Design of a Self-Tuning PID Controller and its Application for a Polymerizing Reactor

Toru Yamamoto, and Kenzo Fujii

Hiroshima University, Higashi-Hiroshima, Japan, e-mail : yama@hiroshima-u.ac.jp

***Abstract*— PID control schemes based on classical control theory, have been widely applied to real control systems. There have been lots of works on tuning PID parameters, since the control performance strongly depends on these parameters. Several self-tuning and auto-tuning PID techniques have been reported for systems with unknown or slowly time-varing parameters. None, however, is applicable to discrete-time system with an unknown dead time. In this paper, a self-tuning PID control algorithm is proposed based on the relationship between the PID control law and the generalized minimum variance control law which is one of self-tuning control strategies. The proposed control scheme is practically evaluated on a temperature control system of a polybutene reactor.**

I. Introduction

Self-tuning control(STC) schemes[1]-[3] have been developed for systems with unknown or slowly time-varying parameters. Furthermore, the rapid progress in digital computer technology has enabled simulation of complex and large-scale systems. Many control techniques have been proposed to improve the performance of dis-crete-time control systems. However, since these techniques are complex, it is difficult to apply them to plants and obtain control parameters. On the other hand, PID[4]-[7] control algorithms still continue to be widely used for most industrial control systems, particularly in the chemical process industry. The reason is mainly because PID controllers have simple control structures, and are simple to maintain and tune. Therefore, it is still attractive to design control systems with PID control structures. Furthermore, one may not be able to get good control performance in the case of time-varying processes. Several techniques for auto-tuning[8]-[11] and self-tuning PID control[12]-[14] have been proposed. To the best of our knowledge, however, no self-tuning PID control scheme has been considered for unknown time-delay systems.

The main motivation in this paper is to present a design scheme of self-tuning PID controllers, and consider the implementation of the proposed control scheme for a temperature control system of a polybutene reactor. In the polybutene reactor, to maintain product quality and the stability, it is important to control temperature. This process is difficult however, because the dynamics of the system varies with catalyst activity, product grade and reactor rate. To date, various PID control schemes have been adopted, the parameters of which have been determined by trial and error. Moreover, the parameters were not necessarily suitable, and fairly conservative values were used to ensure the safety of the reactor. Therefore, a self-tuning PID control scheme is needed.

This paper is organized as follows. The structure of a transfer function model to be considered in the estimation and design of control system is first considered. Furthermore, based on the relationship between PID control and the generalized minimum variance control(GMVC)[15] laws, a design scheme of self-tuning PID controllers is proposed. Finally, an implementation of the proposed control scheme for a temperature control system of a polybutene reactor is considered, and the effectiveness is practically evaluated by applying it to the polybutene process.

II. Self-Tuning PID Controller

A. Mathematical model

Let z^{-1} be the backward shift operator, and consider the following discrete-time SISO model:

$$A(z^{-1})y(t) = z^{-k_m}B(z^{-1})u(t-1) \tag{1}$$

where

$$\left.\begin{array}{l} A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} \\ B(z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_m z^{-m}, \end{array}\right\} \tag{2}$$

and $u(t)$ and $y(t)$ are the input and the output signals, respectively. Furthermore, k_m is the minimum value of the estimated time-delay. If the true time-delay, k, is known exactly in advance, it is better to set as $k_m = k$. On the other hand, where the information about the time-delay is not available, then k_m is set to 1. For the system (1), suppose the following assumptions:

[Assumption]

[A.1] The degree of $B(z^{-1})$, m is known.

[A.2] Parameters, a_i and b_i are unknown.

[A.3] The time-delay k may be unknown, but the following relation is satisfied;

$$k_m \le k \le k_m + m \tag{3}$$

[A.4] Reference input $w(t)$ consists of piecewise constant signals.

