Interfacial Stability In Turbulent Pressure-Driven Channel Flow

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I. INTRODUCTION

We consider the motion of a deformable interface that separates a fully-developed turbulent gas flow from a thin layer of laminar liquid. We outline a linear model to describe the interaction between the turbulent gas flow and the interfacial waves; this consists of the Orr-Sommerfeld equation with the appropriate turbulent mean flow profile, together with a turbulent stress closure scheme. This approach permits us to determine numerically the growth rate of the wave amplitude, as a function of the relevant dimensionless system parameters and turbulence closure relations. It also extends previous work by accounting for the effects of the thin liquid layer on the dynamics.

The growth rate of the wave amplitude depends sensitively on the choice of mean flow. Therefore, it is necessary to derive a mean-flow profile that incorporates the characteristics of the flow observed in experiments. The mean flow profile we obtain demonstrates the features of turbulence in the gas layer: it is linear near the channel wall and interface, and logarithmic in the core. By writing down the functional form of the profile, it is also possible to express the wall and interfacial shear stresses as a function of the applied pressure gradient. The other ingredient necessary to complete the model is a turbulent closure scheme. The simplest possible closure is the mixing-length model: since turbulent eddies are limited in size by the wall and interface, the eddy viscosity can be constituted as a simple function of the vertical coordinate.

Using these inputs, we calculate the growth rate of the interfacial waves. We find that the incorporation of turbulent stresses through this model enhances the growth rate of the interfacial mode, while the growth rate of the internal mode is suppressed. The inclusion of the Reynolds stresses therefore gives rise to a significant correction in the wave growth rates. Previous work on this problem used a boundary-layer mean profile to model the mean flow, and we compare our results with this framework. (See, for example, [4], [1], and [3].) Although the models agree at shorter wavelengths, at long wavelengths it is necessary to take account of the bounded nature of the problem domain to obtain a correct picture of the wave growth rates.

II. BASIC STATE

Figure 1 describes schematically the state of the system wherein a flat interface separates a fully-developed turbulent gas from a laminar liquid layer. The profile shown is an averaged one, obtained by averaging over an ensemble of realizations. The profile can be predicted using an averaged model of turbulent flow, which takes account of the following observed features of the flow:

- The liquid film is thin and produces laminar flow, which is Poiseuille, in view of the pressure gradient $\partial p/\partial x<0$ applied in the x-direction;
- There is fully-developed turbulent flow in the gas;

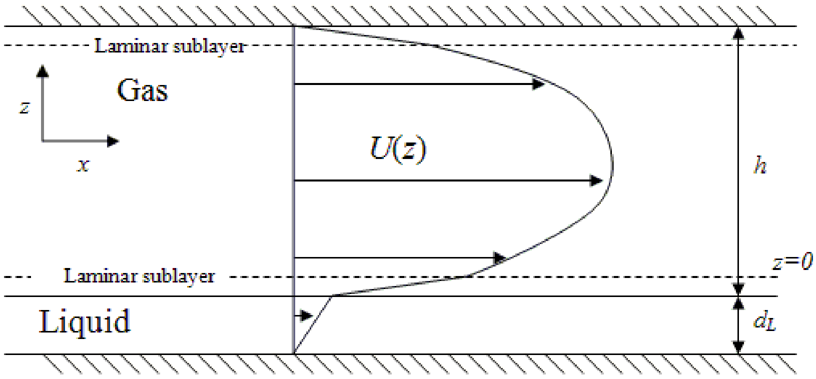


FIG. 1: The turbulent base state

- In the gas, near the gas-liquid interface and the gas-wall boundary, the flow is laminar, since here the viscous scale exceeds the characteristic length scale of the turbulence. These laminar sublayers are located by their z-coordinates $[0, z_{\mathrm{i}}]$, and $[z_{\mathrm{w}}, h]$, respectively;

- In gas core, flow has a logarithmic profile.

We make use of an eddy-viscosity model to describe the turbulent shear stress in the gas core; specifically, this stress term is given by the expression $\tau_{\mathrm{TSS}} = \mu_t\left(\partial U/\partial z\right)$. For pressure-driven channel flow, we make use of the following interpolating function for the eddy viscosity [2]:

$$\mu_t = \frac{\kappa\rho_G h u_{*\mathrm{i}}}{\sqrt{R}}\frac{\eta\left(1-\eta\right)\left(\eta^3 + R^{5/2}\left(1-\eta\right)^3\right)}{\left(R^2\left(1-\eta\right)^2 + R\left(1-\eta\right)\eta + \eta^2\right)} \tag{1}$$

