Optimization of Discrete Time Supply Chain Models with Guaranteed Robust Stability and Feasibility

Darya Kastsian and Martin Mönnigmann,
Technische Universität Braunschweig, Germany

Abstract

This contribution presents a method for the simultaneous economic optimization and optimal design for disturbance rejection of discrete time systems. The method is an extension of a previously presented approach for the optimization of discrete time systems with constraints on robust stability [10]. In this contribution we extend the method to a new set of constraints on the location of poles of discrete time systems.

1 Introduction

Supply chain optimization has recently received a lot of attention. When seeking optimal supply chain designs it is important to consider the stability of the system, since supply chains are known to permit unstable behavior as witnessed, for example, in the bullwhip effect. Beyond mere stability, it is also important to design the system such that disturbances decay quickly enough. In this contribution we discuss a method for the economic optimization of discrete time systems which simultaneously ensures robust disturbance rejection.

The proposed method is an extension of the normal vector methods for the design of nonlinear dynamical systems with guaranteed robust stability and feasibility [4, 7–11]. The normal vector method was originally developed for parametric uncertainty with respect to stability properties in optimization problems. However, it naturally applies to the robust treatment of more general dynamical properties as well as robust feasibility. In Section 2 we discuss the normal vector method and its extension to ensuring disturbance rejection. The proposed approach is applied in Section 3 to the optimization of the supply chain model of an automatic pipeline feedback compensated inventory and order-based production control system (APIOBPCS) embedded within a vendor managed inventory (VMI) supply chain. Finally, in Section 4 we give a conclusion.

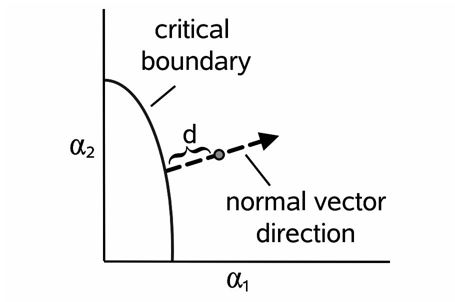

Figure 1: Optimal point of a hypothetical system with a critical boundary. The parametric distance d to the locally closest point on the critical boundary can be measured along the normal vector direction. The symbols α_1 and α_2 denote model parameters. If we ensure the distance between the optimal point and the nearest critical point to be larger than zero, then the optimal point is guaranteed to stay away from the critical boundary.

2 Normal vector method and rejection of disturbances

The normal vector method is based on the fact that equilibrium solutions of dynamical systems can be characterized by their parametric distance to the manifolds of critical points [7]. Typical critical points of interest are bifurcation points or points at which state variable constraints are violated. Normal vectors to the critical manifolds can be used to measure the distance from the nominal point of operation to stability and feasibility boundaries in the space of the system design parameters α_i. In order to keep the optimal point away from the critical boundary we constrain the parametric distance between the critical manifold and critical boundary to be larger than zero. This condition can be added to the optimization problem by posing an appropriate nonlinear inequality constraint. In Figure 1 the critical boundary and the normal direction to it are sketched for a hypothetical system. In Figure 1 we show a case with two parameters, but for higher dimensional cases the idea remains the same. In general more than one critical boundary can exist. By staying sufficiently far away from all critical manifolds we can guarantee robust stability and feasibility of the system.

For discrete time systems three types of bifurcation points exist, namely the fold, Neimark-Sacker and flip bifurcations [6]. These bifurcation points can be characterized by eigenvalues on the unit circle in the complex plane. In order to ensure robust disturbance rejection we introduce a simple but new type of critical point that is an extension and generalization of fold, Neimark-Sacker and flip bifurcations. Applying the normal vector method to this type of critical point amounts to forcing all eigenvalues of the linearized discrete time system into a circle of radius $R < 1$ on the complex plane.