B. PID controller design

Next, the following digital PID control law to be considered in this paper is described as:

$$\begin{aligned} \Delta u(t) = k_c[\{e(t) - e(t-1)\} + \frac{T_s}{T_i}e(t) \\ + \frac{T_d}{T_s}\{e(t) - 2e(t-1) + e(t-2)\}], \end{aligned} \tag{4}$$

where $e(t)$ denotes the control error signal given by

$$e(t) := w(t) - y(t) \tag{5}$$

and k_c, T_i and T_d are the proportional gain, the reset time and the derivative time, respectively. Furthermore, T_s denotes the sampling interval. For convenience, let $C(z^{-1})$ be

$$\begin{aligned} C(z^{-1}) &:= k_c(1 + \frac{T_s}{T_i} + \frac{T_d}{T_s}) \\ &\quad -k_c(1 + \frac{2T_d}{T_s})z^{-1} + \frac{k_c T_d}{T_s} z^{-2} \\ &= c_0 + c_1 z^{-1} + c_2 z^{-2} \end{aligned} \tag{6}$$

then, (4) can be rewritten by

$$C(z^{-1})y(t) + \Delta u(t) - C(z^{-1})w(t) = 0. \tag{7}$$

The tuning of the control parameters in PID control laws (4) or (7), is important, since the performance of the control system strongly depends on them. For systems with unknown parameters and unknown time-delays, however, it is difficult to find the 'optimal' PID gains easily. Therefore, a self-tuning PID control algorithm based on the relationship between PID control and generalized minimum variance control(GMVC) laws, is derived below.

C. PID tuning

Consider the following cost function to derive a GMVC law:

$$\begin{aligned} J = \{&P(z^{-1})y(t + k_m + 1) + \lambda \Delta u(t) \\ &-R(z^{-1})w(t)\}^2 \end{aligned} \tag{8}$$

where λ included in (8) is the weighting factor with respect to the control input, $P(z^{-1})$ is the user-specified polynomial of the form:

$$P(z^{-1}) = 1 + p_1 z^{-1} + p_2 z^{-2}. \tag{9}$$

and $R(z^{-1})$ is determined based on the relationship between PID control and GMVC laws. Δ denotes the differential operator, *i.e.*, $\Delta := 1 - z^{-1}$.

The control input minimizing the cost function (8) is given by the following equation[15]:

$$\begin{aligned} F(z^{-1})y(t) + \{&E(z^{-1})B(z^{-1}) + \lambda\}\Delta u(t) \\ &-R(z^{-1})w(t) = 0, \end{aligned} \tag{10}$$

where $E(z^{-1})$ and $F(z^{-1})$ are obtained by solving the following Diophantine equation:

$$P(z^{-1}) = \Delta A(z^{-1})E(z^{-1}) + z^{-(k_m+1)}F(z^{-1}) \tag{11}$$

$$\left.\begin{aligned} E(z^{-1}) &= 1 + e_1 z^{-1} + \cdots + e_{k_m} z^{-k_m} \\ F(z^{-1}) &= f_0 + f_1 z^{-1} + f_2 z^{-2}. \end{aligned}\right\} \tag{12}$$

Next, based on the relationship between PID control and GMVC laws, a tuning method of PID parameters is derived. Usually the dynamics of the system to be controlled, for example, the time-delays or the time constants, are rarely known precisely in advance. In particular, knowledge of the delay is important. Here, the strategy is adopted, that k_m is under estimated *i.e.*, initially use the upperbound estimate of the delay, or assume that the order of $B(z^{-1})$ is large enough, in order to cope with the above problem. Therefore, the estimates k_m and $B(z^{-1})$, *i.e.*, the second term in (10), $E(z^{-1})B(z^{-1})$, includes some uncertainties. In order to obtain a control law with a PID structure, consider the following control law replaced $E(z^{-1})B(z^{-1})$ by the static gain $E(1)B(1)$:

$$\begin{aligned} F(z^{-1})y(t) + \{&E(1)B(1) + \lambda\}\Delta u(t) \\ &-R(z^{-1})w(t) = 0. \end{aligned} \tag{13}$$

Here, ν is defined as

$$\nu := E(1)B(1) + \lambda \tag{14}$$

then, (13) can be rewritten by

$$\frac{F(z^{-1})}{\nu} y(t) + \Delta u(t) - \frac{R(z^{-1})}{\nu} w(t) = 0. \tag{15}$$

