PLANT-WIDE MPC: A COOPERATIVE DECENTRALIZED APPROACH

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Abstract: Model predictive control (MPC) technology has seen remarkable development since its first appearance. Due to its success in industrial practice, efforts to extend its application from unit-wide to plant-wide control are becoming more widespread. Centralized or monolithic MPC schemes are impractical, and often impossible, for large-scale, plant-wide applications due to reliability, maintainability and computational tractability considerations. In general, industrial practice has tended toward a distributed MPC architecture; however, this common decentralized approach has been shown to produce lower performance than a centralized approach. In our work we propose to adopt the decentralized approach, but to coordinate the individual MPC systems; thereby, gaining the performance advantages of the centralized approach and the reliability, maintainability and computational efficiency of the distributed MPC schemes. A further benefit of the proposed approach is that it requires far less capital investment to gain equal performance increases, in comparison to implementation of a new centralized, plant-wide MPC.

In this work, we focus on the steady-state target calculation layer within an MPC application. We draw on the Dantzig-Wolfe decomposition principle in conjunction with a multi-column generation strategy to yield a coordinating structure for decentralized MPC that realizes an effective trade-off between centralized and decentralized MPC target calculation methods. Our approach provides comparable performance to the centralized scheme, while retaining all the benefits of the decentralized approach. In this paper, we discuss methods of constructing information flow between the coordinator and the individual MPC systems, which effectively deal with constraints that span multiple units.

To illustrate our approach, we use case studies to compare the performance of all three control schemes (*i.e.*, centralized, decentralized and coordinated). The results show that the proposed coordination mechanism significantly improves the performance of the overall decentralized control system.

Keywords: Model predictive control, Target calculation, Dantzig-Wolfe decomposition, Multi-column generation, Coordination

1. INTRODUCTION

Model predictive control (MPC) strategies have gained great success in a wide range of industrial applications. The MPC framework can be further divided into a steady-state calculation and a control calculation (or dynamic optimization) (Qin and Badgewell, 2003). The goal of the steady-state calculation is to calculate the desired targets for output, input, and state variables at a higher frequency than those computed from local economic optimizers. The target calculation provides optimal achievable set-points that are passed to control calculation.

With considerable development in recent years, there has been a trend to extend MPC to large-scale applications, such as plant-wide control. Two commonly used strategies for plant-wide MPC control and optimization are centralized schemes and decentralized schemes. A fully centralized or monolithic MPC for an entire plant is often undesirable and difficult, if not impossible, to implement, particularly for large plants (Lu, 2003). Such a scheme can exhibit poor fault-tolerance, require a centralized computational platform, and can be difficult to tune. Alternatively, in many chemical plants, large-scale control problems are usually solved by using decentralized schemes because of their operability, flexibility and reliability. In this paper, reliability refers to the possibility that some control subsystems or portions thereof are able to function when other subsystems fail.

Most MPC implementations, when considered in a plant-wide context, have a decentralized structured, with individual controllers working in an autonomous manner without communication. Such a decentralized scheme can only provide the optimum of each subsystem with respect to its own objective function, but this solution may not be the plant-wide optimum. It is estimated that the potential global benefit for a typical refinery is 2-10 times more than what decentralized control by itself can achieve (Bodington, 1995).

This potential benefit has garnered increasing interest of many researchers. Camponogara *et al.* (2002) proposed a distributed MPC scheme, where local control agents broadcast their states and decision results to every other agent under some pre-specified rules and this procedure continues until no agent needs to do so. Recently, coordination-based MPC algorithms were discussed in Venkat *et al.* (2004), in which augmented states are used to model interactions, to improve plant-wide performance via the coordination of decentralized MPC dynamic calculation. One common characteristic of the above two schemes is that the communication among decentralized or distributed MPC controllers is completed without a coordination system, and thus controllers stand at equal status within their negotiation. Alternatively, in Lu (2003), a cross-functional integration scheme was developed, in which a coordination "collar" performed a centralized target calculation for decentralized MPC. This idea matches the wide-spread belief among industrial practitioners (Scheiber, April 2004) that the trend toward decentralization will continue until the control system consists of seamlessly collabrating autonomous and intelligent nodes with a supervisory coordinator overseeing the whole process.

