An Approach to the Nonlinear Control of Rolling Mills

Saul Mtakula, smtakula@norpaccontrols.com
NORPAC Controls, Unit 4-556 North Nechako Rd
Prince George, BC, V2K 1A1, Canada

***Abstract*— Traditional attempts at controlling rolling mills have involved some linearization of nonlinear dynamics of the process, and the subsequent employment of linear techniques to solve the regulator problem. This paper illustrates some results in the application of nonlinear techniques to the solution of the rolling control problem. In particular the fairly recent techniques of adaptive backstepping and control based on passivation are shown to be effective with various degrees of success. The structure of the dynamics of the rolling stand and its suitability to the straightforward application of backstepping or feedback passivation is demonstrated. An approximate adaptive mechanism for the identification of an unknown model parameter is shown.**

***Index Terms*— Backstepping, Adaptation, Passivation.**

I. Background

Numerous thickness control strategies have been proposed for metal rolling mills (see for instance [1], [2], [3], [4], [5], [6]). In general the problem statement is related to the set-up depicted in figure 1.

A. Mill system

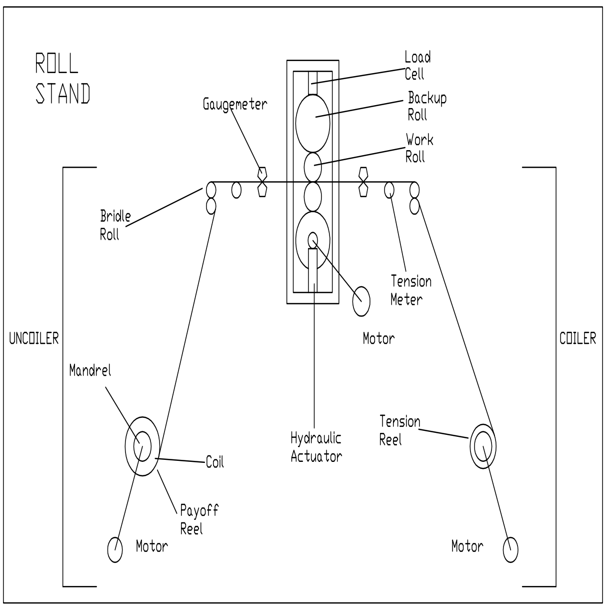

Fig. 1. Typical Single Stand Rolling Mill

The sources cited above liberally explain the control issues pertaining to the rolling mill. We will see in effect that the major nonlinearity may be captured by equation 1 the Gaugemeter equation.

II. Process Description Equations: Sinusoidal Disturbance Free Model

We now briefly state the equations that govern the process without any derivations; those have been done elsewhere in particular [6]. The main control objective is the regulation of exit strip thickness h given an entry strip thickness H. The main control will be a hydraulic input (work roll gap) S that directly acts on the entry strip thickness. Indeed we have

$$h = S + P_a/M \tag{1}$$

where P_a is roll force and M is the plant's mill modulus. P_a is in turn given by $P_a = WkQ_pP_t\sqrt{R(H-h)}$ where k is a resistance to deformation, W the strip width, P_t the effects of tension, Q_p the roll force function and R the radius of the work roll. In practice S is in fact a delayed signal of the applied rolling gap which we call $\hat{S}$; in this discussion time delay issues will not be discussed because of their minimal impact. Slip between rolls and the strip causes a difference between the strip and work roll velocities; for exit strip velocity v_o and work roll velocity v_r we have the forward slip f given by $f = (v_o - v_r)/v_r$. Some spurious disturbance effects manifest themselves as transient changes in the value of the slip. It is assumed there is a means of inferring the slip through some secondary measurements such as the strip and motor velocities.

A. Coiler and Uncoiler Equations

The uncoiler and coiler have similar structure and are effectively symmetric see figure (1). The uncoiler consists of the coil and and the pay-off reel connected to the drive motor via a gear box. For the uncoiler where v_p is circumferential velocity, T_b backward tension, R_p radius, N_u gear ratio, i_p drive current K_u drive motor torque constant, and J_u the moment of inertia one may write $\dot{v}_p = K_uR_p/J_uN_u + {R_p}^2T_b/J_uN_u$. The subscripts u and p represent respectively the uncoiler and payoff reels. Use is made of the tight coupling between velocity and tension to regulate the tension by varying the velocity. The inertia J_u consists of the constant drive motor inertia J_{mp} and a contribution from the varying coil radius. The relationship is in fact

$$J_u = J_{mp} + \pi\rho W({R_p}^4 - {R_{mp}}^4)/2{N_u}^2 \tag{2}$$

where R_{mp} is the mandrel radius. While this relation is time varying, we shall assume that our system is time invariant because for all intents and purposes these radii do not change markedly from the nominal values assumed at the beginning of the analysis. Addition of robustness to the system controller in such a manner as to counteract the time varying effects , now seen as bounded disturbances is instead much more productive. Coiler dynamics are symmetrical to (2) above (the subscript t represents the tension reel).

B. Tension Relationships

Strip tension generated by the difference between the velocity of the strip and that of the reels can be expressed as

$$\dot{T_b} = EHW(v_i - v_p)/L_p \tag{3}$$

$$\dot{T_f} = EhW(v_t - v_o)/L_t. \tag{4}$$

L_p is the distances between the work roll and pay off reel, L_t the distances between the work roll and tension reel, E the Young's modulus of the strip, T_f and T_b the forward and backward tensions respectively. Tension must be maintained within narrow limits to prevent the breaking or tearing of the steel or in the other extreme the bunching up of the steel. Even when this does not happen the profile of the sheet may vary according to the variation in tension. Roll gap adjustment has an immediate effect on upstream tension, and smaller effect on the downstream tension [5].