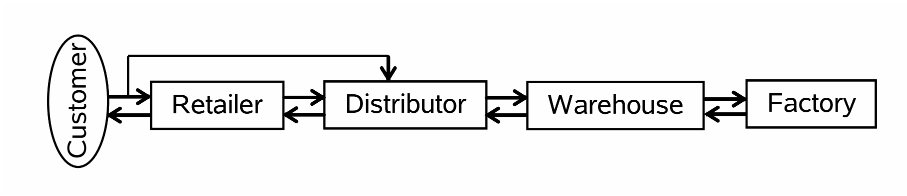

Figure 2: Flow of information in a VMI supply chain.

By specifying the radius R, the user of the method can specify the decay rate by which disturbances are rejected. We stress that while the normal vector method makes use of the linearized system and its eigenvalues, the decay rate constraint holds for the nonlinear system in a finite ball around the nominal optimal system parameters [5].

3 Applications

We present the result of the optimization of a VMI-APIOBPCS supply chain model with constraints on the decay rate for robust disturbance rejection. In contrast to traditional supply chains the vendor managed inventory supply chain distributors share inventory information and/or point of sales data rather than orders with their manufacturers. Alternatively, the manufacturer can determine the inventory from deliveries and sales if the distributor does not have the ability to share inventory information or is hesitant to do so, but is willing to share end sales data. A simple schematic VMI supply chain is illustrated in Figure 2 [3]. The system belongs to the class of automatic pipeline feedback compensated inventory and order-based production control systems (APIOBPCS). In the APIOBPCS the ordering rule is based upon forecast demand and the difference between a fixed target level of inventory and the actual level. The ordering rule also takes into account the work in progress, comparing actual levels with a target value. The pipeline and inventory levels information in the APIOBPCS are incorporated into the production order rate by adding a fraction of the difference between desired work in progress and actual work in progress, and a fraction of the difference between distributors inventory holding and actual inventory level, respectively. A full description of the VMI-APIOBPCS is given in [1]. Figure 3 shows the stability conditions for VMI-APIOBPCS in its parameter space. The line $Ti = 0.5$ shows the stability boundary due to flip bifurcations. All other curves in Figure 3 are sets of Neimark-Sacker bifurcation points. Three sample fixed points are used to illustrate the system's dynamical response. Figures 3 (a), (b) and (c) show stable, critically stable and unstable system response, respectively. In order to ensure the robust disturbance rejection we guarantee that all eigenvalues of the VMI-APIOBPCS lay inside the circle of radius $R = 0.8$ on the complex plane. Figure 4 illustrates