Our previous work (Cheng *et al.*, 2004) aimed at developing a practical approach to coordinated, decentralized control, using Dantzig-Wolfe decomposition to coordinate plant-wide LP-based MPC target calculations. This paper extends the previous work using multi-column generation techniques to increase coordination efficiency in decentralized MPC calculations.

The outline of this paper is as follows. Firstly, we briefly introduce the Dantzig-Wolfe decomposition principle and multi-column generation techniques. Secondly, we discuss a message construction method for coordination mechanism design in decentralized LP-based target calculation. Then, two illustrative case studies are presented. The first compares the computational efficiency of different optimization schemes through a Monte Carlo simulation study, while the second demonstrates one application of the proposed coordination approach. Finally, we end this paper with conclusions and discussions.

2. DANTZIG-WOLFE DECOMPOSITION

2.1 Decomposition Principle

The Dantzig-Wolfe decomposition principle (Dantzig and Wolfe, 1960; Dantzig and Thapa, 2002) is depicted in Figure 1. A large-scale linear pro-

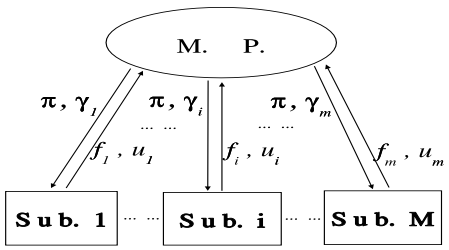

Fig. 1. Mechanism of D-W decomposition

gramming problem can be decomposed into independent subproblems, which are coordinated by a master problem (MP). The solution to the original

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large-scale problem can be shown to be equivalent to solving the subproblems and the MP through a finite number of iterations. During each iteration, the MP handles the linking constraints that connect the subproblems, using the information supplied by the subproblems $[f_i, \mathbf{u}_i]$. f_i is the objective function value and $\mathbf{u}_i$ is the solution of the i^{th} subproblem. Then the MP sends its solution $[\pi, \gamma_i]$ as price multipliers to all the subproblems for updating their objective functions. Consequently, the subproblems with updated objective functions are re-solved. The iterative procedure continues until convergence, which solves the original large-scale problem.

Dantzig-Wolfe decomposition hinges on the theorem of convex combination and column generation techniques (Lasdon, 2002; Dantzig and Thapa, 2002). Although any large-scale linear program problem can be decomposed and solved by Dantzig-Wolfe decomposition (Chvatal, 1983), the approach is particularly powerful for structured linear programs. Consider a block-wise linear programming problem that has been converted to Simplex standard form:

$$\min \quad z_1 = \sum_{i=1}^{n} \mathbf{c}_i^T \mathbf{x}_i$$

$$\text{s.t.} \quad \sum_{i=1}^{n} \mathbf{A}_i \mathbf{x}_i = \mathbf{b}_0 \tag{1}$$

$$\mathbf{B}_i \mathbf{x}_i = \mathbf{b}_i \tag{2}$$

$$\mathbf{x}_i \geq \mathbf{0} \quad i = 1, 2, ..., n \tag{3}$$

where the constraints in (1) represent the linking constraints associated with n subproblems, and the constraints in (2) are the local constraints of independent subproblems. Via the theorem of convex combination, the master problem (MP) can be formulated as follows using the linking constraints in (1) and the convex combination of the extreme points from (2), assuming that the feasible regions of subproblems are bounded [2].