III. Open Loop Equations

A. Sinusoidal Disturbance Free Model

We begin by recasting the equations above to a more convenient form. We introduce the more familiar state space x and shift the equilibrium so that it's at the origin of the new state space. Essentially the problem at hand is a regulation problem with the desired exit strip thickness given by h_r. One may then define an error in the strip thickness h_e given by $h_e = h - h_r$. The takeup and payoff velocities may also be referred to values v_{pr} and v_{tr} to give the error signals respectively $v_{te} = v_t - v_{tr}$ and $v_{pe} = v_p - v_{pr}$. Tension errors T_{fe} and T_{be} may be similarly defined to result in $\dot{T_{fe}} = \beta h(v_t - v_o)$ from (4), where $\beta = EW/L_t$ is a constant. This may be further rewritten $\dot{T_{fe}} = \beta h_r(v_t - v_o) + \beta h_e(v_t - v_o)$ which may be expanded out into $\dot{T_{fe}} = \beta h_r v_{te} + \beta h_r v_{tr} - \beta h_r v_o + \beta h_e v_{te} + \beta h_e v_{tr} - \beta h_e v_o$. Equation (3) may be written

$$\dot{T_{be}} = \alpha H(v_i - v_p) \tag{5}$$

where $\alpha = EW/L_p$ is a constant. We are given the relation

$$v_i = hv_o/H \tag{6}$$

This may be substituted into (5) to yield

$$\dot{T_{be}} = \alpha v_o h_e - \alpha H v_{pe} - \alpha H v_{pr} + \alpha v_o h_r. \tag{7}$$

Equation (1) may be written

$$h = S + \mu(\sqrt{H - h})/M = S + \mu(H - h_r - h_e)^{1/2}/M \tag{8}$$

Now (8) may be written $h = S + \theta(H - h)^{1/2}$ where $\theta = \mu/M$, M being a constant known to within a ten percent confidence range. This equation can be differentiated on both the left and right hand sides to yield $\dot{h} = \dot{S} - \theta\frac{1}{2}(H - h)^{-1/2}\dot{h_e}$. After some algebraic manipulation it is clear that $\dot{h}_e = g(h_e)\dot{S}$ where $g(h_e) = 1/(1 + \frac{\theta}{2\sqrt{(H-h)}})$. One at this stage may introduce a new set of variables thus

$$\begin{aligned} x_1 &= h_e \\ u_1 &= \dot{S} \\ x_2 &= T_{be} \\ x_3 &= v_{pe} \\ u_2 &= i_p \\ x_4 &= T_{fe} \\ x_5 &= v_{te} \\ u_3 &= i_t. \end{aligned} \tag{9}$$

The system dynamics may then be summarised by the set of differential equations in (10)

$$\begin{aligned} \dot{x_1} &= g(x_1)u_1 \\ \dot{x_2} &= k_1 x_1 - k_2 x_3 + k_3 \\ \dot{x_3} &= b_0 x_2 + a_0 u_2 + k_4 \\ \dot{x_4} &= -k_5 x_1 + \beta x_1 x_5 + k_6 x_5 - k_7 \\ \dot{x_5} &= d_0 x_4 + c_0 u_3 + k_8. \end{aligned} \tag{10}$$

The constants that appear in (10) are defined by the following set of equations

$$\begin{aligned} a_0 &= K_u R_p/(J_u N_u) \\ b_0 &= {R_p}^2/(J_u N_u) \\ c_0 &= K_c R_t/(J_c N_c) \\ d_0 &= {R_t}^2/(J_c N_c) \\ k_1 &= \alpha v_0 \\ k_2 &= \alpha H \\ k_3 &= \alpha v_0 h_r - \alpha H v_{pr} \\ k_4 &= b_0 T_{br} \\ k_5 &= \beta(v_0 - v_{tr}) \\ k_6 &= \beta h_r \\ k_7 &= \beta h_r(v_t - v_0) \\ k_8 &= d_0 T_{fr} \\ v_r &= v_0/(1 + f). \end{aligned}$$

It's then a simple matter to compute the constants once the motor parameters are given. From the expressions for k_3 and k_7 it is not difficult to see that under ideal conditions these constants equal zero. We may in fact consider them zero except at instants that is, they may be generalised into disturbance signals $k_3(t)$ and $k_7(t)$. Slip has been known to vary that is, it suffers disturbance effects. The conservation of mass equations suffer impulse type disturbances. We will not always make explicit the time dependence of k_3 and k_7 and will frequently drop them in analyses.

B. Model With Disturbance

The sinusoidal disturbance effects have not been considered above. To incorporate persistent disturbance effects it is sufficient to replace the entry strip thickness H with $H + \Delta H$

where ΔH accounts for the persistent disturbance effects. We shall make a distinction between the former and a more spurious and transient type of disturbance. The main type of disturbance on the system is a persistently exciting sinusoid of the form $\delta(t) = A_d \sin(\omega t + \phi)$ where $A_d = 0.2$ mm, $\omega = \frac{2\pi v_i}{L_d}$. A_d and L_d are the magnitude and "period" of the thickness deviation respectively. Assuming v_i has been satisfactorily controlled it's clear this thickness deviation is a simple sinusoid of known frequency and magnitude. We shall employ the internal model principle to reject this disturbance. For successful rejection, knowledge only of the frequency will suffice; the controller in effect identifies both the phase and magnitude of the disturbance to reject it's effects. The model of the system that incorporates the sinusoidal disturbance is

$$\begin{aligned} \dot{x_1} &= g_1(x_1, \theta, t)u_1 + g_2(t) \\ \dot{x_2} &= k_1x_1 - k_2x_3 + k_3 - \tilde{A}_d \sin(\omega t + \phi) \\ \dot{x_3} &= b_0x_2 + a_0u_2 + k_4 \\ \dot{x_4} &= -k_5x_1 + \beta x_1x_5 + k_6x_5 - k_7 \\ \dot{x_5} &= d_0x_4 + c_0u_3 + k_8 \end{aligned} \tag{11}$$

We note that [6] employs the internal model for disturbance rejection for a linear scheme. The functions g_1 and g_2 that appear in (11) are defined precisely in the following equations $g_1(h_e, \theta, t) = 1/(1 + \frac{\theta}{2\sqrt{(H-h)+\sin(\omega t+\phi)}})$ and $g_2(h_e, \theta, t) = \theta\omega\cos(wt)/(1 + \frac{\theta}{2\sqrt{(H-h)+\sin(\omega t+\phi)}})$.

