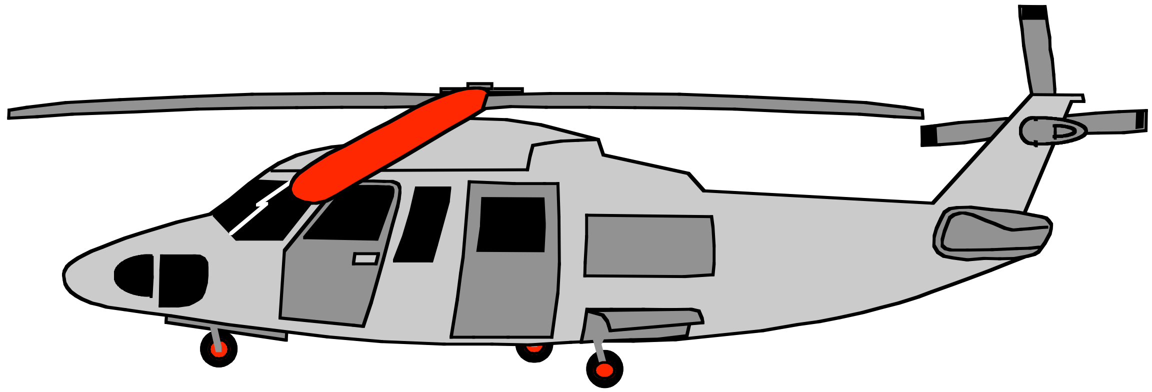


Identification for Control

A rather extreme example in helicopter vibration control

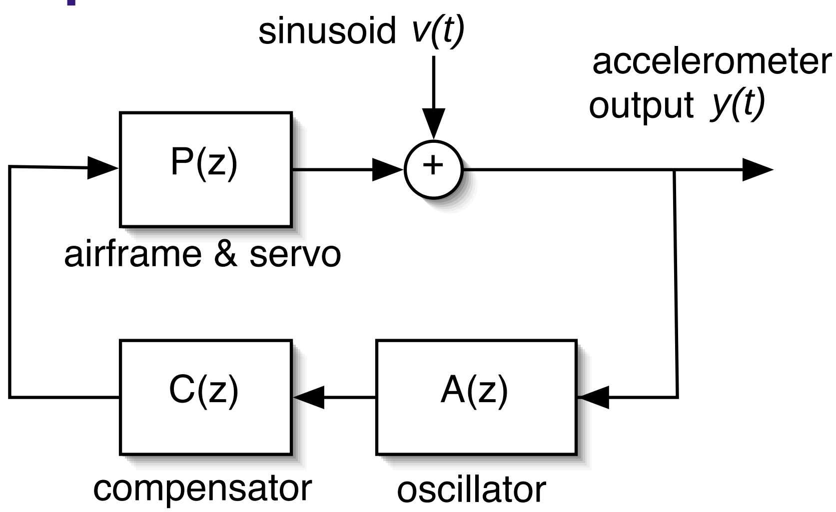
Problem statement

- Vibration from rotors leads to pilot fatigue
  - Passive vibration damping system needs active control
    - Feedback from accelerometers near pilot to active dampers
      - Hydraulic system coupled with spring plates
        - Under-actuated but expect improvement over passive control

System information a priori

- Range of vibration frequencies is very limited 17Hz ±5%
  - An under-damped mode lies around 40Hz
    - System is stable in open-loop
      - No control produces no craziness
    - The vibration frequency changes only slightly in flight
      - The airframe dynamics change with configuration and load
- Look for a stable model with a simple parametrization
  - Control design will use open-loop stability - low gain solution
    - Adaptive solution for changing dynamics and frequency
      - Slow changes