Furthermore, if the following relations are satisfied:

$$\left.\begin{aligned} R(z^{-1}) &= F(z^{-1}) \\ C(z^{-1}) &= \frac{F(z^{-1})}{\nu} \end{aligned}\right\} \tag{16}$$

then (15) becomes identical to (7). Therefore, based on (6) and (16), PID parameters can be calculated as follows:

$$\left.\begin{aligned} k_c &= -\frac{1}{\nu}(f_1 + 2f_2) \\ T_i &= -\frac{f_1 + 2f_2}{f_0 + f_1 + f_2} T_s \\ T_d &= -\frac{f_2}{f_1 + 2f_2} T_s \end{aligned}\right\} \tag{17}$$

Note that the parameter λ is related to only k_c. In other words, T_I and T_D are independent of λ. Thus, the proposed scheme has a feature such that after selecting $P(z^{-1})$ in (8) or (9), λ can be determined or chosen independently by considering the stability of the control system based on *a priori* information. For example, the Bode diagram can be utilized to determine λ. Such a design method for selecting λ is discussed in the latter section.

The design method of $P(z^{-1})$ is discussed below. $P(z^{-1})$ is designed based on the following two features which allow one to select the response shape, overshoot, settling time, *etc.*

1) the rise time
2) the damping property.

The reference [16] has presented a practical method to design the reference model based on the above features in the continuous-time systems. By transforming these features into the discrete-time, the following coefficients can be obtained:

$$p_1 = -2e^{-\frac{\rho}{2\mu}} \cos(\frac{\sqrt{4\mu - 1}}{2\mu}\rho) \tag{18}$$

$$p_2 = e^{-\frac{\rho}{\mu}} \tag{19}$$

where ρ and μ are defined by

$$\rho := T_s/\sigma \tag{20}$$

$$\mu := 0.25(1 - \delta) + 0.51\delta \tag{21}$$

and σ and μ denote the rise-time and the damping index, respectively. Here, the binomial model response can be obtained for $\delta = 0$, and the Butterworth model response for $\delta = 1.0$. Although the response shape can be chosen to one's liking by changing the parameter δ, δ should be set to between 0.0 and 2.0 practically. On the other hand, σ_i corresponding to the rise time, can be set to between $1/3$ and $1/2$ of the time constant depending on the practical needs.

D. Self-tuning PID controller

Based on the control scheme discussed above, a self-tuning PID controller is designed in this section.

First, unknown parameters included in (1) are estimated by using the following least squares(RLS) algorithm with a dead zone [17]:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{\Gamma(t-1)\psi(t-1)}{1+\psi^T(t-1)\Gamma(t-1)\psi(t-1)}\eta(t) \quad (22)$$

$$\Gamma(t) = \frac{1}{\omega}[\Gamma(t-1) - \frac{\Gamma(t-1)\psi(t-1)\psi^T(t-1)\Gamma(t-1)}{\omega+\psi^T(t-1)\Gamma(t-1)\psi(t-1)}] \quad (23)$$

$$\varepsilon(t) = y_f(t) - \hat{\theta}^T(t-1)\psi(t-1) \quad (24)$$

$$\eta(t) = \begin{cases} \varepsilon(t) > d & \rightarrow \quad \eta(t) = \varepsilon(t) - d \\ |\varepsilon(t)| \le d & \rightarrow \quad \eta(t) = 0 \\ \varepsilon(t) < -d & \rightarrow \quad \eta(t) = \varepsilon(t) + d \end{cases} \quad (25)$$

where ω is a forgetting factor given by $0 < \omega < \infty$, and $\varepsilon(t)$ is a prediction error. Also, $2d(>0)$ in (25) is the width of the dead zone. $\hat{\theta}(t)$ and $\psi(t-1)$ are the unknown parameter and data vectors, respectively, *i.e.*,

$$\hat{\theta}(t) = [\hat{a}_1(t), \hat{a}_2(t), \hat{b}_0(t), \hat{b}_1(t), \cdots, \hat{b}_m(t)]^T \quad (26)$$

$$\psi(t-1) = [-y_f(t-1), -y_f(t-2), u_f(t-k_m-1), \cdots, u_f(t-k_m-m-1)]^T \quad (27)$$

In (25) and (27), the subscript '*f*' denotes the filtered values of the inputs and outputs, and they are given by

$$\left.\begin{array}{l} y_f(t) := W_f(z^{-1})y(t) \\ u_f(t) := W_f(z^{-1})u(t). \end{array}\right\} \quad (28)$$

$W_f(z^{-1})$ is designed as the low-pass filter or the band-pass filter. The filters are usually used for the purpose of improving reliability of the parameter estimation in real systems.