IV. Stabilisation

This paper applies results explained in [7] and [8]. Clearly equation 10 may be considered a cascade of three subsystems, namely the $\dot{x}_1$, the $\dot{x}_2, \dot{x}_3$ and the $\dot{x}_4, \dot{x}_5$ subsystems. These cascades may be stabilised individually, with the hope that interconnected individually stabilized cascades would be stable. Indeed assuming an invertible (an examination indicates that g is invertible in the range of interest) $g(\theta, x_1)$ the control law $u_1 = -x_1/g(\theta, x_1)$ stabilizes the first subsystem. For the rolling mill

$$g(\theta, x_1) = \frac{1}{1 + \frac{\theta}{2\sqrt{(c_1 - x_1)}}} \tag{12}$$

with θ is unknown. This paper will show that an approximate adaption law is able to give satisfactory results.

Assuming x_1 has been stabilised the remaining dynamics for the $\dot{x}_2, \dot{x}_3$ subsystem may be written

$$\begin{matrix} \dot{x}_2 \\ \dot{x}_3 \end{matrix} = \begin{matrix} -k_2x_3 + k_3 \\ b_0x_2 + a_0u_2 + k_4 \end{matrix} \tag{13}$$

With the substitution $\hat{u} = a_0u_2 + k_4$ we may write

$$\begin{matrix} \dot{x}_2 \\ \dot{x}_3 \end{matrix} = \begin{matrix} -k_2x_3 + k_3 \\ b_0x_2 + \hat{u}_2 \end{matrix} \tag{14}$$

realising a lower triangular subsystem that is stabilisable via backstepping. Any lower triangular system may be summarised

$$\begin{matrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{matrix} = \begin{matrix} f_1(x_1x_2) \\ f_2(x_1, x_2, x_3) \\ \vdots \\ f_n(x_1, x_2, ...x_n, u) \end{matrix} \tag{15}$$

The backstepping procedure begins with the construction of a control Lyapunov function (clf), for example in this case $V_1 = \frac{1}{2}x_2^2$. Its derivative $\dot{V}_1 = x_2\dot{x}_2 = x_2(-k_2x_3 + k_3)$ may be rendered negative definite to make the dynamics of x_2 globally asymptotically stable. A choice of $V_1 = -x_2^2$ and substution is sufficient to compute a desired psuedo (or virtual) control $x_{3des} = (k_3 + x_2)/k_2$. If k_2 and k_3 are known, a straighforward application is viable, whereas if they are unknown but bounded to known bounds, robustifying control laws based on domination are effective. At this expository stage we assume that k_2 and k_3 are known. The control for the augmented system $\hat{u}_2$ may be computed as follows; assume that x_3 deviates from x_{3des} by z_1. We may then write

$$x_3 = x_{3des} + z_1 = (x_3 + x_2)/k_2 + z_1 \tag{16}$$

Introducing a new composite clf $V_2 = \frac{x_2^2}{2} + \frac{z_1^2}{2}$ and rendering its derivative negative definite $\dot{V}_2 = -x_2^2 - z_1^2$, one may then compute a suitable control law

$$\hat{u}_2 = -2x_{3des} + z_1 = -2x_3 + (k_2 + 1/k_2 - b_0)x_2 + 2k_3/k_2 \tag{17}$$

One can similarly repeat the procedure for the $(\dot{x_4}, \dot{x_5})$ subsystem (or one may use the symmetry) to result in the stabilising control $\hat{u}_3 = -2x_5 + (-k_6 - 1/k_6 - d_o)x_2 - 2k_7/k_6$. By further using the relations $u_2 = (\hat{u_2} - k_4)/a_o$ and $u_3 = (\hat{u_3} - k_8)/c_o$ one may compute the actual controls, u_2 and u_3 to input into the system. We note that u_1 as computed is actually a differential of the desired control law and integral action is therefore required to generate the desired control effort on the actual system, $S = \int u_1 dt$.

The treatment above gives a flavour of the methods available for nonlinear control; it is true a linear method could have stabilised the $\dot{x}_2, \dot{x}_3$ or $\dot{x}_4, \dot{x}_5$ subsystems, and that approach of combining nonlinear and linear methods for stabilisation is not unusual. In this paper we demonstrate that methods based on passivation (of which backstepping is an instance) are viable for the achievement of integrated solution to the control problem. This paper focuses on the single stand with a persistant sinusoidal disturbance. Simulation results of the resultant controllers are shown.

V. Simulation Results

A. Summary of Simulation Constants

Herewith is a summary of the values of the physical constants used in the simulation. The entry strip thickness is $H = 2$ mm, the forward slip $f = 0.02$. $R_p = R_t = 1$ m, $a_0 = b_0 = c_0 = d_0 = 1$ m^{-1}, $v_0 = 5.95$ m s^{-1}, $h_r = 1.24$ mm, $T_{br} = 7270$ kg, $T_{fr} = 8283$ kg. The sinusoidal disturbance ΔH is given by $\Delta H = A_d \sin(2\pi v_i t/L_d)$ where $L_d = 5$ m is the period of the thickness disturbance, and $A_d = 0.2$ mm is the magnitude of the disturbance.

B. Open Loop Results: Disturbance Free Model

We begin first by showing the time history of the system in open loop, in particular on the quantities h, T_f and T_b respectively the exit strip thickness, the forward tension and the backward tension. Simultaneous unit step inputs of S, i_p

and i_t respectively the roll gap, payoff reel motor current and take-up reel motor current are applied at time $t = 0$. The evolution of the states of interest in that case is shown in figure 2. The deterministic sinusoidal disturbance on H is ignored in this instance. From figure 2 it is observed that there does not seem to be any obvious unbounded instability. Instead h settles with a steady state error of 0.235 mm, while T_f and T_b oscillate sinusoidally with frequencies of approximately 1 rad s^{-1} and 1.4 rad s^{-1} respectively. (T_f has the larger magnitude.)