$$\min \quad z_2 = \sum_{i=1}^{n} \sum_{j=1}^{N(i)} f_{ij} \lambda_{ij}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} \sum_{j=1}^{N(i)} \mathbf{p}_{ij} \lambda_{ij} = \mathbf{b}_0 \tag{4}$$

$$\sum_{j=1}^{N(i)} \lambda_{ij} = 1, \quad \lambda_{ij} \geq 0, \quad i = 1, 2, ..., n \tag{5}$$

where *N(i)* represents the number of extreme points of the feasible region in the i^{th} LP subproblem, and

$$\mathbf{x}_i = \sum_{j=1}^{N(i)} \lambda_{ij} \mathbf{u}_i^j \tag{6}$$

$$f_{ij} = \mathbf{c}_i^T \mathbf{u}_i^j \tag{7}$$

$$\mathbf{p}_{ij} = \mathbf{A}_i \mathbf{u}_i^j \tag{8}$$

with $\mathbf{u}_i^j$ the j^{th} extreme point of i^{th} subproblem.

The resulting master problem has fewer rows in the coefficient matrix than the original problem. However, the number of columns in the MP is larger due to the increase in the number of variables associated with the extreme points of all subproblems. The column generation method discussed below provides an efficient approach to dealing with the increase in the number of columns.

2.2 Algorithms with Multi-column Generation

For a large-scale problem, it can be a formidable task to obtain all the extreme points and formulate a full master problem. If the MP is solved via the Simplex method, we only need a basic set which has the same number of basic variables as the number of rows. Thus we do not need to explicitly know all the extreme points of subproblems. This leads to solving an equivalent problem, the *restricted master problem* (RMP), which can be dynamically constructed at a fixed size by incorporating column generation techniques (Gilmore and Gomory, 1961; Dantzig and Thapa, 2002).

Assume that we have a starting basic feasible solution to the RMP and it has a unique optimum. The optimal solution provides us with Simplex multiplier $[\pi, \gamma]$ for the basis in the current RMP, with π associated with (4) and γ with (5), respectively. Then, subproblems are formulated and solved to find the priced-out column associated with λ_{ij}:

$$f_{ij} = (\mathbf{c}_i^T - \pi \mathbf{A}_i) \mathbf{u}_i^j - \gamma_i \tag{9}$$

and the i^{th} subproblem is:

$$\min z_i^0 = (\mathbf{c}_i^T - \pi \mathbf{A}_i) \mathbf{x}_i$$

$$\text{s.t.} \quad \mathbf{B}_i \mathbf{x}_i = \mathbf{b}_i, \quad \mathbf{x}_i \geq \mathbf{0} \tag{10}$$

Therefore, we reach an optimal solution when the following condition is satisfied:

$$\min_{i,j} f_{ij} = \min_{i} (z_i^0 - \gamma_i) \geq 0, i = 1, ..., n \tag{11}$$

Optimality and finite convergence have been proved in Dantzig and Thapa (2002). When condition (11) is not satisfied, we have different column generation strategies to determine the column or columns to enter the basis.

In the single-column generation scheme, assume the minima of problems (11) occur for $i = s$ and

[2] Unbounded cases are also discussed in the related references (Lasdon, 2002; Dantzig and Thapa, 2002)

$\mathbf{x}_s(\pi)$ solves subproblem s, the column to enter the basis is given by

$$\begin{bmatrix} \mathbf{A}_s \mathbf{x}_s(\pi) \\ \mathbf{i}_s \end{bmatrix} \tag{12}$$

where $\mathbf{i}_s$ is a n-component vector with a "1" in position s and zeros elsewhere. Although the generated column is associated with the most favorable subproblem (i.e., that with the most negative reduced cost), any other subproblem with a negative reduced cost has the potential to generate a column to enter the basis of the master problem. Several variants of multi-column generation techniques are discussed in Lasdon (2002) and Dantzig and Thapa (2002). In this work, we use the multi-column generation scheme suggested in Lasdon (2002). Thus, to incorporate all potential favorable proposals, a "new" column is generated in the RMP for each subsystem by applying (12):