The putative control solution



$$y(t) = [1 + PCA]^{-1} v(t)$$

Performance: need *1+PCA* very large at vibration frequency 17Hz

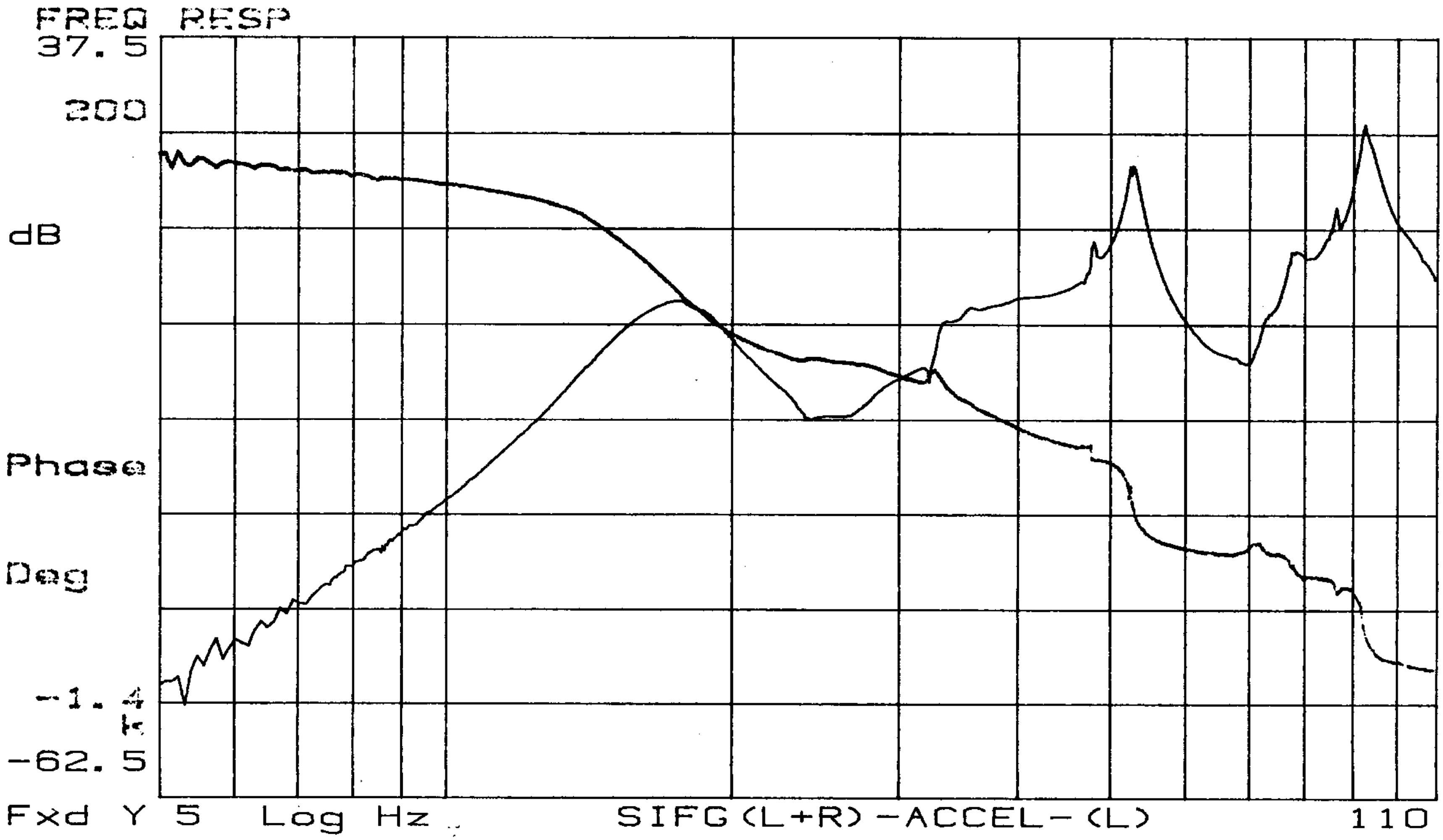
- Need *CA* small near the unmodeled resonance at 40Hz
  - Robust stability given by *CA* almost zero
    - Look for a perturbation on *CA=0*
      - *A(z)* an oscillator
      - *C(z)* a phase compensator