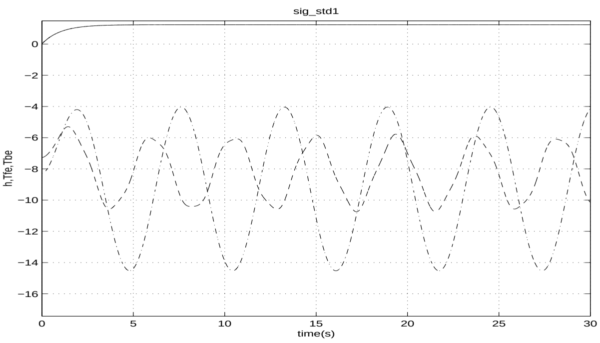

Fig. 2. Open loop time histories of h, T_f and T_b

C. Open Loop Results: Model incorporating sinusoidal disturbance

When the deterministic sinusoidal disturbance is added to the H signal there is evidence of a transient oscillation about the steady state values of h (as indeed there is around the transient values of T_b and T_f).

D. Closed Loop Results

1) Back-stepping Control Laws: Application of the control laws computed by the method of back-stepping in conjunction with the control that stabilises x_1 achieves the results shown in figure 3 . It is observed in figure 3 that perfect regulation is

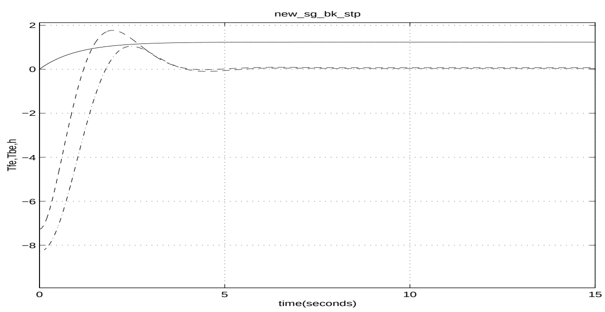

Fig. 3. Closed loop time histories of h, -T_{fe} and -T_{be} The effect of back-stepping control law

achieved for h, T_b and T_f within about 7 seconds of start up of the closed loop process. The tensions experience overshoots of less than 12 per-cent momentarily before settling. We will seek in the following sections to guarantee convergence by the use of other methods in particular adaption.

2) Internal Model Principle: The internal model principle suggests that to reject a persistent disturbance, it's dynamics must exist in the closed loop control. In the case at hand the dynamics of the sinusoid may be incorporated into the closed loop by cascading a sinusoid (in fact the Laplace equivalent is included) to the respective plant outputs. This adds at least two states to the open loop plant for each output of interest. In general any output which requires the suppression of the sinusoid would be augmented with two additional states.

Any method of stabilisation may then be attempted on the augmented system. The dynamics of a sinusoid are lower triangular and their addition to the system does not alter the lower triangular dependence of the dynamics. As such the whole system cascaded with a sinusoid is still amenable to stabilisation via backstepping. References [9] ,using [10], [11] and [12] demonstrates that for the problem at hand, the suppression of sinusoids in the non-linear functions g_1 and g_2 is straightforward, an example of the non-linear output regulation problem with full information. Indeed having the variable h_e as an input into a sinusoidal filter $\frac{{\omega_n}^2}{(s^2+2\epsilon\omega_n+{\omega_n}^2)}$ completes the solution. The two additional states from the second order oscillatory filter must be part of the controller to achieve output regulation.

VI. Adaptive Control

We recall that there exists an unknown parameter θ that figures prominently in all control laws formulated. This parameter is known to within ten percent of a certain nominal value $\hat{\theta}$. The problem of adaptive control in the non-linear setting has received a lot of attention in recent years. Results in the literature appear most frequently for the type of equation that can be stated as

$$\dot{x} = f(x)\theta^T + g(x)u \tag{18}$$

where θ is an unknown constant. Suppose when θ is known, the appropriate control law is given by $u = k(x, \theta)$. The problem of adaptive control is whether the control law may actually be framed as

$$\begin{aligned} u &= \hat{k}(x, \hat{\theta}) \\ \dot{\hat{\theta}} &= \lambda(x, \hat{\theta}) \end{aligned} \tag{19}$$

where $\hat{\theta}$, an estimate of the actual parameter θ, is employed in the control law in conjunction with some dynamic adaptation mechanism, λ to ensure that the estimate converges to the true value of the parameter. Now it will be remembered our dynamic system in the absence of the sinusoidal disturbance may be summarised as

$$\begin{aligned} \dot{x_1} &= g(x_1)u_1 \\ \dot{x_2} &= k_1x_1 - k_2x_3 + k_3 \\ \dot{x_3} &= b_0x_2 + a_0u_2 + k_4 \\ \dot{x_4} &= -k_5x_1 + \beta x_1x_5 + k_6x_5 - k_7 \\ \dot{x_5} &= d_0x_4 + c_0u_3 + k_8 \end{aligned} \tag{20}$$

with g given by

$$g(x_1, \theta) = 1/(1 + \frac{\theta}{2\sqrt{(c_1 - x_1)}}). \tag{21}$$

This relation (21) does not have the usual form for this type of problem (as typified by equation (18)). Fortunately results in [7], [13], [8] exist showing how an attempt at a solution may be made.

