Modeling from data: Physics + System Identification

plus a little Philosophy

Key Ideas

Models should be in a useful form

- Linear system difference equations are good for design
  - Linear system plus memoryless nonlinearities OK too
    - Interconnections of simple components

Simplicity is a major goal

- Occam's razor, parsimony or even simpler
  - Complexity hurts us downstream

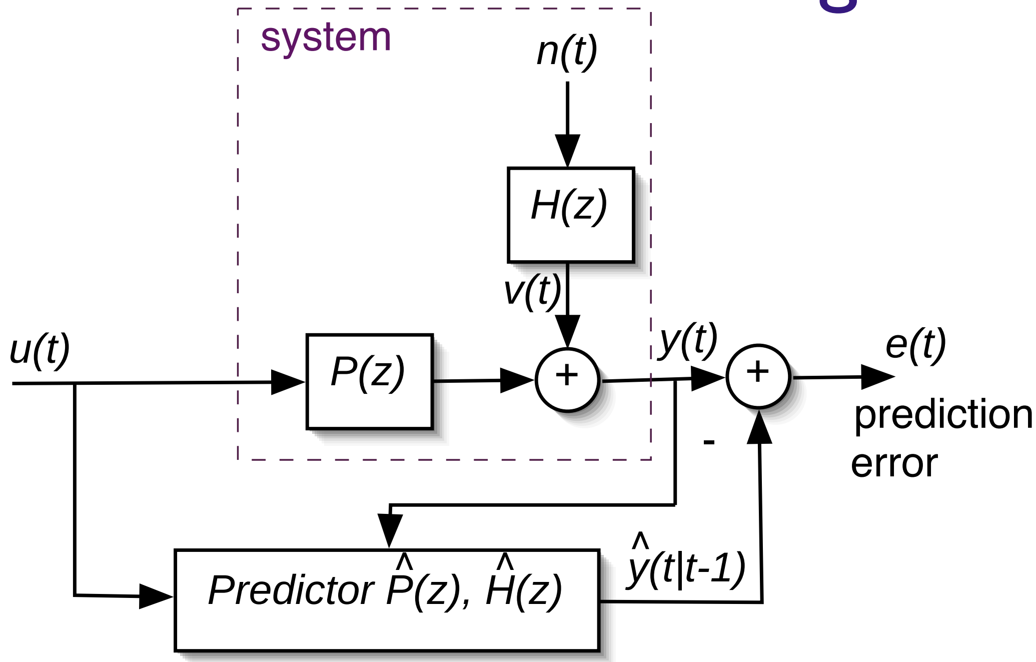
Approximation is a necessity and is desirable

- No exact match is possible

Characterize model performance in a sensible way

- Try to reflect the ultimate model usage

Prediction error modeling



Prediction error formalism for model selection

Good model = small prediction error

Lots of existing (linear) theory - statistics

Lots of good (linear) software - matlab toolbox

Especially useful if we want the model for prediction

What about for control?

Prediction error methods PEM

- A good model predicts the plant system output well
  - Need to test this outside the current application
    - Extrapolation and not just repetition
      - Changing experimental conditions
        - Input signal
        - Feedback control
      - Acid tests to determine two things
        - The best model fit to the data
        - The quality of the fit to the data

Prediction error - some math

model
$$\begin{aligned} y_t &= \hat{P}(z)u_t + \hat{H}(z)n_t \\ \hat{y}_{t+1|t} &= \hat{H}(z)^{-1}\hat{P}(z)u_t + [1 - \hat{H}(z)^{-1}]y_t \end{aligned}$$

Associated predictor

Leads to the prediction error frequency-domain formula

Changing the input spectrum alters the prediction task
- For control design want inputs similar to eventual controlled system
  - Circular problem

Sometimes prediction needs to be formulated without an input
- Example coming up in combustion instability modeling
  - Different formulation
    - Similar ideas

Affecting PEM model fits

$$\lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} \{L(z)[y_k - \hat{y}_{k|k-1}]\}^2 =$$

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} \left\{ \left| P(e^{j\omega} - \hat{P}(e^{j\omega}, \theta) \right|^2 \Phi_u(\omega) + \left| H(e^{j\omega}) \right|^2 \right\} \frac{\left| L(e^{j\omega}) \right|^2}{\left| \hat{H}(e^{j\omega}, \theta) \right|^2} \, d\omega$$