$$\min \quad z_3 = \sum_{i=1}^{n} \sum_{j=1}^{m_0+n} f_{ij}\lambda_{ij} + \sum_{i=1}^{n} f_i^* \lambda_i^*$$

$$\text{s.t.} \quad \sum_{i=1}^{n} \sum_{j=1}^{m_0+n} \mathbf{p}_{ij}\lambda_{ij} + \sum_{i=1}^{n} \mathbf{p}_i^* \lambda_i^* = \mathbf{b}_0 \tag{13}$$

$$\sum_{j=1}^{m_0+n} \lambda_{ij} + \lambda_i^* = 1, \quad i = 1, 2, ..., n \tag{14}$$

$$\lambda_{ij} \geq 0, \quad \lambda_i^* \geq 0 \tag{15}$$

where m_0 is the number of linking constraints in (1). The above problem has n more variables than constraints, rather than one more as in the single-column generation case. If we use the size of the coefficient matrix in Simplex standard form to represent the size of the problem, the RMP with multi-column generation has a size of $(m_0 + n) \times (m_0 + n + n)$ while the RMP with single-column generation has a size $(m_0 + n) \times (m_0 + n + 1)$. One would expect a greater decrease in z_3 through every iteration, and thus significantly reduce the number of iterations.

It has been discussed in Lasdon (2002) and verified by our direct experience that the advantage of having more columns in the RMP outweighs the disadvantage of increased RMP size. This is investigated in the first case study.

In addition, it is also noticed that, within the scheme described above, redundant columns may be generated for the subproblems with unfavorable proposals at some iterations. When the number of subproblems is large, the number of redundant columns in the RMP may be large, which will reduce the efficiency of the coordination mechanism. To develop a more efficient multi-column generation strategy, instead of generating a new column for each subproblem, we only generate columns for subproblems with strictly negative reduced costs. Therefore, the number of columns in the RMP will be a number between (m_0+n+1) and $(m_0 + n + n)$, or:

$$\min \quad z_3 = \sum_{i=1}^{n} \sum_{j=1}^{m_0+n} f_{ij}\lambda_{ij} + \sum_{k=1}^{K} f_k^* \lambda_k^*$$

$$\text{s.t.} \quad \sum_{i=1}^{n} \sum_{j=1}^{m_0+n} \mathbf{p}_{ij}\lambda_{ij} + \sum_{k=1}^{K} \mathbf{p}_k^* \lambda_k^* = \mathbf{b}_0 \tag{16}$$

$$\sum_{j=1}^{m_0+n} \lambda_{ij} + \lambda_k^* = 1, \quad i = 1, 2, ..., n \tag{17}$$

$$\lambda_{ij} \geq 0, \quad \lambda_i^* \geq 0, \quad 0 \leq K \leq n \tag{18}$$

where k in (16) is associated with the corresponding subproblem. Without loss of information, the new formulation of the RMP can have fewer columns than in (13) and (14), and thus may save much computational effort. Such an algorithm is under development, and is the focus of future work.

3. COORDINATION IN PLANT-WIDE MPC

3.1 General Idea

Typically, in a process plant, the overall process model used by MPC has a block-wise structure. Therefore, most process plants are potential candidates for the application of Dantzig-Wolfe decomposition to coordinate independent MPC calculation. A coordinator can be designed by considering different kinds of interactions among operating units, which can be formulated as the linking constraints. Since we are going to follow the coordinator-individuals working mode, a number of challenges still remain in synthesizing efficient coordination, among which two key factors are: (i) computational efficiency of the coordination mechanism; and (ii) information flow load throughout the communication network in a plant.

In this work, we focus on the design of the coordination mechanism for steady-state target calculation in MPC systems. The target calculation problem commonly takes the form of an LP or a QP problem (Kassmann *et al.*, May 2000; Rao and Rawlings, 1999). Here, we assume an LP formulation for the target optimization.

3.2 Constructing Messages for Coordination

In this section, we are going to discuss a method of *off-diagonal message abstraction* for coordination system design. Here the term message refers to the information that is available, but cannot be used by current decentralized MPC. The abstracted message will be used to design the coordinator.