VII. Test Cases

A. Direct Estimation of Unknown Constant Gain

Consider

$$\dot{x} = \theta u \tag{22}$$

where θ is an unknown constant of known sign. This is the case of linear parameterisation for which firm results are most frequently encountered. If θ were known then the control $u = -cx/\theta$ with $c > 0$ would result in GAS dynamics. If θ were unknown it is reasonable as a first step to suggest a control law of the form

$$u = -cx/\hat{\theta} \tag{23}$$

instead. Indeed the justification for this step is the certainty equivalence principle. Defining an error $\tilde{\theta}$, the difference between the parameter estimate and the true value of the parameter we can write

$$\theta = \tilde{\theta} + \hat{\theta}. \tag{24}$$

Substituting (23) into (22) we have

$$\begin{aligned} \dot{x} &= \hat{\theta}u + \tilde{\theta}u \\ &= -cx - cx\tilde{\theta}\hat{\theta}^{-1}. \end{aligned} \tag{25}$$

We are unable to conclude immediately from equation (25) whether the equilibrium point $x, \tilde{\theta} = 0, 0$ is GAS. A Lyapunov analysis has to be carried out to enable firm conclusions to be drawn. To that end we choose the Lyapunov function candidate

$$V = \frac{x^2}{2} + \frac{\tilde{\theta}^2}{2} \tag{26}$$

whose time derivative is equal to

$$\begin{aligned} \dot{V} &= x\dot{x} + \tilde{\theta}\dot{\tilde{\theta}} \\ &= -cx^2 - cx^2\tilde{\theta}\hat{\theta}^{-1} + \tilde{\theta}\dot{\tilde{\theta}} \\ &= -cx^2 + \tilde{\theta}(-cx^2\hat{\theta}^{-1} + \dot{\tilde{\theta}}). \end{aligned} \tag{27}$$

By choosing the so-called parameter update law of the form

$$\dot{\tilde{\theta}} = cx^2\hat{\theta}^{-1} \tag{28}$$

the time derivative of the Lyapunov function is rendered NSD. That condition in conjunction with invariance property may be used to conclude the GAS properties of the equilibrium point $(x, \hat{\theta}) = (0, 0)$. Now (28) may be written $\dot{\hat{\theta}} = -cx^2\hat{\theta}^{-1}$ which indicates that there is the danger of $\hat{\theta}$ going to zero with time, resulting in jumps in the value $\dot{\hat{\theta}}$. Some example simulations indicate the potential pitfalls. This jump is usually dependent on the initial estimate for the parameter, $\theta(0)$. A lot of times in the control literature this initial estimate is set to zero, highlighting the pitfalls that may therefore arise. Figure 4 is the simulation for the case where $\theta = 6$, $c = 1$ $x(0)$=4 and $\hat{\theta}(0) = 1.33$. The value of $\hat{\theta}$ equals zero for brief instances which then destroys any meaningful control effort.

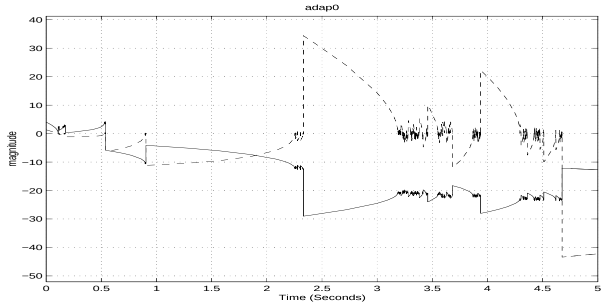

Fig. 4. x and $\hat{\theta}$. Unfortunate choice of $\hat{\theta}(0)$ (Simulink)

It's suggested in [8] that instead of estimating θ it might be productive to instead estimate it's reciprocal θ^{-1} with some estimate $\hat{r}$. For this case the representative equation may be written

$$\theta^{-1} = \tilde{r} + \hat{r}. \tag{29}$$

The corresponding control law in analogy with the previous analysis may be written

$$u = -\hat{r}cx. \tag{30}$$

Following the steps for the previous case, we obtain

$$u = -(\theta^{-1} - \tilde{r})cx. \tag{31}$$

To make conclusions about stability the Lyapunov function

$$V = \frac{x^2}{2} + \theta\frac{\tilde{r}^2}{2} \tag{32}$$

is differentiated to yield the relations

$$\begin{aligned} \dot{V} &= x\dot{x} + \theta\tilde{r}\dot{\tilde{r}} \\ &= -cx^2 + \theta cx^2\tilde{r} + \theta\tilde{r}\dot{\tilde{r}} \\ &= -cx^2 + \theta\tilde{r}(cx^2 + \dot{\tilde{r}}). \end{aligned} \tag{33}$$

One may chooses a parameter update law that guarantees a NSD Lyapunov derivative. This law is shown to be $\dot{\tilde{r}} = -cx^2$. This update mechanism has a redeeming feature in that because $\dot{\tilde{r}} = -\dot{\hat{r}}$, the derivative in the update law is greater than or equal to zero. This adaptive control law is therefore stable for any initial estimates $\hat{r}$ that are greater than zero. There is a caveat however in that for actual physical systems signals never perfectly converge to zero. The net effect of this on the parameter update law is to have a residual positive derivative. The danger is then of a drift to infinity of the estimate $\hat{r}$. In practice this is solved by switching off the update laws when required. Figure 5 is the corresponding simulation with $\theta = 6$, $c = 1$ $x(0)$=4 and $\theta\hat{(0)} = 0$. Indeed one may choose any value for the initial parameter estimate with satisfactory results. An attempt to extend the results for the following cases [9]

$$\dot{x} = g(x)\theta u \tag{34}$$

$$\dot{x} = g(\theta)u \tag{35}$$

$$\dot{x} = g(x, \theta)u. \tag{36}$$

was unsuccessful in coming up with parameter update laws that had no dependence on θ.