Big effects on model fit over frequency

- Input spectrum
  - Feedback controller
    - Correlation between input and output
- Data filter
- Disturbance model
  - Assumed known or estimated
- Model structure

Closed-loop PEM formulae

$$\lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} \left\{ L(z)[y_k - \hat{y}_{k|k-1}] \right\}^2 =$$

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} \left\{ \frac{\left| P(e^{j\omega}) - \hat{P}(e^{j\omega},\theta) \right|^2}{|1 + P(e^{j\omega})C(e^{j\omega})|^2} \left| C(e^{j\omega}) \right|^2 \Phi_r(\omega) \right.$$

$$\left. + \frac{\left| 1 + \hat{P}(e^{j\omega},\theta)C(e^{j\omega}) \right|^2}{|1 + P(e^{j\omega})C(e^{j\omega})|^2} \left| H(e^{j\omega}) \right|^2 \right\} \frac{\left| L(e^{j\omega}) \right|^2}{\left| \hat{H}(e^{j\omega},\theta) \right|^2} \, d\omega$$

- Accounts for the correlation between input and output
- More complicated than open-loop PEM formula
  - But still comprehensible
    - The controller is even more evident
- Connects to robust control criteria

Closed-loop modeling and control

$$\lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} \left\{L(z)[y_k - \hat{y}_{k|k-1}]\right\}^2 =$$

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} \left\{ \frac{\left|P(e^{j\omega}) - \hat{P}(e^{j\omega},\theta)\right|^2}{|1+P(e^{j\omega})C(e^{j\omega})|^2} \left|C(e^{j\omega})\right|^2 \Phi_r(\omega) \right.$$

$$\left. + \frac{\left|1+\hat{P}(e^{j\omega},\theta)C(e^{j\omega})\right|^2}{|1+P(e^{j\omega})C(e^{j\omega})|^2} \left|H(e^{j\omega})\right|^2 \right\} \frac{\left|L(e^{j\omega})\right|^2}{\left|\hat{H}(e^{j\omega},\theta)\right|^2} \, d\omega$$

$$\left| \frac{P(e^{j\omega}) - P(e^{j\omega})}{P(e^{j\omega})} \times \frac{C(e^{j\omega})P(e^{j\omega})}{1 + C(e^{j\omega})P(e^{j\omega})} \right| < 1$$

The main issue is to understand which controller is C(z)

Current controller for identification, next controller for control

Acid tests of models

Falsificationism

Karl Popper

- Propose a new experiment to test the model
  - Corroboration or invalidation
    - Hypothesis testing approach

- Model invalidation
  - Poor prediction
    - Strongly correlated residuals/errors
      - Systematic errors
        - Statistical tests

Building models - some Philosophy

What can you do if the model fails?

- Modify it to perform better
  - Deductive reasoning to include Physics
    - New model structure
      - Inductive reasoning fits the model to data

Deduction:

deriving conclusions from general or universal principles

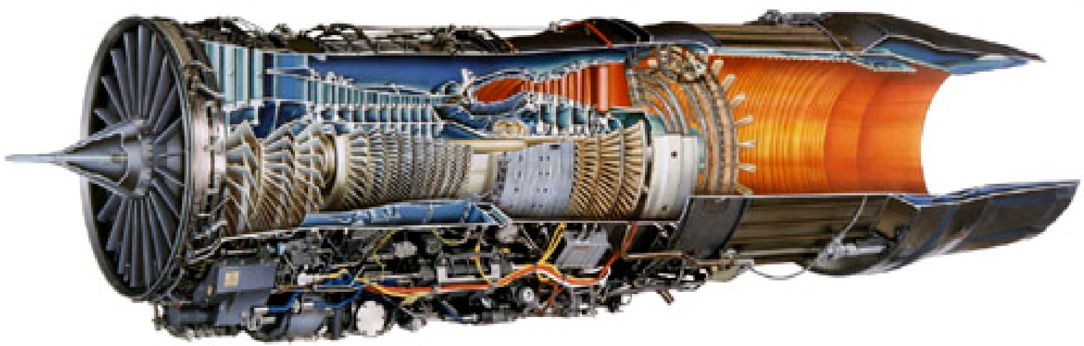
- Adjusting model structure to accommodate new experiments
  - Determining model structure from Physics