Next, by solving the Diophantine equation (11) based on estimates included in $\hat{\theta}(t)$, and calculating (14) and (17), PID parameters can be obtained.

The proposed self-tuning PID control algorithm is then realized via the following steps.

[Self-Tuning PID control algorithm]

1. Choose $P(z^{-1})$ and λ.
2. Design the estimator filter $D(z^{-1})$.
3. Estimate $\hat{\theta}(t)$ by using the RLS algorithm in (22)-(28).
4. Solve the Diophantine equation (11).
5. Calculate PID parameters based on (14) and (17).
6. Calculate the control input $u(t)$ based on (4).
7. Update t and return to 3.

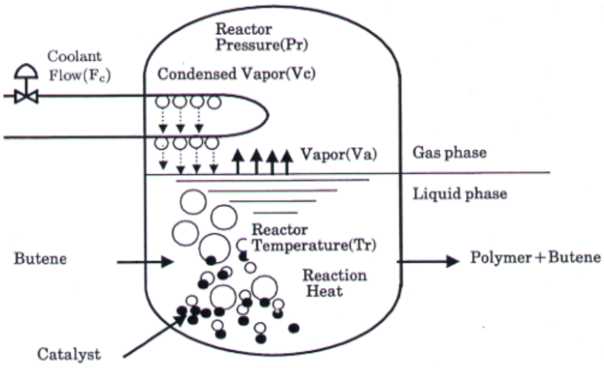

Fig. 1. Schematic diagram of the polybutene reactor.

III. Implementation

In this section, the proposed self-tuning controller is practically evaluated on the temperature control system of a polybutene reactor.

A. Outline of the polybutene process

In a polybutene plant, the object system products of C_4 distillation (butene, isobutene and so on) are polymerized by injection of a catalyst. Since the thermal polymerization reaction is exothermic, the temperature in the reactor must be strictly controlled. However, since a change in the operating conditions or catalyst activity disturbs the temperature control, the constant human intervention and monitoring are required currently.

A schematic diagram of the polybutene reactor is shown in Fig.1.

The action of the reactor can be summarized as follows:

i) The products of C_4 distillation (butene, iso-butene and so on) are supplied into the reactor.

ii) A catalyst is injected and the products are polymerized at a specified temperature.

iii) The products are vaporized or condensed with a coolant.

iv) Temperature changes in the reactor due to a change in the operating conditions, a change in catalyst activity, the heat of the exothermic reaction and so on, are controlled by the coolant flow rate.

v) The mixed fluids, some of which are polymerized, are sent for further processing.

The reactor with respect to the temperature control is modeled by considering iv).

B. Modeling

Suppose the following conditions:

[1] The equilibrium between the gas and the liquid phase is maintained.

[2] Changes in exothermic heat owing to changes of temperature in the reactor are neglected, since the activation energy for the reaction rate is small.

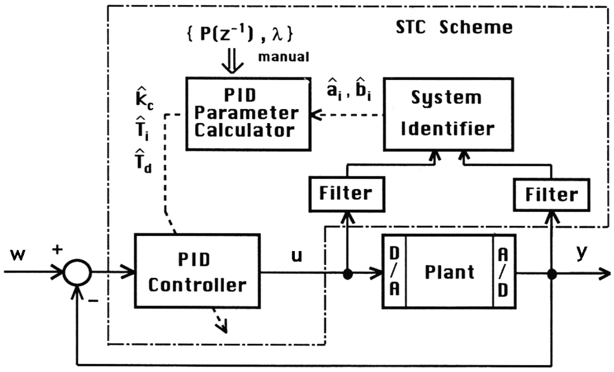

Fig. 2. Block diagram of the temperature control system of a polybutene reactor using the self-tuning PID control scheme.

[3] The inside of the reactor is in a state of boiling owing to the reaction heat, and is stirred.
[4] The flow of coolant involves two phases.
[5] The heat capacity of the steal parts of the reactor and the heat exchanger for cooling is neglected.