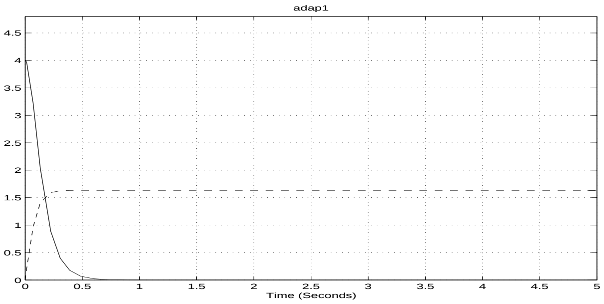

Fig. 5. x and $\hat{\theta}$. Stable parameter update law. (Simulink)

VIII. Approximate Adaptation

Here we seek to investigate whether an adaptive law in which one uses the estimate $\hat{\theta}$ of θ in the update algorithm has redeeming features, or alternately whether it might illuminate a new method of attack toward the problem. We recall (**??**) restated below for convenience

$$\dot{x} = \frac{u}{(1+\theta f(x))}. \tag{37}$$

Consider the control law

$$u = -(1+\hat{\theta} f(x))cx \tag{38}$$

where $\hat{\theta}$ is an estimate of the unknown parameter and c is a constant that is strictly positive. Clearly then (37) can be restated

$$\dot{x} = -\frac{(1+\hat{\theta} f(x))cx}{(1+\theta f(x))}. \tag{39}$$

We assume (24) holds and can then write after some algebra

$$\dot{x} = -cx + \frac{\tilde{\theta} f(x)cx}{(1+\theta f(x))}. \tag{40}$$

We use for analysis a Lyapunov function $V = \frac{x^2}{2} + \frac{\tilde{\theta}^2}{2}$ to result in the time derivative

$$\begin{aligned} \dot{V} &= x\dot{x} + \tilde{\theta}\dot{\tilde{\theta}} \\ &= -cx^2 + \frac{\tilde{\theta} f(x)cx^2}{(1+\theta f(x))} + \tilde{\theta}\dot{\tilde{\theta}}. \end{aligned} \tag{41}$$

For a parameter update law we seek, as is the norm to force the two last terms in (41) to cancel. The parameter update law that would achieve this would be

$$\dot{\tilde{\theta}} = -\frac{cx^2 f(x)}{(1+\theta f(x))}. \tag{42}$$

Unfortunately that adaptive law cannot be implemented because it depends on the unknown parameter θ. At this stage we endeavour to find how the alternative parameter update (43) would fare. It differs from (42) only in that $\hat{\theta}$ substitutes for θ in the adaptive algorithm.

$$\dot{\tilde{\theta}} = -\frac{cx^2 f(x)}{(1+\hat{\theta} f(x))}. \tag{43}$$

Substituting (43) in (41) we can write

$$\begin{aligned} \dot{V} &= x\dot{x} + \tilde{\theta}\dot{\tilde{\theta}} \\ &= -cx^2 + \frac{\tilde{\theta} f(x)cx^2}{(1+\theta f(x))} - \frac{\tilde{\theta} cx^2 f(x)}{(1+\hat{\theta} f(x))} \\ &= -cx^2 + \tilde{\theta} cx^2 f(x)\left(\frac{1}{1+\theta f(x)} - \frac{1}{1+\hat{\theta} f(x)}\right) \\ &= -cx^2 + \tilde{\theta} cx^2 f(x)\frac{\{(1+\hat{\theta} f(x)) - (1+\theta f(x))\}}{(1+\hat{\theta} f(x))(1+\theta f(x))} \\ &= -cx^2 - \frac{\tilde{\theta}^2 cx^2 f(x)}{(1+\hat{\theta} f(x))(1+\theta f(x))} \end{aligned} \tag{44}$$

If $\hat{\theta}$ is rendered always positive(both positive), then from fact that $f(x)$ is always positive it turns out that this approximate adaptation (for lack of a better term) results in a negative definite derivative of the Lyapunov function, assuring the convergence of both x and $\tilde{\theta}$ to zero. Now this is not always true and a mechanism to guarantee the positiveness of $\hat{\theta}$ would be required.

A. A Simulation

From (21) we know the form of $f(x)$ (with h_e denoted as x we have $f(x) = \frac{1}{2\sqrt{H-h_r-x}} = 12\sqrt{c_1 - x}$). We set up the in Simulink the structure for the adaption suggested by (43) and arbitrarily choose values of θ, $\hat{\theta}(0)$ and the constants c and c_1. With the following values $\theta = 4$, $\hat{\theta}(0) = 0$, $x(0) = 1$, $c_1 = 3$ and $c = 1$ the simulation results appear as shown in figure 6 While adaptation shows promise, methods based on

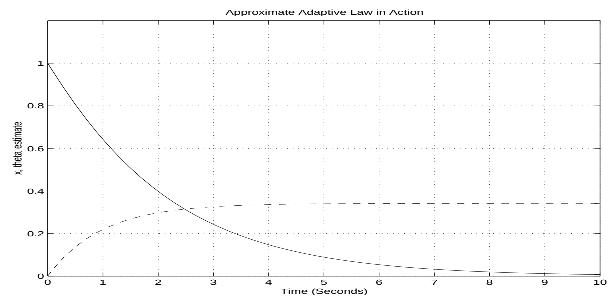

Fig. 6. x and $\hat{\theta}$. Approximate adaptive control.

robust domination of the unknown θ have better results in that they provide guaranteed stability.

IX. Control By Passivation

We will illustrate a powerful passivation method to achieve GAS of the origin of the system

$$\begin{aligned} \dot{x_1} &= g(x_1)u_1 \\ \dot{x_2} &= k_1 x_1 - k_2 x_3 + k_3 \\ \dot{x_3} &= b_0 x_2 + a_0 u_2 + k_4 \\ \dot{x_4} &= -k_5 x_1 + \beta x_1 x_5 + k_6 x_5 - k_7 \\ \dot{x_5} &= d_0 x_4 + c_0 u_3 + k_8 \end{aligned} \tag{45}$$

assuming there is knowledge of θ. It will be remembered that we anticipated that a credible control could be formulated as follows: design a stabilising control u_1 for the $\dot{x_1}$ sub-system and if the decay of x_1 is rapid enough, it may be ignored completeley in the design for the $\dot{x_2}, \dot{x_3}$ and the $\dot{x_4}, \dot{x_5}$ subsystems. However as the simulation in figure 7 shows if the control action u_1 is not strong enough the whole system may go unstable. In other words the system is not GS.

