \ 0 \ 0.1 \ 0 \ 0.005]$ , two biases simultaneously changing with time are successfully estimated as seen in Fig. 8.

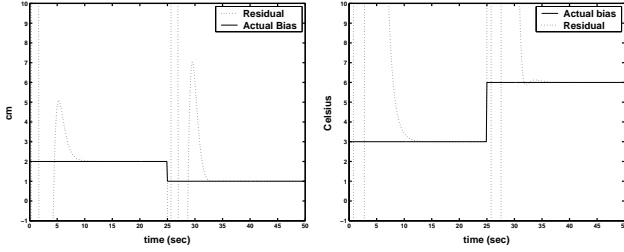


Fig. 8. (a) Bias estimation for  $y_1$  (b) Bias estimation for  $y_2$ .

The state and output estimation errors also converge to zero in this case.

**Case study 2:** In this case, we consider the case of low frequency sensor faults (in the range  $[0, 5 \text{ rad/sec}]$ ). Using the  $H_\infty$  design introduced in section 4.1 (and with the weighting selections as  $a = 0.01$ ,  $b = 5$  and  $c = 0.001$ ), the optimal observer gain is obtained by solving the  $H_\infty$  problem in Definition 7 using the command *hinfsyn* in MATLAB, with minimum  $\gamma$  as 0.1 and with  $\beta = 1$ . Using this observer for the faulty outputs in Fig. 9, a correct estimation of the low frequency sensor faults is shown in Fig. 10. The maximum error in that case was 0.0839 for the first fault estimation, and 0.2787 for the second fault estimation.

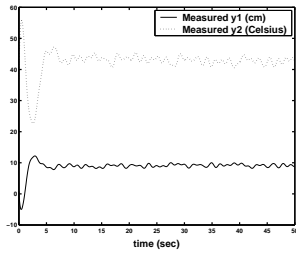


Fig. 9. Outputs  $y_1$  and  $y_2$  affected by low frequency sensor faults.

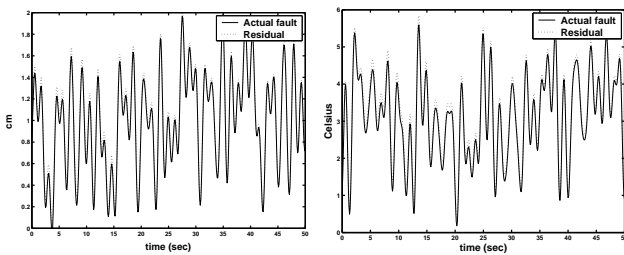


Fig. 10. (a) Fault estimation for  $y_1$  (b) Fault estimation for  $y_2$ .

## 6. CONCLUSION

In this paper, we considered the use of a dynamic observer structure for the sensor faults diagnosis problem. This structure offers extra degrees of freedom over the classical Luenberger structure and we showed how this freedom can be used for the sensor faults and state estimations problems. For the narrow frequency band case, the problem was shown to be equivalent to an output zeroing problem for which a dynamic gain is necessary. The use of appropriate weightings to transform this problem into a standard  $H_\infty$  optimal control problem was also demonstrated. The introduced strategies were applied through simulations to the *PROCON<sup>TM</sup>* level/flow/temperature process training system.

## APPENDIX I SYSTEM MATRICES

$$A = \begin{bmatrix} -0.0084 & -0.0012 & 0.0155 & 0.0280 & 0.0017 \\ -0.0046 & -0.0352 & -0.0227 & 0.0150 & 0.0082 \\ -0.0825 & -0.0122 & -0.0773 & 0.0661 & 0.3209 \\ -0.2105 & 0.0336 & -0.0929 & -0.3418 & -0.1551 \\ 0.0388 & -0.0754 & -0.1532 & 0.0126 & -0.1602 \end{bmatrix}$$

$$C = \begin{bmatrix} -6.7795 & -0.7974 & 0.0766 & 0.1585 & 0.0444 \\ -0.1862 & 19.2450 & -0.4087 & -0.0602 & -0.3102 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.0028 & -0.0022 \\ -0.0016 & 0.0118 \\ -0.1157 & 0.2819 \\ -0.2818 & -0.1153 \\ 0.0552 & -0.2418 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0.0090 \\ 0 & -0.0388 \end{bmatrix}$$