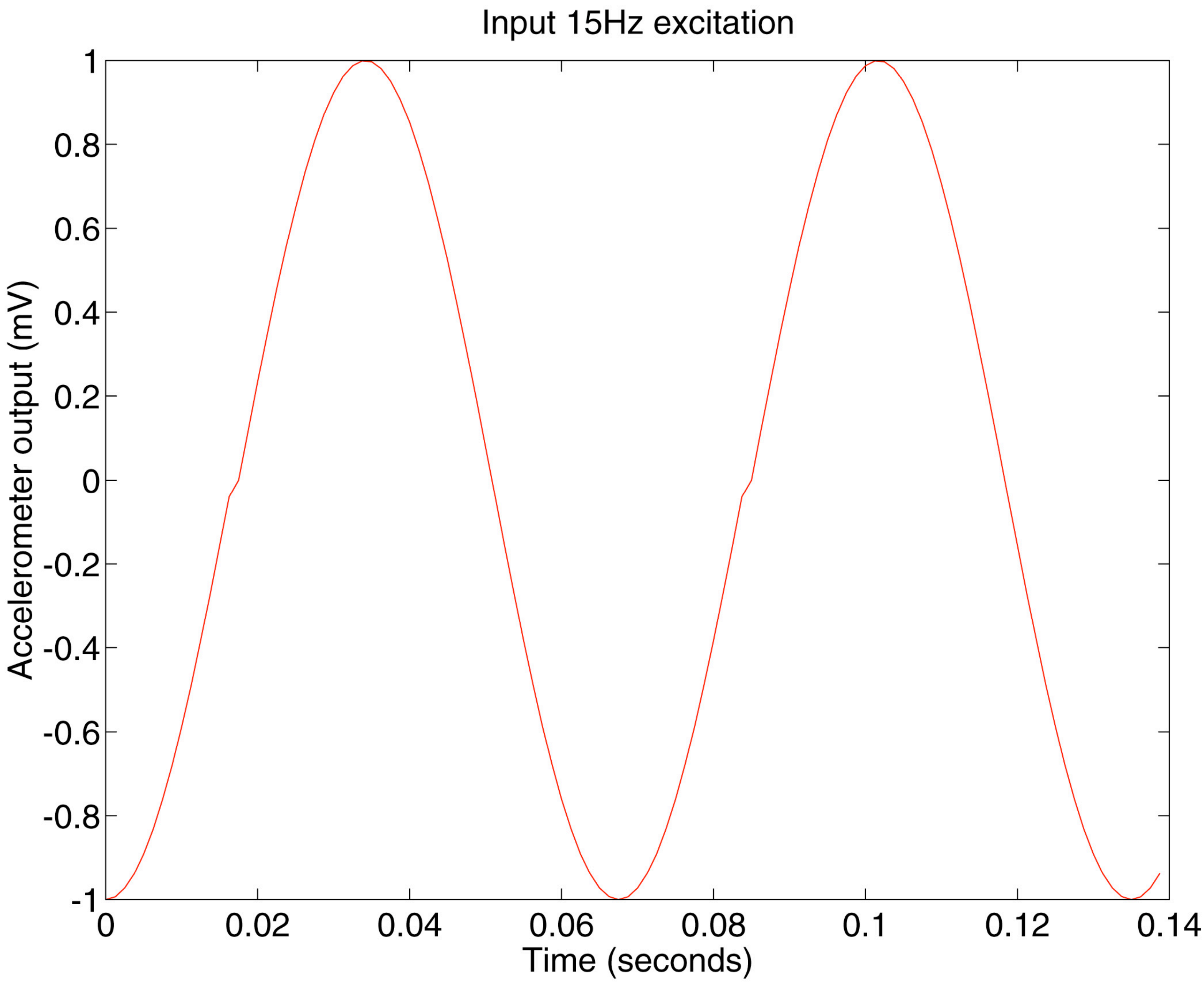
Stability tied to the phase of *C(z)* at the oscillation frequency