First, the experience equation with respect to the dynamics of the heat exchanger is as follows:

$$G_{p1}(s) = \frac{T_c(s)}{F_c(s)} = \frac{k_{ex}}{(1+\tau_1 s)} e^{-\tau_1 s}. \quad (29)$$

The notation is summarized as follows:

T_c : the temperature conducted at the heat exchanger [K]
F_c : the flow of the coolant [kmol/s]
k_{ex} : the temperature gain outside of the heat exchanger to the change of the coolant flow
τ_1 : the time lag by holding up the temperature inside of the heat exchanger [s]

Next, from the pressure balance in the reactor and the heat balance owing to the heat exchanger, it is found that the temperature T_r in the reactor is expressed as the first order element to the temperature T_c of the heat exchanger.

$$G_{p2}(s) = \frac{T_r(s)}{T_c(s)} = \frac{k_r}{1+\tau_2 s} \quad (30)$$

T_r : the temperature in the reactor [K]
k_r : the overall heat transfer gain
τ_2 : the time lag by conducting heat at the heat exchanger [s]

Therefore, the total dynamics $G_p(s)$ of the polybutene reactor with respect to the temperature can be expressed as follows.

$$\begin{aligned} G_p(s) &= G_{p1}(s)G_{p2}(s) \\ &= \frac{k_{ex}k_r}{(1+\tau_1 s)(1+\tau_2 s)} e^{-\tau_1 s} \end{aligned} \quad (31)$$

Note that the flow of the coolant, F_c, and the temperature in the reactor T_r, correspond to the input $u(t)$ and the output $y(t)$ in (1), respectively.

C. Control strategy

The self-tuning PID control scheme has been applied to the temperature control system of the polybutene reactor. The control scheme is shown in Fig.2.

In Fig.2, the solid lines represent the signals which are transferred at each sampling time $T_s = 1$[min], and the broken lines the signals transferred at each STC period $T_{sc} = 60$[min]. Furthermore, the part enclosed by the dotted-broken lines is the STC scheme. The following strategy is employed in adopting the STC scheme as shown in Fig.2.

[1] Though the data-sampling period for the system identification is the same as for the sampling time T_s, the STC period T_{sc} during which the PID parameters are changed is set to sixty times T_s for safety. And the system identification and PID parameter calculation are carried out by using the last 250 sampled data at each T_{sc}.

[2] The following band-path filter $D(z^{-1})$ is employed for the input and output sampled data of the plant in order to smooth the observation noise, and to remove the trend owing to the change of the catalyst activity.

$$D(z^{-1}) = (\frac{1-\alpha}{2}\frac{1+z^{-1}}{1-\alpha z^{-1}})(\frac{1+\beta}{2}\frac{1-z^{-1}}{1-\beta z^{-1}}) \quad (32)$$

Here, α and β are respectively set to 0.05 and 0.95 from the roughly estimated time constant of the system.

D. $P(z^{-1})$ and λ

¿From historical data of the polybutene process, the time constant of the process is calculated as $T = 566$[sec]. Based on the time constant, the rise-time is determined as $\sigma = 250$[sec], and $\delta = 0.0$ in this case. ¿From (18)-(21), $P(z^{-1})$ is designed as:

$$P(z^{-1}) = 1 - 1.238z^{-1} + 0.383z^{-2}. \quad (33)$$

Next, a scheme how to set suitable λ is considered. As mentioned in section 2.3, λ is related only to the proportional gain k_c. To ensure stability, one can adjust λ by using a simple Bode analysis based on *a priori* or approximate knowledge of the process. In the Bode diagram, only the gain curve is affected by λ and the phase curve is invariant to λ. Assuming the following equation as the true system, which has been obtained from historical data:

$$\begin{aligned} y(t) &= 1.575y(t-1) - 0.654y(t-2) \\ &\quad +0.023u(t-2) - 0.019u(t-3), \end{aligned} \quad (34)$$

the Bode diagram of the control system is shown in Fig.3, which is constructed by using (34) and the PID controllers whose PID parameters are calculated by (11), (14) and (17). In **Fig.3**, λ is changed from 0.0 to 0.5 in increments 0.1.

¿From Fig.3, λ is set to 0.1 to obtain a gain margin of approximately 10[dB].