Quite often, advanced control strategies are designed and installed at different times for different operating units. For example, in a copper ore concentrator plant, model-based and model-assisted APC controllers were separately implemented in the crushing, grinding and flotation sections (Herbst and Pate, 1996). In this case, certain controlled variables (CVs) and manipulated variables (MVs) have been specified and paired in a unit-wide sense. These unit-wide implementa-

$$\mathbf{G} = \begin{bmatrix} \mathbf{G1} & \text{G12} & \text{G13} \\ \text{G21} & \mathbf{G2} & \text{G23} \\ \text{G31} & \text{G32} & \mathbf{G3} \end{bmatrix}$$

Fig. 2. A plant-wide process model

tion of APC strategies can only use the block-diagonal information of the plant-wide model. For instance, assuming a full plant-wide process model $\mathbf{G}$ is available through identification, three existing decentralized MPC systems are designed based on block-diagonal elements $\mathbf{G}_1$, $\mathbf{G}_2$, and $\mathbf{G}_3$ shown in Figure 2. Thus the off-diagonal elements $\mathbf{G}_{ij}$, as one representation of interaction, are not considered. Ignoring the off-diagonal information can cause significant loss in performance.

Given the coordination mechanism of Dantzig-Wolfe decomposition, if we can abstract off-diagonal messages from the overall model for the design of linking constraints, the coordination only requires minor modifications to the objective function and constraints of each decentralized MPC problem. Assume in the i^{th} decentralized MPC system the LP-based target calculation has the form:

$$\begin{aligned} \min \quad & c_i^T X_i \\ \text{s.t.} \quad & Y_i = \mathbf{G}_i U_i + E^i && (19) \\ & U_{lb}^i \le U_i \le U_{ub}^i && (20) \\ & Y_{lb}^i \le Y_i \le Y_{lb}^i && (21) \end{aligned}$$

where U_i and Y_i are the deviation variables of MVs and CVs, respectively. X_i contains vector U_i and Y_i. Vector E^i represents the unmeasured disturbances. U_{lb}^i and Y_{lb}^i are the lower bounds, while U_{ub}^i and Y_{ub}^i are the upper bounds for the decision variables. To design the coordinator, we can model the interaction as :

$$E_m^i = \sum_{i \neq j} G_{ij} U_j \qquad (22)$$

which will be Incorporated into the coordination problem as the linking constraints introduced in (1). This approach requires minor modifications to the existing decentralized control systems by adding the *measured disturbance term* E_m^i to each individual model shown in (23).

$$Y_i = G_i U_i + E_m^i + E^i \qquad (23)$$

Thus the decision variables of each individual MPC can be augmented as $[U_i, Y_i, E_m^i]$. We can treat E_i as an unrestricted variable or impose rather loose bounds on it. Only the original CVs or MVs will be implemented when the optimization is accomplished. Furthermore, the objective function of each MPC system gets dynamically updated based on the sensitivity information $[\pi, \gamma]$ from the coordinator which solves a master problem.

The coordination scheme is particularly computationally efficient for coordinating control systems using a plant-wide model with sparse off-diagonal matrices.

3.3 Algorithmic Coordination Strategies

Since we have discussed the method to construct useful messages that can be used for plant-wide coordination of decentralized MPC, together with our previous discussion on the Dantzig-Wolfe decomposition algorithms, we may be able to draw a complete picture of the proposed coordination scheme. Reduced to the simplest form, when we focus on MPC target calculation, the proposed coordination strategy is :

(1) Obtain existing decentralized MPC configurations:

$$MPC_i = \{\min\ z_i = \mathbf{c}_i^T \mathbf{x}_i \mid \mathbf{B}_i \mathbf{x}_i = \mathbf{b}_i\};$$

(2) Message construction for coordination and linking constraints modeling: $\{\mathbf{A}_i \mathbf{x}_i = \mathbf{b}_0\}$;