X. Passivation Control of Cascades

We will seek to design a control system in which the interaction of x_1 with the two other sub-systems is taken into account to achieve GAS. We will appeal mostly to results that appear in [7] for answers to this problem. We briefly note that the most severe interaction with the x_1 system occurs in the $\dot{x_4}, \dot{x_5}$ sub system. We will consider this interaction initially and aim to resolve it. Our system is in the form

$$\begin{aligned} \dot{z} &= f(z) + \psi(z,\xi), \\ \dot{\xi} &= A\xi + Bu. \end{aligned} \tag{46}$$

We briefly recap on theorems and definitons that are going to be useful for our passivation and stabilisation procedure. We begin with a definition of passivity taken from [7] slightly modified. Consider a square system i.e, with equal number of inputs and outputs, H defined by

$$(H) \quad \left\{ \begin{array}{lcl} \dot{x} &=& f(x,u),\ x \in R^n \\ y &=& h(x,u),\ u,y \in R^m \end{array} \right.$$

Assume that associated with H is a bilinear supply rate $w(u,y) = u^T y$, $w: R^m \times R^m$ such that w is locally integrable for all $u \in U$ where U is a set of admissible controls. Integrability is captured in the relation $\int_{t_0}^{t_1} \|w(u(t), w(y(t))|dt < \infty$ for all $t_0 \leq t_1$. Let X be a connected set of R^n containing the origin. Then H is passive in X if there exists a positive semidefinite storage function $S(x)$, $S(0) = 0$, such that for all $x \in X$

$$S(x(T)) - S(x(0)) \leq \int_0^T w(u(t), y(t))dt.$$

If the storage function is differentiable we write $\dot{S}(x(t)) \leq u^T y$. Next we state two theorems from [7] relating to the interconnection of passive systems and the stabilisation of passive systems. Suppose that systems H_1 and H_2 are passive. Then the systems, one obtained by the parallel interconnection, and the other obtained by feedback interconnection, are both passive. Let H be the system

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= h(x) \end{aligned} \tag{47}$$

and let it be passive with a C^1 storage S. Then the feedback $u = -y$ acheives asymptotic stability of $x = 0$ if and only if H is zero state detectable (ZSD). For a linear system without throughput $\dot{x} = Ax + Bu, y = Cx$ it can be shown [7] that its passivity means there exists a storage function $S(x) = x^T P x$ where P is a positive definite matrix satisfying

$$\begin{aligned} PA + A^T P &\leq 0 \\ B^T P &= C. \end{aligned} \tag{48}$$

A theorem which links the algebraic conditions of (48) with frequency domain characteristics follows below. (KYP lemma) If the linear system (A,B,C) is passive, i.e., there exists a matrix P satisfying (48) then the transfer function $H(s) = C(sI - A)^{-1}B$ is positive real. Conversely, if $H(s)$ is positive real, then for any minimal realisation of $H(s)$ there exists $P > 0$ which satisfies (48). Consider now (46) and suppose the equilibrium $z = 0$ of $\dot{z} = f(z)$ is GS and a C^2 radially unbounded positive definite function $W(z)$ is known such that $L_f W \leq 0$. Suppose the interconnection term can be factored as follows

$$\psi(z,\xi) = \tilde{\psi}(z,\xi)C\xi. \tag{49}$$

If the linear system H_1 with the transfer function

$$H_1(s) = C(sI - A)^{-1}B \tag{50}$$

can be formed such that H_1 is positive real then it is passive. The nonlinear block H_2 below may then be formed

$$\dot{z} = f(z) + \tilde{\psi}(z,\xi)u_2 \tag{51}$$

with it's input given by $u_2 = C\xi = y_1$. We can freely choose as a design choice, an output y_2 of H_2 that renders H_2 passive. Employment of theorem X leads to a passive interconnected system. Using $W(z)$ as a storage function for for H_2 we require that

$$\dot{W} = \frac{\partial W}{\partial z}(f(z) + \tilde{\psi}(z,\xi)y_1) \leq y_2^T u_2. \tag{52}$$

Knowing that $L_f W \leq 0$, we satisfy (52) by selecting

$$y_2 = h_2(z,\xi) := (L_{\tilde{\psi}} W)^T (z,\xi) = \tilde{\psi^T}(\frac{\partial W}{\partial z}^T) \tag{53}$$

we make H_2 passive. Via a feedback transformation

$$u = -h_2(z,\xi) + v \tag{54}$$

we obtain an system that is passive from v to y_1. Global stability is then achieved by feedback

$$v = -cy_1 \tag{55}$$

where $c > 0$ is a constant.

XI. Application of Passivity Results

Consider our system (45). Let's make the substitutions $z_1 = x_5$, $z_2 = x_4$ and $\xi = x_1$. Then momentarily ignoring the $\dot{x_2}, \dot{x_3}$ dynamics (45) may be rewritten

$$\begin{aligned} \dot{z_1} &= d_0 z_2 + \hat{u_3} \\ \dot{z_2} &= -k_5\xi + \beta\xi z_1 + k_6 z_1 - k_7 \\ \dot{\xi} &= g(\xi)u_1 = v \end{aligned} \tag{56}$$

If we consider the output of the "linear" block to be $y_1 = \xi$ then all that is interposed between v and ξ is an integrator, i.e., a passive system exists. (Note the square root nonlinearity has been subsumed in the control signal v). Now (56) may be written as

$$\begin{aligned} \dot{z_1} &= d_0 z_2 + \hat{u_3} \\ \dot{z_2} &= k_6 z_1 - k_7 + \xi(-k_5 + \beta z_1) \\ \dot{\xi} &= g(\xi)u_1 = v \end{aligned} \tag{57}$$

with the interconnection term obviously defined and

$$\tilde{\psi}(z,\xi) = \begin{bmatrix} 0 \\ \xi(-k_5 + \beta z_1). \end{bmatrix} \tag{58}$$