Induction:

deriving general conclusions from specific examples

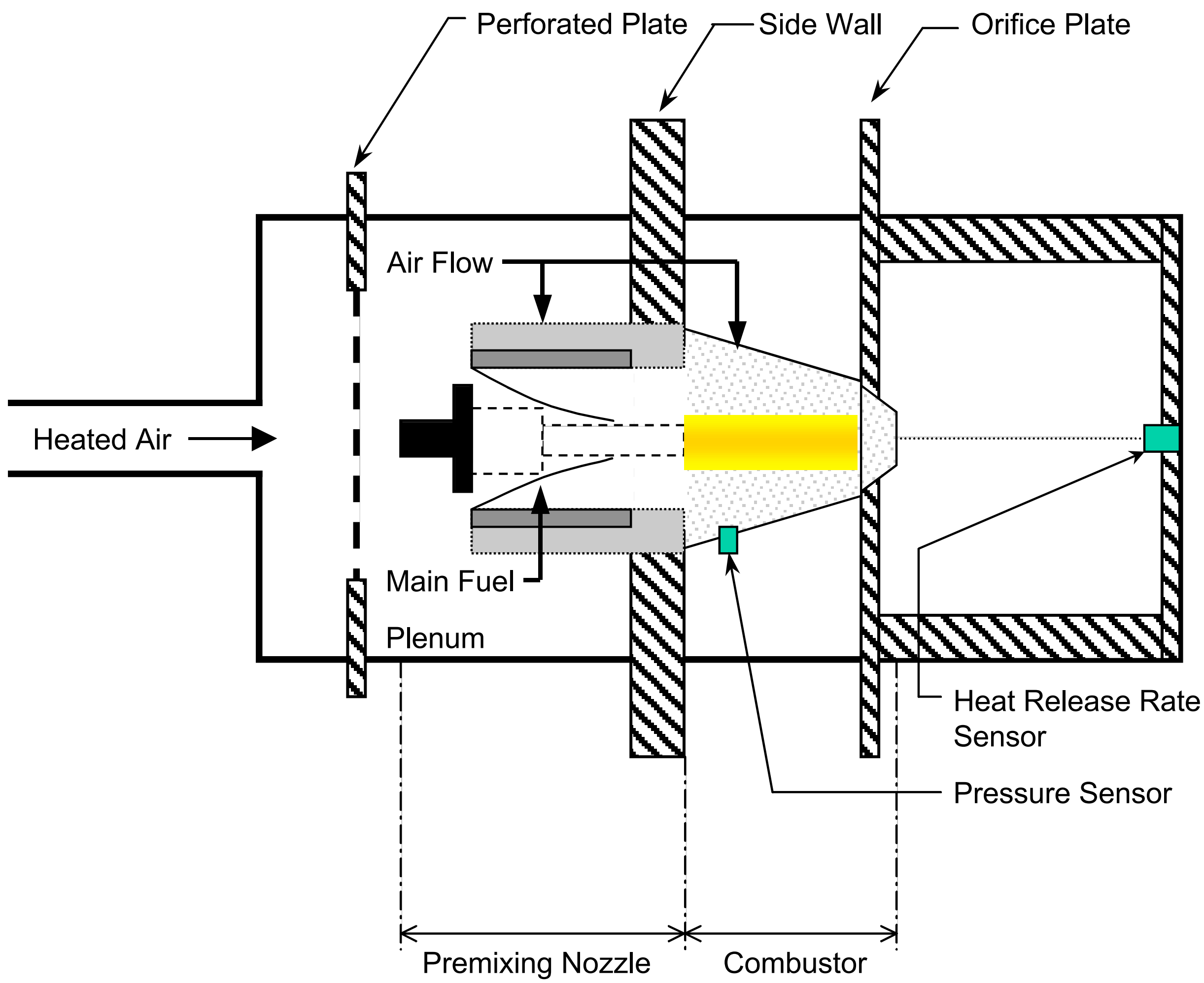
- "Let the data speak for themselves"
  - Fitting models and parameters to experimental data