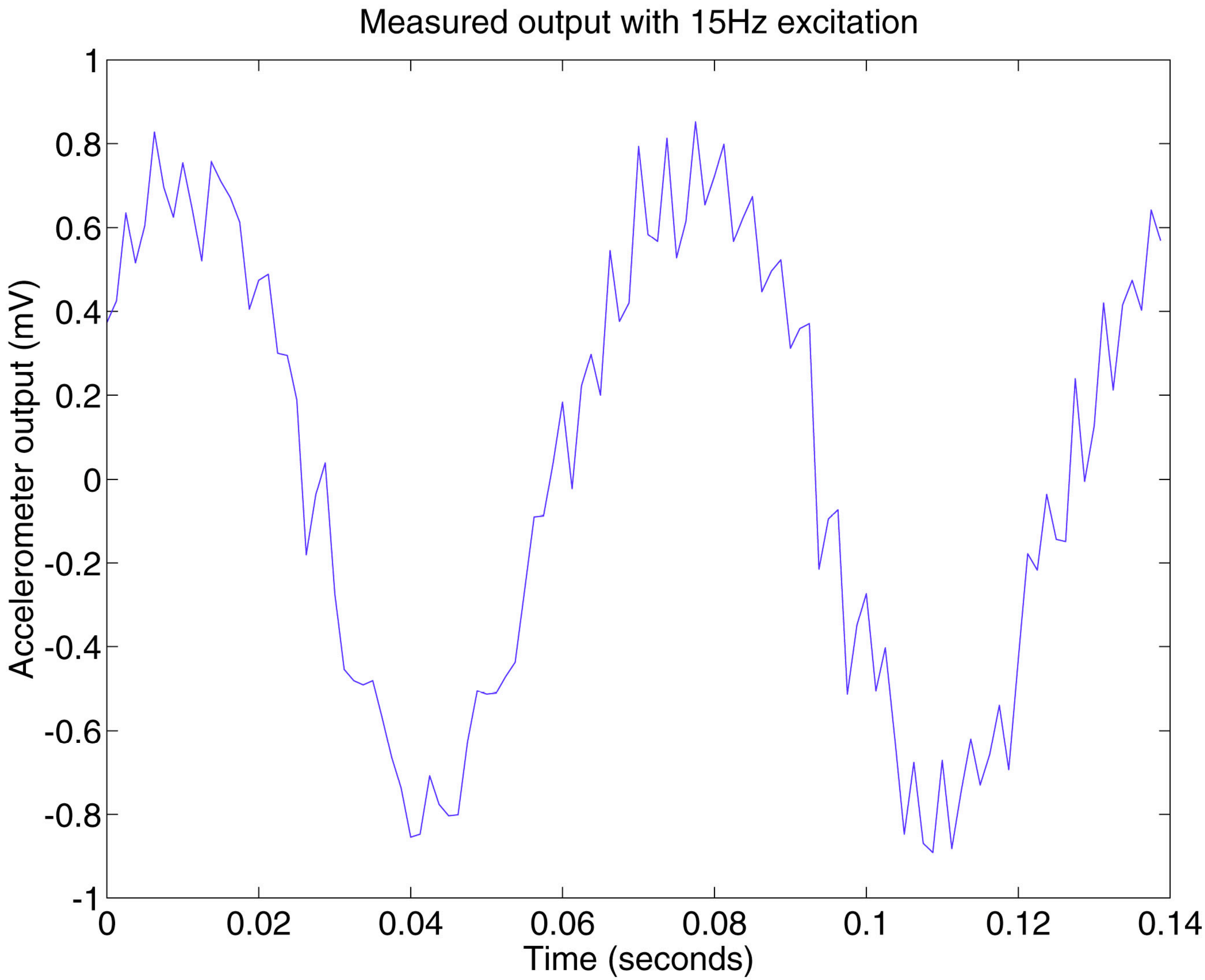
Model requirements

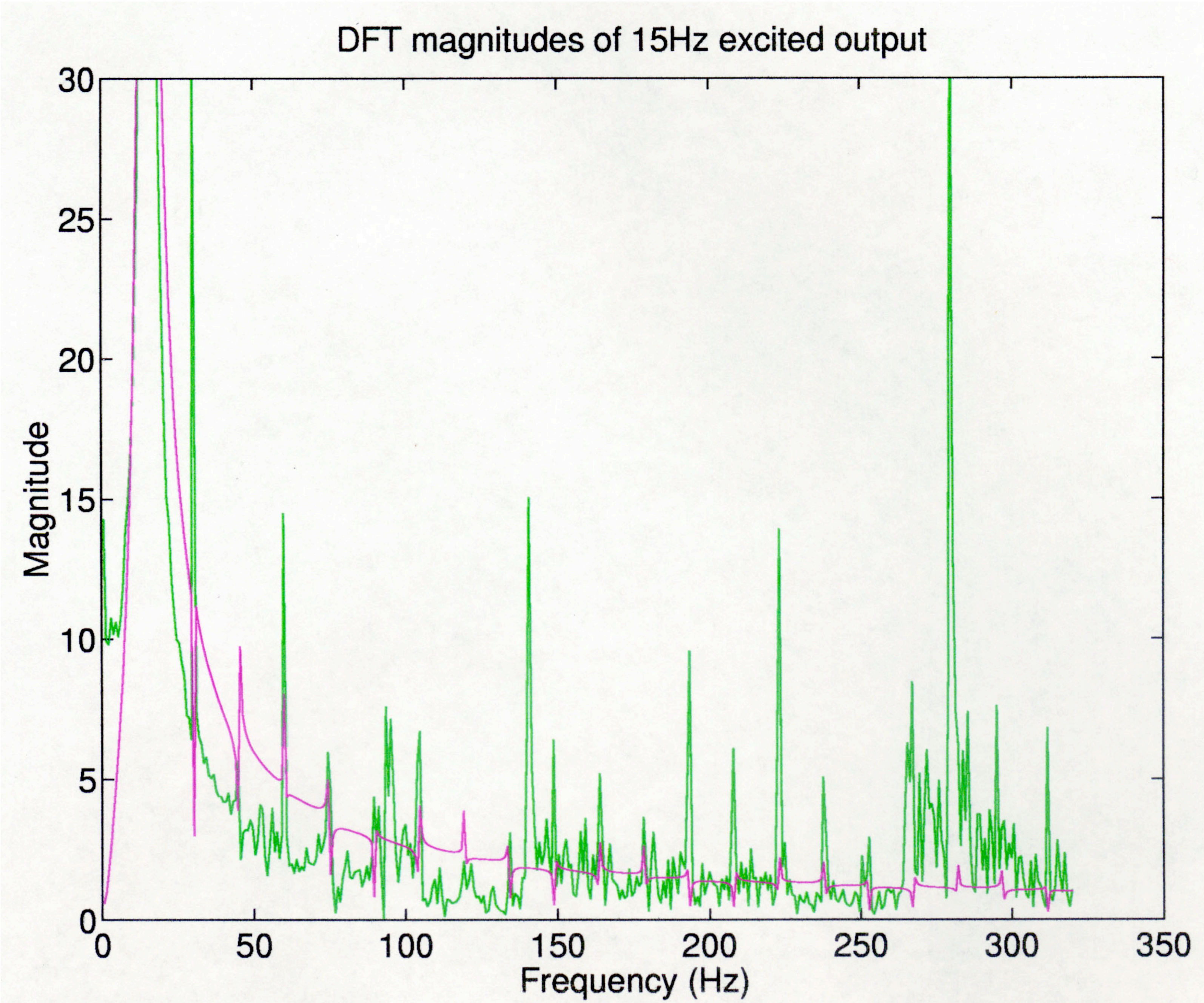
- Low-order simple model
  - High frequency detail managed by the controller not modeling
    - Parametric model is good foe adaptation
      - Want few parameters to estimate
        - Data is very sinusoidal - not informative
- The model should be stable just like the real plant
  - A low-gain control strategy is adopted
    - Matches the robust stability approach
- The fit in the neighborhood of 15Hz to 20Hz is most important
  - Actuation incapable of addressing other modes
    - Accommodation of model mismatch by the controller