(3) Modifications to existing decentralized MPC local constraints, if necessary,

$$\{\mathbf{B}_i \leftarrow \mathbf{B}_i \cup \mathbf{E}_m^i\};$$

(4) Online **coordination**:
INI : $[\pi, \gamma] = \mathbf{0}, [f_i, \mathbf{u}_i] = arg\{MPC_i\}$;
WHILE any $f_i \le 0$
col $\leftarrow$ column generation $(f_i, \mathbf{u}_i, \mathbf{A}_i)$;
$[\pi, \gamma, \lambda] \leftarrow arg\{RMP(col, \mathbf{u}_i)\}$;
$[f_i, \mathbf{u}_i] \leftarrow arg\{MPC_i(\pi, \gamma)\}$; (In Parallel)
END

(5) Generate the plant-wide optimal solution:

$$[CV_i, MV_i] \leftarrow X_i \leftarrow [\mathbf{u}_i, \lambda_i]$$

4. ILLUSTRATIVE CASE STUDIES

4.1 Case 1: Efficiency of Decomposition Algorithm

To investigate the problem size scaling behavior of the proposed coordination strategy, a set of Monte

Carlo simulations have been carried out as a step toward understanding how the decomposition algorithm will perform for large-scale LP problems.

In our experiments, several sets of test LP problems are constructed by randomly generating elements for the coefficients such that all of problems have optimal solutions. The problem size reported is determined as the coefficient matrix dimension in the centralized LP. In the centralized scheme, we use MATLAB's "*linprog*" large-scale algorithm, which employs an interior point method, to solve the LP problems; while in the decentralized scheme, the Dantzig-Wolfe decomposition algorithms are applied.

During the course of constructing LP problems, we follow the structure of Dantzig-Wolfe decomposition. Firstly generate n feasible subproblems of dimension $m \times N$, and then construct m_0 linking constraints where $\mathbf{A}$ has a dimension of $m_0 \times (N \times n)$. Thus we have a problem of dimension:

$$\begin{aligned} & m_0 \times (N \times n) + (m \times n) \times (N \times n) \\ & = (m_0 + m \times n) \times (N \times n) \end{aligned} \quad (24)$$

Note that, we generate subproblems of identical sizes to balance the computational effort.

Five groups of Monte Carlo simulations have been done, and the computational effort comparisons are reported in Table 1 and Figure 3. In the simulations, we increase the number of subproblems indicating that we can integrate more decentralized control systems into the coordinated system. The computational time for the decomposition algorithms is estimated by summing up the time for solving the master problem and the most time-consuming subproblem, assuming a distributed computational environment.

Table 1. Computational Effort

Problem Size	Central	Coordinated (s)	
$[m_0, m, N] = [20, 30, 50]$	(s)	Single	Multi
$n = 10$	1.364	5.034	3.608
$n = 20$	3.263	8.289	4.389
$n = 30$	5.791	7.708	2.273
$n = 40$	8.757	7.622	1.570
$n = 50$	12.51	9.792	1.546

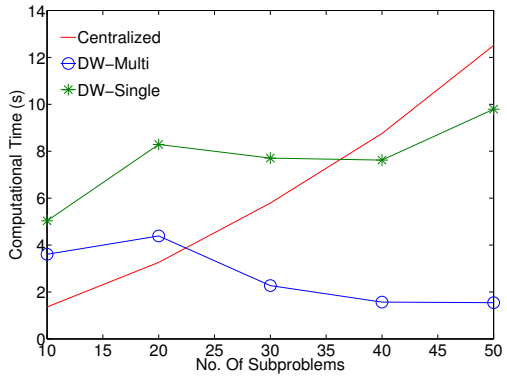

Fig. 3. Computational effort comparison

Evidently, compared with the single-column generation, the multi-column generation technique leads to a significant reduction in the computational time of the decomposition algorithm, and thus a more efficient coordination mechanism. In addition, we also can notice that the computational time for the centralized scheme increases monotonically with the problem size, while the decomposition algorithms have good scaling behavior, which shows the decomposition and coordination algorithms can be suitable for solving large-scale problems.

