An Approach to the Nonlinear Control of Rolling Mills

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1 Abstract

The dynamic equations describing a disturbance free single stand rolling mill may be summarized equation 1

$$\begin{array}{ll} \dot{x}_1 & g(\theta, x_1)u_1 \\ \dot{x}_2 & k_1x_1 - k_2x_3 + k_3 \\ \dot{x}_3 = & b_0x_2 + a_0u_2 + k_4 \\ \dot{x}_4 & -k_5x_1 + \beta x_1x_5 + k_6x_5 - k_7 \\ \dot{x}_5 & d_0x_4 + c_0u_3 + k_8 \end{array} \tag{1}$$

with $\mathbf{x}$ the state, $\mathbf{u}$ the input vector, $g(\theta, x_1)$ a nonlinear function, and everything else a constant. This paper applies results explained in [1] and [2]. Clearly equation 1 may be considered a cascade of three subsystems, namely the $\dot{x}_1$, the $\dot{x}_2, \dot{x}_3$ and the $\dot{x}_4, \dot{x}_5$ subsystems. These cascades may be stabilised individually, with the hope that interconnected individually stabilized cascades would be stable. Indeed assuming an invertible $g(\theta, x_1)$ the control law $u_1 = -x_1/g(\theta, x_1)$ stabilizes the first subsystem. For the rolling mill

$$g(\theta, x_1) = \frac{1}{1 + \frac{\theta}{\sqrt{(c_1 - x_1)}}} \tag{2}$$

with θ is unknown. This paper will show that an approximate adaption law is able to give satisfactory results.

Assuming x_1 has been stabilised the remaining dynamics for the $\dot{x}_2, \dot{x}_3$ subsystem may be written

$$\begin{array}{c} \dot{x}_2 \\ \dot{x}_3 \end{array} = \begin{array}{c} -k_2x_3 + k_3 \\ b_0x_2 + a_0u_2 + k_4 \end{array} \tag{3}$$

With the substitution $\hat{u} = a_0u_2 + k_4$ we may write

$$\begin{array}{c} \dot{x}_2 \\ \dot{x}_3 \end{array} = \begin{array}{c} -k_2x_3 + k_3 \\ b_0x_2 + \hat{u}_2 \end{array} \tag{4}$$

realising a lower triangular subsystem that is stabilisable via backstepping. Any lower triangular system may be summarised

$$\begin{array}{c} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_n \end{array} = \begin{array}{c} f_1(x_1x_2) \\ f_2(x_1, x_2, x_3) \\ \vdots \\ f_n(x_1, x_2, ...x_n, u) \end{array} \tag{5}$$

The backstepping procedure begins with the construction of a control Lyapunov function (clf), for example in this case $V_1 = \frac{1}{2}x_2^2$. Its derivative $\dot{V}_1 = x_2\dot{x}_2 = x_2(-k_2x_3 + k_3)$ may be rendered negative definite to make the dynamics of x_2 globally asymptotically stable. A choice of $V_1 = -x_2^2$ and substution is sufficient to compute a desired psuedo (or virtual) control $x_{3des} = (k_3 + x_2)/k_2$. If k_2 and k_3 are known, a straighforward application is viable, whereas if they are unknown but bounded to known bounds, robustifying control laws based on domination are effective. At this expository stage we assume that k_2 and k_3 are known. The control for the augmented system $\hat{u}_2$ may be computed as follows; assume that x_3 deviates from x_{3des} by z_1. We may then write

$$x_3 = x_{3des} + z_1 = (k_3 + x_2)/k_2 + z_1 \tag{6}$$

Introducing a new composite clf $V_2 = \frac{x_2^2}{2} + \frac{z_1^2}{2}$ and rendering its derivative negative definite $\dot{V}_2 = -x_2^2 - z_1^2$, one may then compute a suitable control law

$$\hat{u}_2 = -2x_{3des} + z_1 = -2k_3 + (k_2 + 1/k_2 - b_0)x_2 + 2k_3/k_2 \tag{7}$$

The treatment above gives a flavour of the methods available for nonlinear control; it is true a linear method could have stabilised the $\dot{x}_2, \dot{x}_3$ or $\dot{x}_4, \dot{x}_5$ subsystems, and that approach of combining nonlinear and linear methods for stabilisation is not unusual. In this paper we demonstrate that methods based on passivation (of which backstepping is an instance) are viable for the achievement of integrated solution to the control problem. This paper focuses on the single stand with a persistant sinusoidal disturbance. We show how to deal with time delay, unknown parameters, and nonlinearity. Simulation results of the resultant controllers are shown.

References

[1] M. J. R. Sepulchre and P. Kokotovic, *Constructive Nonlinear Control.* Springer, 1997.

[2] M. Krstic and P. K. I. Kanellakopoulos, *Nonlinear and Adaptive Control Design.* Prentice-Hall, 1991.