Non-parametric model data




Input 15Hz excitation
Accelerometer output (mV)
Time (seconds)
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14


Measured output with 15Hz excitation
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
Accelerometer output (mV)
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
Time (seconds)


DFT magnitudes of 15Hz excited output
Magnitude
Frequency (Hz)
0
5
10
15
20
25
30
0
50
100
150
200
250
300
350

Cleaning the data - filtering

$$\hat{P}(e^{j\omega_i}) = \frac{\sum_{k=1}^{1000} y_{i,k} \exp\left(j\omega_i k/T\right)}{\sum_{k=1}^{1000} u_{i,k} \exp\left(j\omega_i k/T\right)}$$

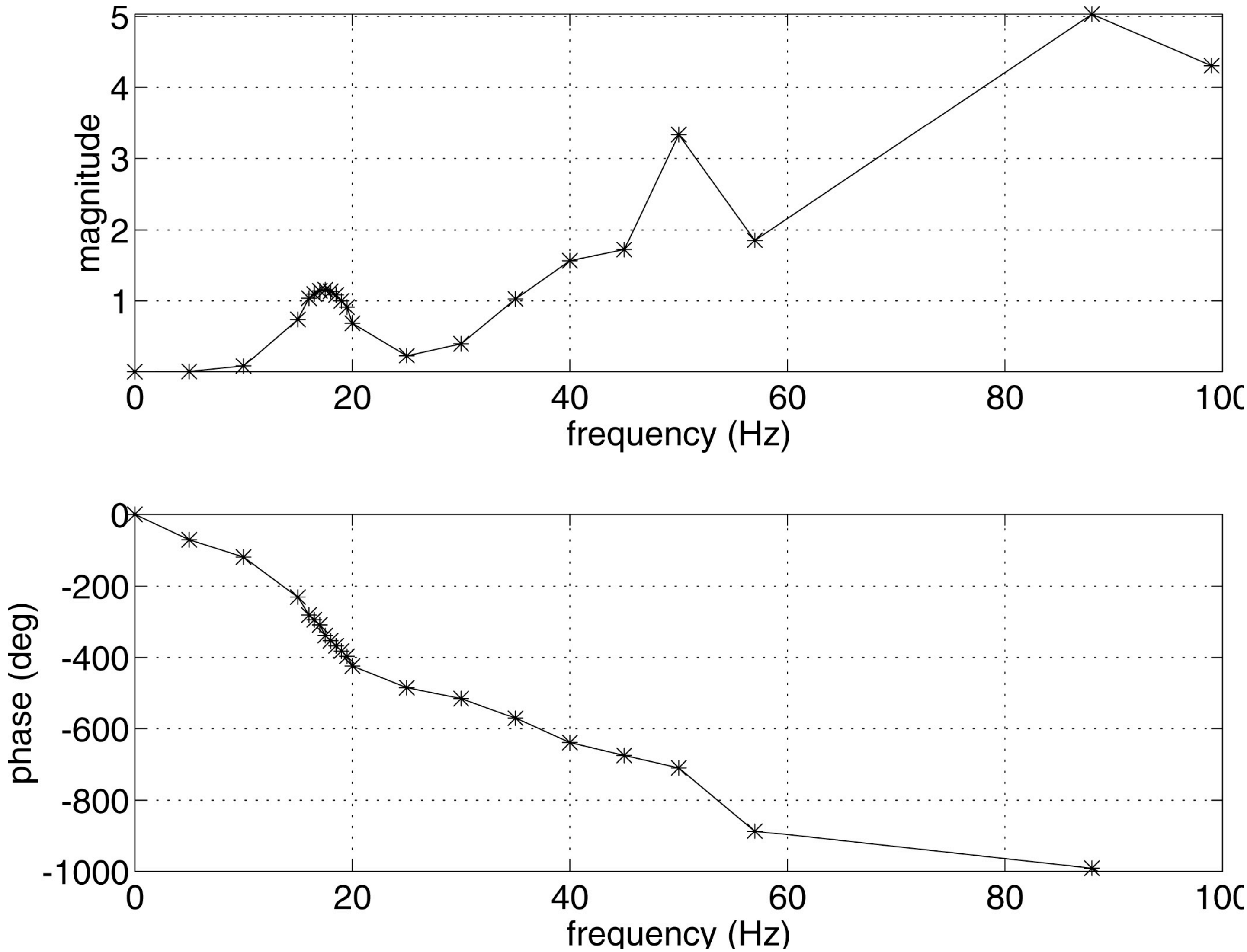
Sinusoidal experimental data collected at 21 frequencies