Combustion instability modeling

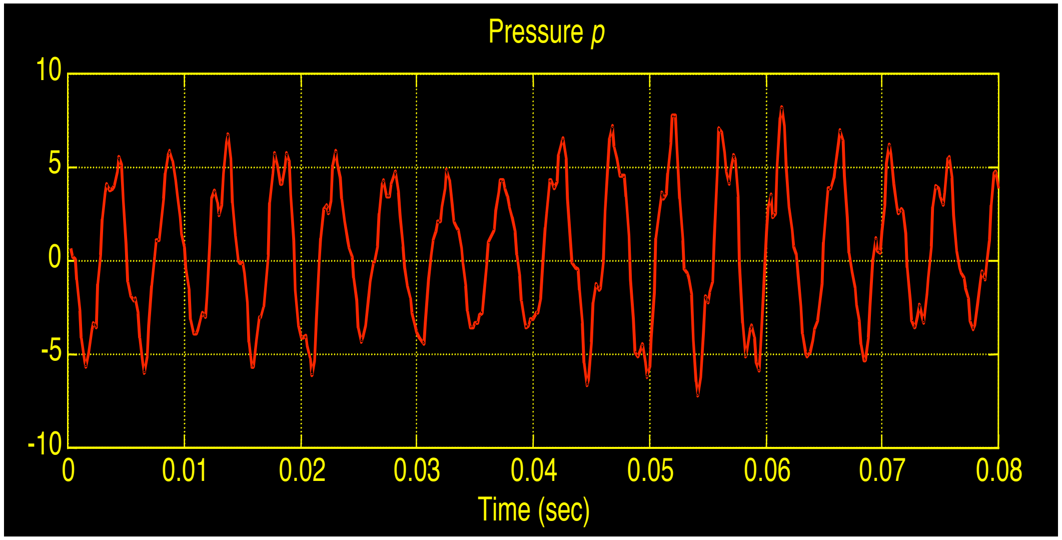


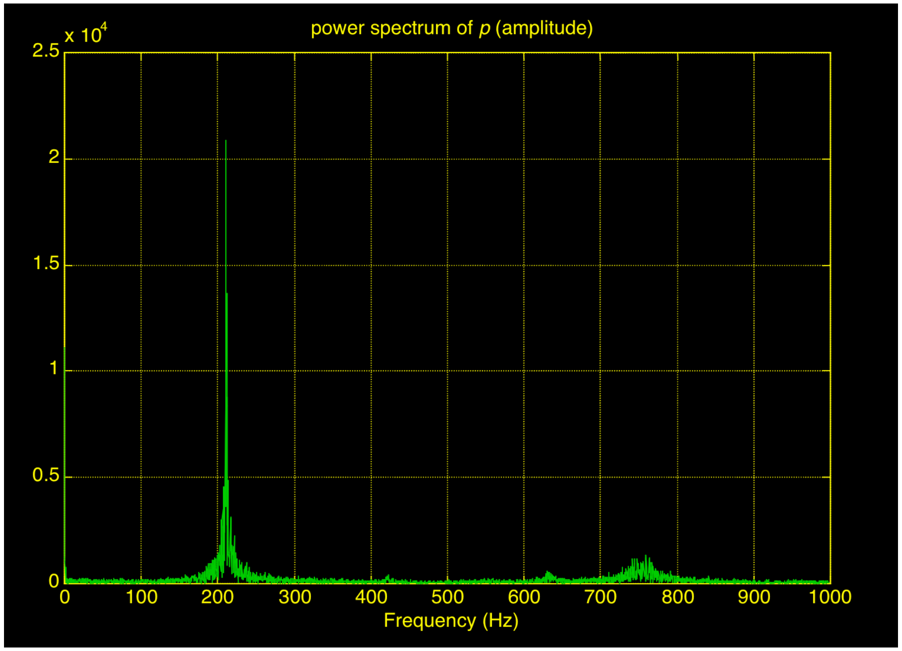
- Jet engines and gas turbines
  - Lean combustion yields economic and environmental benefits
    - Limited by appearance of limit cycling at low fuel-to-air ratios
      - Benefits are lost
- Build a model for control of the combustion instability
  - Alternating deductive and inductive stages
    - Stressful experimental tests of models' predictive powers