Because u_3 has not yet been defined, we can flexibly achieve the goal of a stable linear cascade $z = 0$ for $\dot{z}$=$f(z)$ by the judicious use of any linear stabilisation method (pole placement, LQR etc). Indeed we can assume a control of the form

$$\hat{u_3} = -a_1 z_1 - a_2 z_2 \tag{59}$$

and a corresponding Lyapunov function

$$W(z_1, z_2) = \frac{z_1^2}{2} + \frac{z_2^2}{2}. \quad (60)$$

We know from (54) that a feedback transformation given by

$$v = -h_2(z, \xi) + w \quad (61)$$

achieves passivity from w to y_1 where

$$h_2(z, \xi) = \tilde{\psi}^T (\frac{\partial W}{\partial z}^T). \quad (62)$$

Noting that

$$\frac{\partial W}{\partial z} = \begin{bmatrix} z_1 & z_2 \end{bmatrix} \quad (63)$$

and substituting in (53) we have

$$h_2(z, \xi) = -k_5 z_2 + \beta z_1 z_2. \quad (64)$$

Using (61) we obtain

$$v = k_5 z_2 - \beta z_1 z_2 + w. \quad (65)$$

The transformation $w = -\xi$ completes the design. In the original state space we have

$$v = k_5 x_4 - \beta x_5 x_4 - x_1 \quad (66)$$

or

$$\begin{aligned} u_1 &= \frac{k_5 x_4 - \beta x_5 x_4 - x_1}{1/(1 + \frac{\theta}{2\sqrt{(H - h_r - x_1)}})} \\ &= (k_5 x_4 - \beta x_5 x_4 - x_1)(1 + \frac{\theta}{2\sqrt{(H - h_r - x_1)}}). \end{aligned} \quad (67)$$

The other control is given by

$$u_2 = \frac{-a_1 x_1 - a_2 x_2 - k_4}{b_0}. \quad (68)$$

It must be remembered k_3 may be set equal to zero for analysis.

XII. Simulation

If we place poles as follows: x_5 at -1 and x_4 at -2 we obtain $a_1 = -3$ and $a_2 = -2.6129$. The simulation for a gain $k = 5$ is shown in figure 7; that gain may be used as a means of tuning controller action. Care must be taken though not to make it too aggressive and bring into play unmodelled dynamics.

XIII. Conclusion

This paper demonstrates that the design of non-linear controllers is viable for the rolling mill. The theory has matured enough in recent times to allow a straightforward application of the results, and the explosion in computing power should enable the implementation of some of these algorithms.

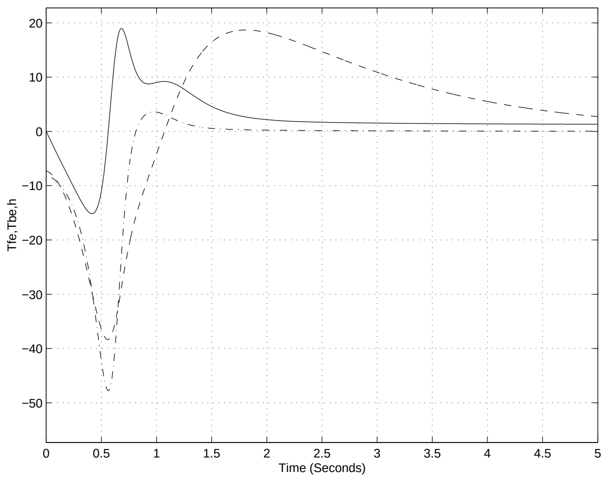

Fig. 7. Nonlinear control law (Simulink)

References

[1] W. J. Edwards, G. C. Goodwin, and P. J. Thomas, "A multi-roll eccentricity controller for strip rolling mills," *The Third Conference on Control Engineering*, 1986.

[2] S. Nakagawa, H. Miura, S. Fukushima, and J. Amasaki, "Gauge control system for hot strip finishing mill," *Proceedings of the 29th Conference on Decision and Control*, pp. 1573–1578, 1990.

[3] I. J. Ferguson and R. F. de Tina, "Modern hot-strip mill thickness control," *IEEE Transactions on Industry Applications*, vol. IA-22, no. 5, pp. 934–940, 1986.

[4] M. Waltz and L. Reed, "Eccentricity filter for rolling mills," *Proceedings of American Control Conference*, vol. 21, pp. 1–9, 1971.

[5] E. J. M. Geddes and I. Postlethwaite, "Improvements in product quality in tandem cold rolling mill using robust multivariable control," *IEEE Transactions on Control Systems Technology*, vol. 6, pp. 257–269, 1998.

[6] H. A. D. Hwang and D. Kim, "Design of a robustness thickness controller for a single stage cold rolling mill," *Proceedings of American Control Conference*, 1996.

[7] R. Sepulchre, M. Jankovic, and P. Kokotovic, *Constructive Nonlinear Control*. Springer, 1997.

[8] M. Krstic and P. K. I Kanellakopoulos, *Nonlinear and Adaptive Control Design*. Prentice Hall, 1991.

[9] S. E. Mtakula, "Control of a nonlinear rolling mill," *Department of Systems and Control*, 1999. Master's Thesis.

[10] W. M. Wonham, *Linear Mulitivariable Control: A Geometric Approach*. Springer-Verlag, 1979.

[11] B. A. Francis and W. M. Wonham, "The internal model principle of control theory," *Automatica*, vol. 12, pp. 457–465, 1976.

[12] A. Isidori, "Nonlinear output regulation," in *The Control Handbook* (W. S. Levine, ed.), pp. 923–932, CRC Press Inc., 1996.

[13] D. J. Clements, "Introduction to backstepping: State feedback of strict feedback systems," 1997. (University of New South Wales Control Systems Department working paper).