Input sinusoid contains some small harmonic distortion

- Accelerometer signal contains very significant harmonic distortion
  - Pass both through a very narrow-band-pass filter
    - This is achieved with the above computations
      - This is akin to the discrete Fourier transform
  - Divide the the results to get the frequency response values

21 complex frequency response estimates

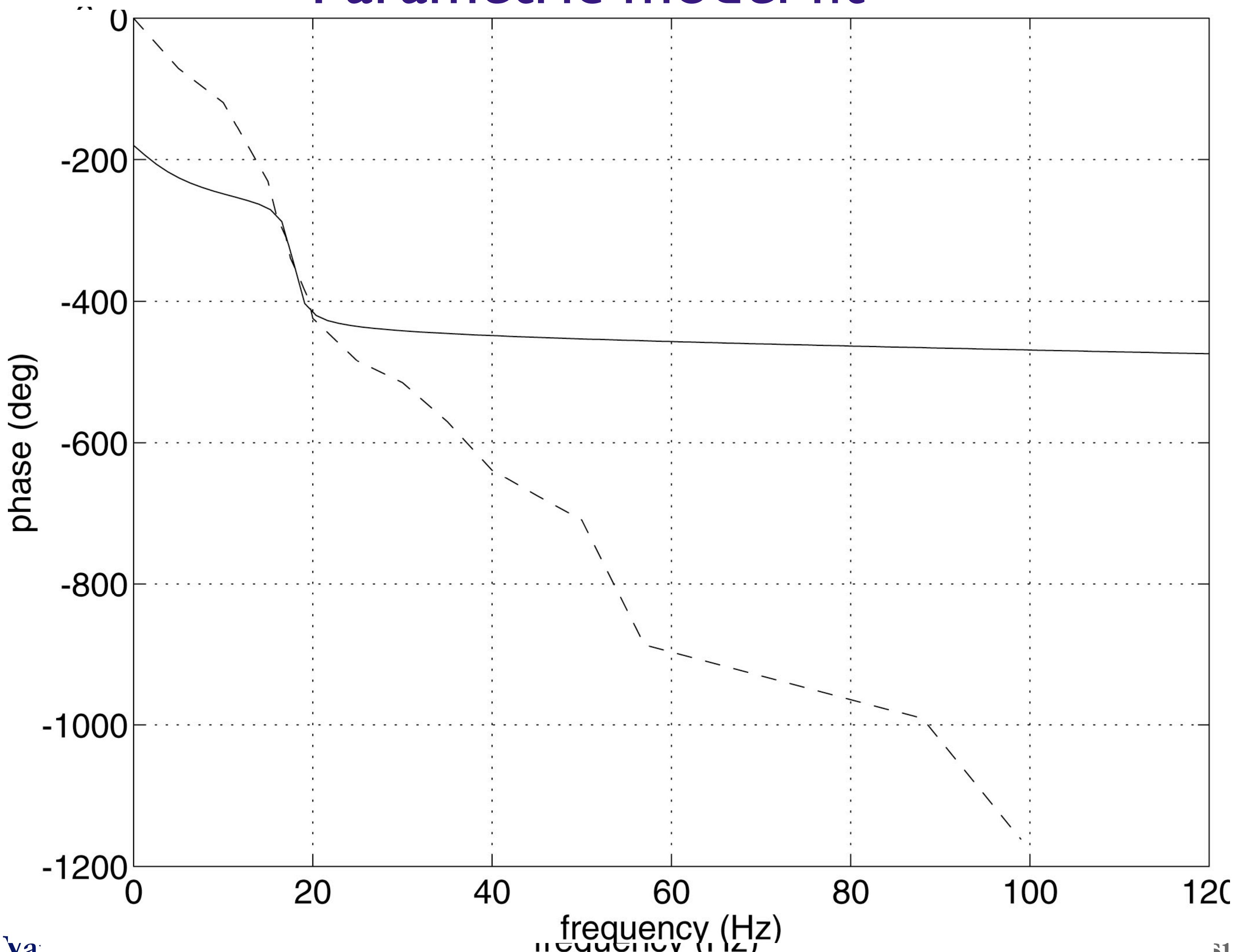
Estimated frequency response



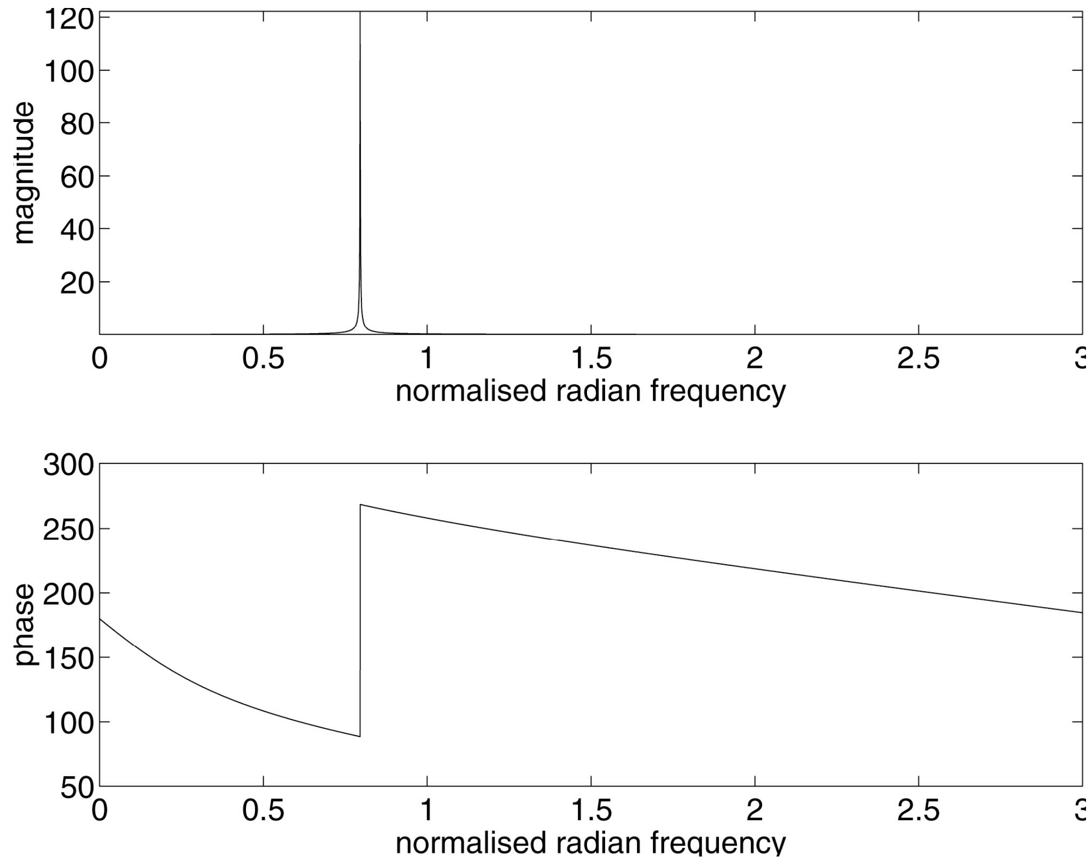
Parametric model development

- Start with 21 frequency response values centered around 17Hz
  - Fit a stable low-order model to these values
    - Direct frequency-domain fit
      - Difficult to guarantee stability
        - We actually want a weighted fit to emphasize 17Hz
    - Use the estimated frequency response values to cook up some fake data with the correct frequency distribution.
      - Fit the model using this data record and Output Error model structure
        - Guaranteed stable