E. Control results

The newly proposed self-tuning PID control scheme has been applied to the temperature control system of the polybutene reactor. The control results are shown in Fig.4. In the system description (1), k_m and m were respectively set to 0 and 3. Furthermore, $\omega = 0.99$(the forgetting factor) and $2d = 0.1$(the dead zone width) in the parameter estimation. The results of the estimator to compute PID parameters are shown in Fig.5.

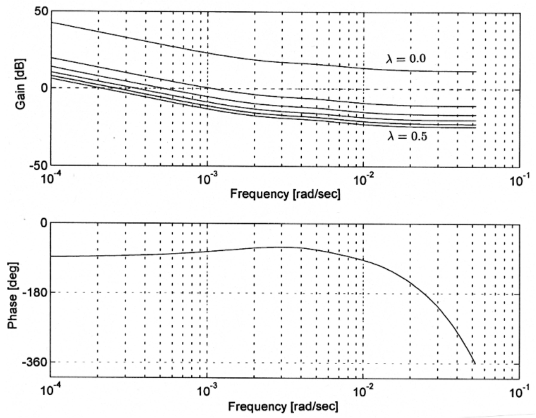

Fig. 3. Bode diagram of the temperature control system of the polybutene reactor to indicate the effect of λ.

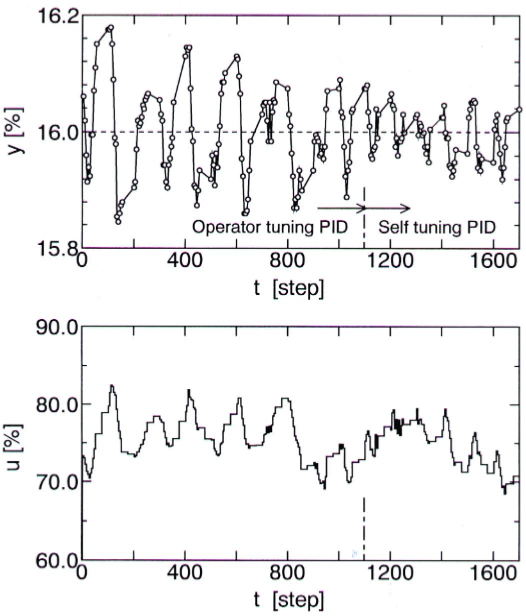

Fig. 4. Control results where the operator tuning parameters were used in the interval $0 \leq t < 1100$ and the self-tuning PID parameters were used in the interval $t \geq 1100$.

IV. CONCLUSIONS

In this paper, a self-tuning PID control scheme has been proposed based on the relationship between the PID control and the GMVC laws. Also, a method for designing the parameter λ and the polynomial $P(z^{-1})$ included in the cost function of the GMVC law has been discussed, which are selected based on some *a priori* information about the process. Finally, the effectiveness of the newly proposed control scheme has been shown by practical evaluation on a temperature control system of a polybutene reactor. Especially, the control results illustrate that the proposed scheme is applicable to a real plant whose dynamics varies with the catalyst activity and the change of product grade. Transition control in the change of product grade is currently under investigation.

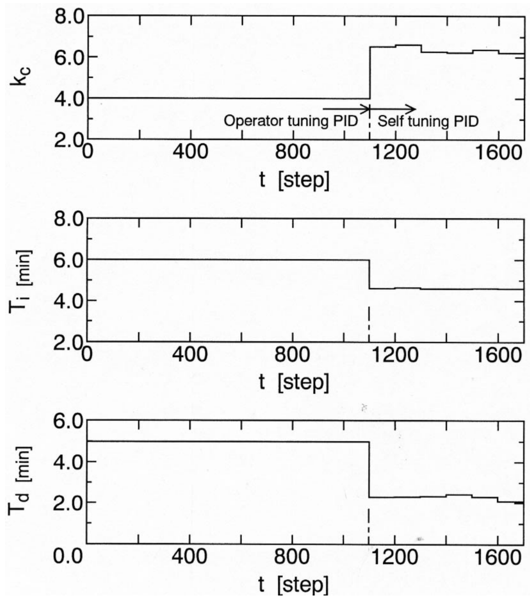

Fig. 5. PID parameters trajectories corresponding Fig.4.

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