4.2 Case 2: Method of Interaction Abstraction

This case study is carried out to illustrate the application of the message construction method for coordinating decentralized MPC. Shown in Figure 4, a generic process network is used for this case study. Described as follows, the overall process network can be represented by an 8-input and 6-output model $\mathbf{G}$, which is a linearization of the process around the operating point $[\mathbf{y}_0, \mathbf{u}_0]$:

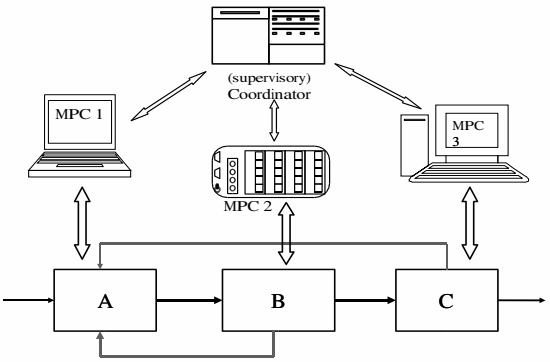

Fig. 4. An interacting MIMO unit network

$$\begin{aligned} y_0 &= [5, 3, 4, 2, 8, 10]; \\ u_0 &= [0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5]; \end{aligned} \quad (25)$$

$$G_0^T = \begin{pmatrix} -0.88 & 1.49 & 0 & -2.36 & 0 & -1.4 \\ 1.13 & -0.5 & 0 & 0.24 & 0 & -0.26 \\ 1.49 & 2.59 & 0 & -1.19 & 0 & 0.77 \\ 0 & 0.55 & -0.42 & -0.32 & 0 & 1.48 \\ 0 & 3.03 & 0.4 & -0.97 & 0 & 1.12 \\ 0 & 2.56 & 0 & 0 & -0.25 & 0.06 \\ 0 & 0.66 & 0 & 0 & -2.1 & -0.55 \\ 0 & 0.29 & 0 & 0 & -0.28 & -0.61 \end{pmatrix}$$

where G_0 is the steady-state gain matrix of the process model G. The flowsheet was originally decomposed into three operating units, each of which has two output variables. Further, unit

A and unit C have three manipulated variables, while unit B has two. Each operating unit has its own objective, which is a subset of information used by plant-wide optimizers. In this maximization problem, the profit function cost coefficients are

$$c_A^T = \begin{bmatrix} 2 & 3 & 0 & 0 & 0 \end{bmatrix} \quad c_B^T = \begin{bmatrix} 1 & 3 & 0 & 0 \end{bmatrix}$$
$$c_C^T = \begin{bmatrix} 4 & 7 & 0 & 0 & 0 \end{bmatrix} \tag{26}$$

where the objective functions are only related to output variables.

The decentralized MPC controllers use incomplete process information and ignore the interactions. Using *off-diagonal message abstraction* method, we can model the interactions as the measured disturbances and augment them in the model of each decentralized MPC system as discussed in (23). Note that, since the off-diagonal block matrices are relatively sparse, the dimension of E_m is only three. In this case study, we set the upper and lower bounds of the decision variables within $\pm 10\%$ interval of the nominal operating point, while treating the measured disturbance variables as unrestricted variables.

Within this configuration, we can specify an information package which flows on the communication network that connects the coordinator and individual control systems. Similar to Figure 1, at every coordination round, each decentralized MPC system submits to the coordinator a unit-wide optimal solution $X_i = [Y_i, U_i, E_m^i]$ and corresponding objective function value f_i. As soon as it receives all the proposals, the coordinator executes a linear program to solve the master problem. Then the coordinator records the solution λ_{ij} and sends sensitivity information $[\pi, \gamma]$ to the decentralized MPC. Here, π is related to the measured disturbance E_m^i, which can reflect the gap between the plant-wide optimal solution and unit-wide solution. In other words, when an optimal E_m^i is obtained through coordination, the plant-wide optimum is reached. Note that the information flow on the communication network is not heavy and real-time communication is quite possible.