Parametric model fit

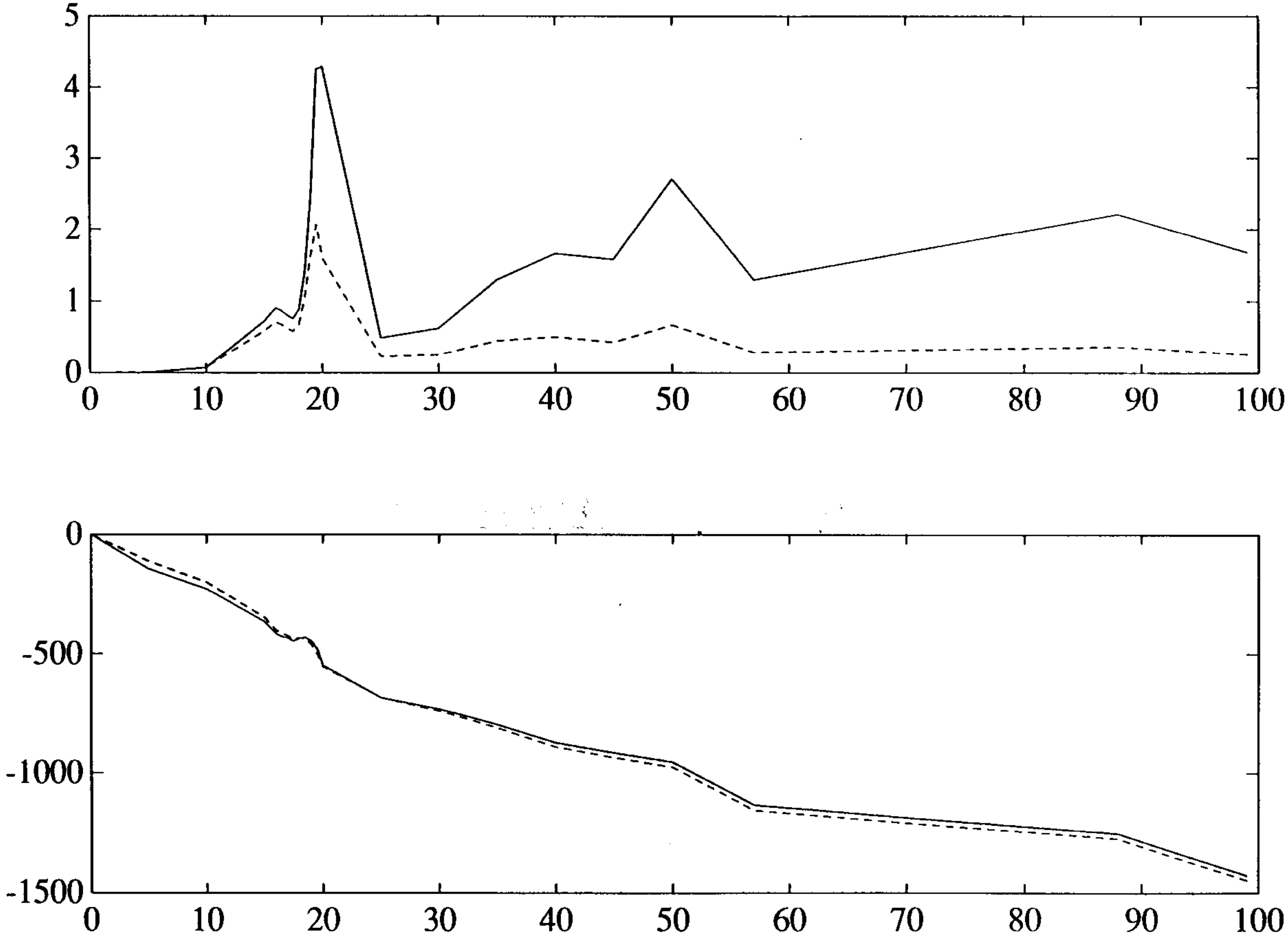


Model-based control design



Frequency-weighted Linear Quadratic Gaussian Control for disturbance rejection

Controller robustness check



Conclusion

The problem specification and our prejudice about the solution have colored the whole modeling and control design

- Modeling has reflected the robust control requirements
  - Experiment design
    - Data concentrated where phase accuracy needed
    - Sinusoidal excitation avoided rate limiters
  - Data preparation before modeling
    - Harmonic distortion needed to be removed via filtering
  - Parametric model has low order and is stable
    - Heavy frequency weighting in model fit stage
- Control design
  - Frequency weighted against model mismatch at HF
  - Don’t care at LF