## REFERENCES

- [1] J. Chen, R. J. Patton, Robust model-based fault diagnosis for dynamic systems, Kluwer Academic Publishers, 1999.
- [2] E. Y. Chow, A. Willsky, "Issues in the development of a general algorithm for reliable failure detection," *Proc. of the 19th conf. on decision and control*, Albuquerque, NM, 1980.
- [3] A. S. Willsky, "A survey of design methods for failure detection in dynamic systems," *Automatica*, vol. 12, 601-611, 1976.
- [4] P. M. Frank, S. X. Ding and T. Marcu, "Model-based fault diagnosis in technical processes". Transactions of the Institute of Measurement and Control 22,1 (2000), 57-101.
- [5] P. M. Frank, "Fault diagnosis in dynamic systems using analytical and knowledge-based redundancy - A survey and some new results," *Automatica*, vol. 26, No. 3, 459-474, 1990.
- [6] P. M. Frank, "Analytical and qualitative model-based fault diagnosis - A survey and some new results," *European journal of control*, vol. 2, 6-28, 1996.
- [7] R. V. Beard, "Failure accomdation in linear system through self reorganization," *PhD thesis*, Massachusetts Institute of Technology, USA, 1971.
- [8] R. J. Patton, J. Chen, "Observer-based fault detetction and isolation: robustness and applications," *Control Engineering Practice*, vol. 5, No. 5, 671-682, 1997.
- [9] R. K. Mehra and J. Peschon, "An innvations approach to fault detection and diagnosis in dynamic systems," *Automatica*, vol. 7, 637-640, 1971.
- [10] A. Zolghardi, "An algorithm for real-time failure-detection in Kalman filters", *IEEE transations on automatic control*, vol. 41, No. 10, 1537-1539, 1996.
- [11] D. van Schrick, "A comparison of IFD schemes: a decision aid for designers", *Proceedings of the third IEEE conference on control applications*, Glasgow, Scotland, 889-894, 1994.
- [12] K. Watanabe, D. M. Himmelblau, "Instrument Fault Detection in Systems with Uncertainty", *Int. J. Sys. Sci.*, Vol 13, pp. 137-158, 1982.
- [13] P. M. Frank, J. Wünnenberg, "Robust Fault Diagnosis using Unknown Input Schemes", *System Fault Diagnosis in Dinamical Systems: Theory and Application*, R.J. Patton, P. M. Frank and R. N. Clark (eds), pp. 47-98, Prentice Hall, 1989.
- [14] J. Chen, R. J. Patton, H. Zhang, "Design of unknown input observers and robust fault detection filters," *International Journal of Control*, vol. 63, No.1, pp. 85-105, 1996.
- [15] R. J. Patton, J. Chen, "Robust fault detection using eigenstructure assignment: a tutorial consideration and some new results", *Proceedings of the 30<sup>th</sup> IEEE Conference on Decision and Control*, Brighton, UK, pp 2242-2247, 1991.
- [16] S. X. Ding, T. Jeansch, P. M. Frank, E. L. Ding, A unified approach to the optimization of fault detection systems, *International journal of adaptive control and signal processing*, 2000, 14, 725-745.
- [17] H. J. Marquez, A frequency domain approach to state estimation, *Journal of the Franklin Institute* 340 (2003) 147-157.
- [18] H. J. Marquez, M. Riaz, Robust state observer design and its application to an industrial boiler system, *Control Engineering Practice*, vol. 13, No. 6, 713-728, 2005.
- [19] K. Zhou, J. C. Doyle, "Essentials of robust control", Prentice-Hall, NY, 1998.