To simplify the discussion in our case study, we assume accurate modeling and noise-free simulation. Therefore, in terms of plant-wide optimum, a centralized controller provides benchmark performance. With the above information, one execution of plant-wide target calculation is performed with three optimization schemes, the *centralized, decentralized, and coordinated* schemes, respectively. The following tables provide simulation results for comparison. Table 2 compares the performance of different control strategies. We can see that the centralized and the coordinated target calculation schemes give the same achievable profit as the benchmark optimum, while the fully decentralized target calculation only captures around 96.56% of the maximum profit.

Table 2. Performance Comparison

Strategy	Profit	Achievability Ratio	Computational Effort (s)
Central	134.674	100	0.0359
Decentral	130.035	96.56	0.0304
Coordin	134.674	100	0.1990

*All the simulations were performed in Matlab 7.0 on a Pentium III 1.0G Hz, 512M RAM machine.

The computational effort is also reported in Table 2 for completeness. Here the coordinated scheme uses the multi-column generation based Dantzig-Wolfe decomposition. The method used to evaluate the computational efficiency has been discussed in case study 1. We would like to point out that, since the problem size in this case study is very small, the decomposition/coordination strategy may not outperform the centralized scheme. This is consistent with our previous discussion in computational efficiency study. When the number and the size of subproblems are very large, which is common in industry, the centralized problem could be very large and require significant computation.

In addition, we also can obtain information from the simulation for simple comparison between the two column generation schemes. Computational times spent on the decentralized MPC (T-Sub) and the coordination (T-RMP), as well as the number of iterations to complete coordination, are reported in Table 3. One purpose to introduce Table 3 is associated with an implementation issue. Note that the computational time spent on subproblems has already been significantly reduced by a large reduction of the iteration number (more than 50%) for coordination. Keep in mind that we are improving the plant-wide control system performance with minimum modification to the existing systems. A more powerful computing resource or a better solver could be used to reduce the computational time for solving the master problem, without expensive upgrading of computing for current decentralized MPC controllers.

Table 3. Single vs Multi-col. Generation

Column Generation	T-total	T-Sub	T-RMP	Iteration Number
Single	0.199	0.187	0.012	17
Multiple	0.185	0.084	0.101	8

A key point drawn from this study is that the proposed approach may require far less capital investment to gain equal performance increases, in comparison to implementation of a new centralized, plant-wide MPC. As such, it provides an approach to plant-wide control that does not require centralized computing environment.

5. CONCLUSION AND DISCUSSION

Industrial practice has revealed the deficiencies of existing decentralized MPC systems in finding plant-wide optimal operations. Using a coordinator with decentralized controllers can address these issues. This work introduces a novel approach to coordinating decentralized MPC target calculation by taking advantage of the Dantzig-Wolfe decomposition algorithms. It also proposes a message construction method for coordination system design, in which the constraints associated with multiple units can be Incorporated.

Our work shows that the proposed coordinated target calculation scheme substantially improves the performance of the existing decentralized control scheme, while it can utilize decentralized computing environment to ensure acceptable real-time calculation speeds. The proposed scheme only requires a minor modification to the existing decentralized MPC structure. In addition, we have verified that the multi-column generation technique significantly improves the computational efficiency of Dantzig-Wolfe decomposition, which makes promising the industrial applications of the coordinated scheme.

A number of challenges remain. One issue that should be investigated is how to integrate subsystems to ensure high performance with minimal computation, e.g., for the decomposition should one balance the computational load of each subproblem. In addition, a full understanding of the complexity analysis of Dantzig-Wolfe decomposition is desired to determine the efficacy of the proposed approach on industrial-scale problems.

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