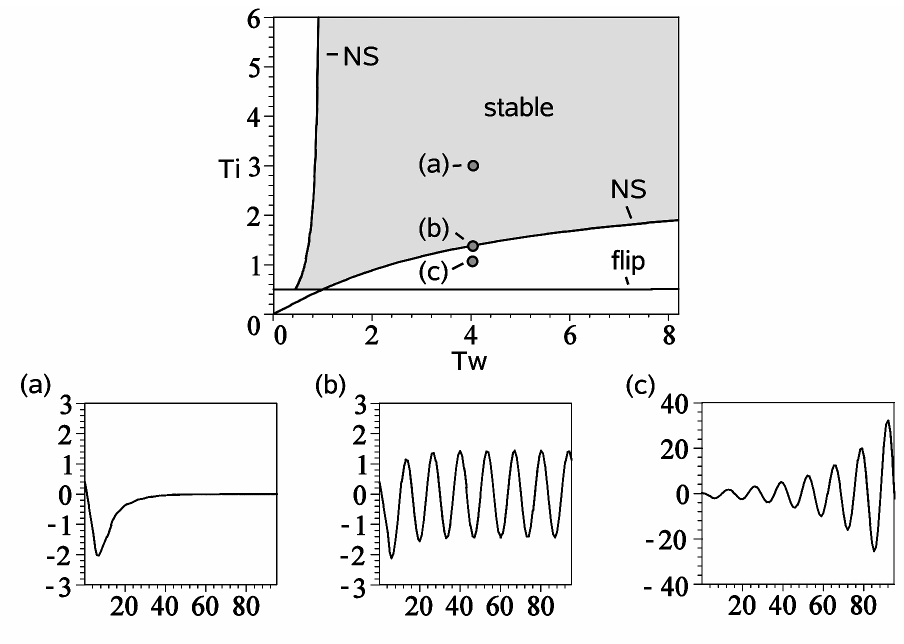

Figure 3: Bifurcation picture of the VMI-APIOBPCS. Diagram (a), (b) and (c) show unstable, stable, and critically stable dynamic response of the actual inventory, respectively.

the modified critical boundaries where the region with radius less than 0.8 is marked by gray color. The dynamic response of the actual inventory is shown in Figure 4 (a).

We will optimize the VMI-APIOBPCS supply chain model according to the following three criteria. The motivation and detailed description for these criteria is given in [2]. The first criterion is the noise of bandwidth, which is traditionally a useful measure to characterize the frequency response of a system and hence the production adaptation costs. The second and third criterion are scaled quadratic errors in inventory and in virtual consumption. The normal vector method is used to ensure robust disturbance rejection such that all eigenvalues lay inside the circle of radius 0.8. The result of the optimization procedure with the normal vector method is shown in Figure 5.

4 Conclusion

We showed that the concept of normal vector method can be applied to the simultaneous economic optimization and optimal design for disturbance rejection of discrete time systems. This concept was successfully demonstrated on the optimization of the VMI-APIOBPCS supply chain model.

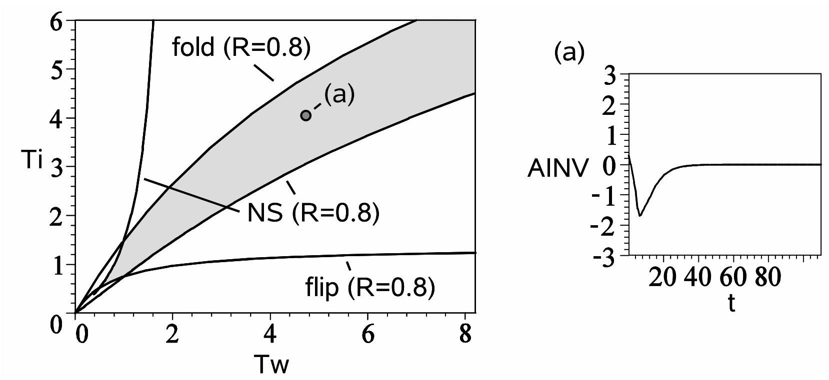

Figure 4: Modified critical boundaries with $R = 0.8$ of the VMI-APIOBPCS. Diagram (a) shows the dynamic response of the actual inventory.

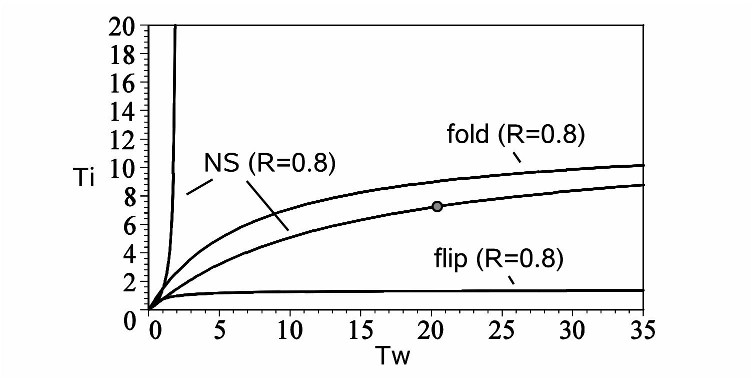

Figure 5: The optimal point of the VMI-APIOBPCS obtained with the normal vector method and modified critical boundaries with $R = 0.8$.

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