Experimental set-up



Experimental data





Highly periodic

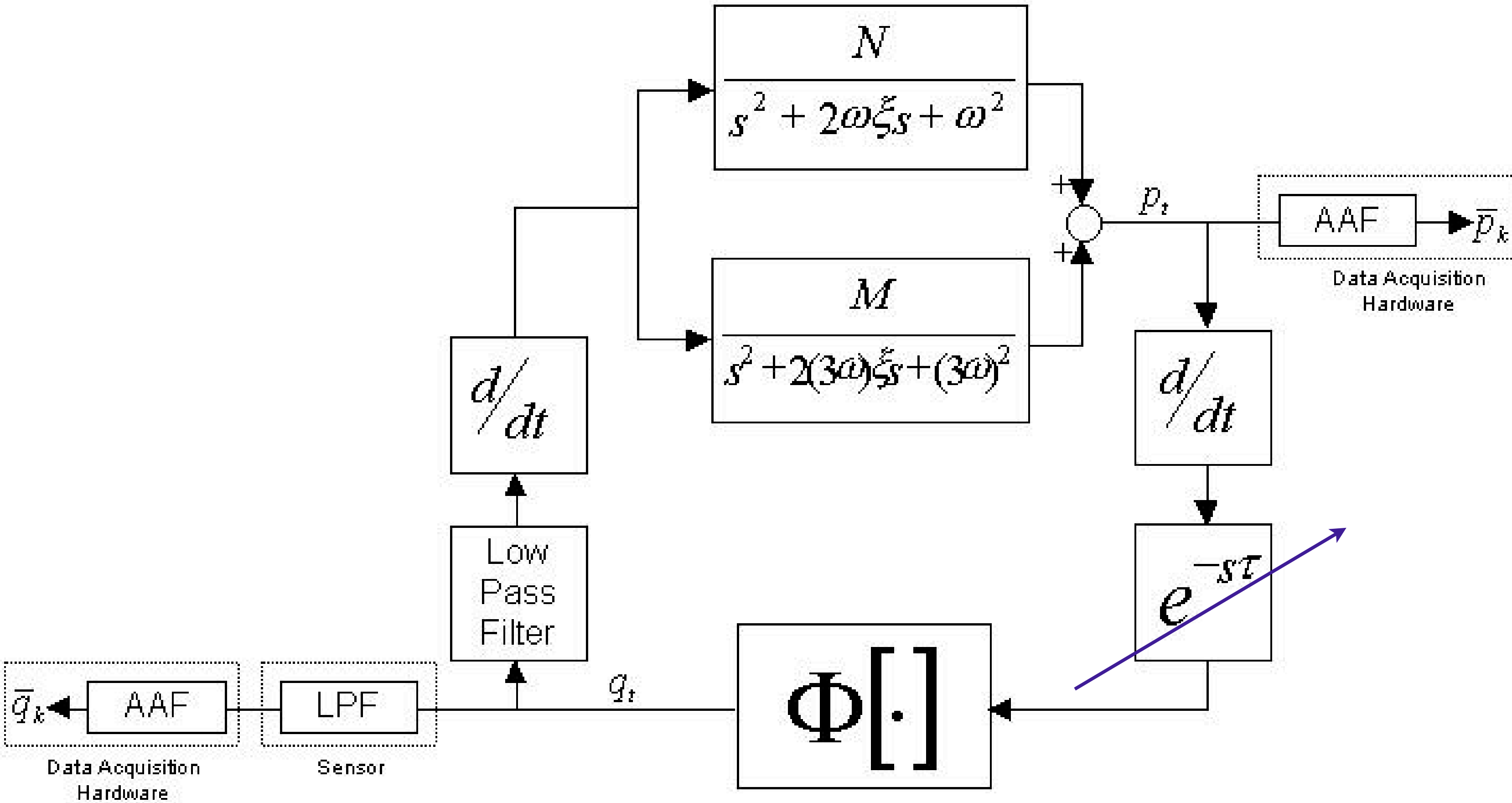
- Not very informative for modeling

Harmonics at 210Hz, 420Hz and 630Hz

Non-harmonic component at 720Hz

Nonlinear phenomena

Model structures from Physics



Combustion chamber acoustics meet heat release rate function

Improved fidelity with model development

Parsimonious model adjustments

Model development

Peracchio & Proscia — *Deduction*

First-order acoustics and model fit to data — *Induction*

Incapable of explaining multiple frequencies — *Test*

More complex Physics — *Deduction*

Third-order acoustics and model fit to data — *Induction*

Corroboration simulation test passed — *Test*

Invalidated at multiple operating points — *Test*

740Hz frequency changes with fuel-to-air ratio

More simple Physics - another phenomenon included — *Deduction*

Variable delay with fuel-to-air ratio and fit — *Induction*

Multiple-operating-point corroboration test passed — *Test*

Message

- Modeling involves a number of processes
  - Deduction, induction, testing
    - Much of this might be classified as "Prejudice"
      - This embodies our understanding of the process
        - I call this "Idiot testing" Does the model make sense?

- Modeling for use in control design has a special set of prejudices
  - Extraordinary simplicity
    - Control systems operate over only a couple of decades of frequency
  - Stability properties are important
    - Unlike when modeling for prediction

- We could really model well if we know what the final controller was