where $\eta = \frac{z-z_{\mathrm{i}}}{z_{\mathrm{w}}-z_{\mathrm{i}}}$ is the coordinate in the gas core, and $R = \tau_{\mathrm{i}}/\tau_{\mathrm{w}}$. Finally, using the momentum balance equation in the gas,

$$\frac{\partial}{\partial z}\left(\mu_G\frac{\partial U}{\partial z} + \tau_{\mathrm{TSS}}\right) = -\frac{\partial p}{\partial x},$$

the gas velocity profile can be determined by quadrature of the equation

$$\left(\mu_G + \mu_t\right)\frac{\partial U}{\partial z} = \tau_{\mathrm{i}} + \frac{\partial p}{\partial x}z, \tag{2}$$

where μ_G is the gas molectular viscosity and τ_{i} is the interfacial shear stress. The approach taken has the added advantage of predicting this stress value through the matching condition

$$U_{G,\mathrm{core}}\left(z_{\mathrm{w}}\right) = U_{G,\mathrm{laminar}}\left(z_{\mathrm{w}}\right). \tag{3}$$

The results of the calculations in Eqs. (2) and (3) are shown in Fig. 2

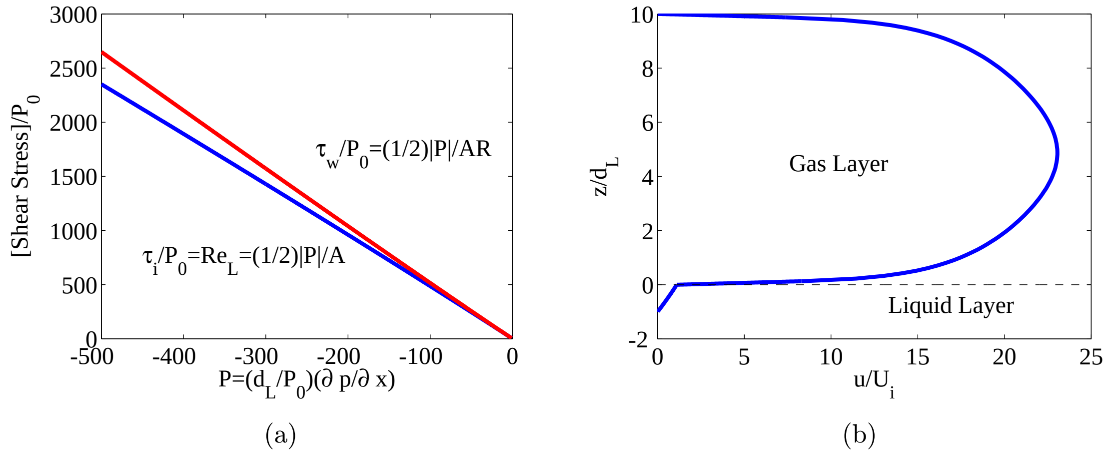


FIG. 2: (a) The relationship between the interfacial and wall shear stresses as a function of non-dimensional pressure gradient (an equivalent relationship between stresses and flow rate can be derived); (b) The average turbulent velocity profile for an air-water system with $Re_L = \tau_{\rm i}/P_0 \approx 400$ and $\delta = d_L/h = 0.1$. Here the non-dimensional unit of pressure is $P_0 = \mu_L^2/\rho_L d_L^2$, and $U_{\rm i} = \tau_{\rm i} d_L/\mu_L$.

III. LINEAR STABILITY: RESULTS

We perform a linear-stability analysis around the base state just outlined. Conceptually, we imagine introducing a tiny sinusoidal perturbation at the interface, which produces a disturbance in the velocity and pressure proportional to the amplitude. This amplitude grows or decays exponentially in time. This procedure is shown schematically in Fig. 3. Mathematically, we solve a linearized version of the Reynolds-averaged Navier–Stokes equation, which accounts for the turbulence in the gas layer. As before, the turbulent stresses are modelled through an eddy-viscosity approach. This is an eigenvalue problem whose solution gives the growth rate of the perturbation.

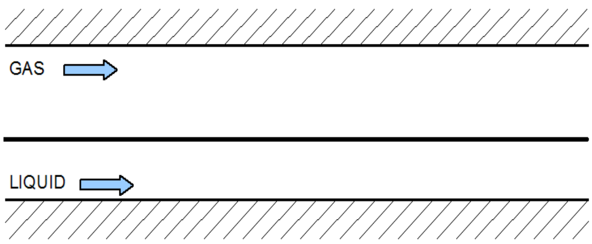


FIG. 3: A schematic diagram showing the perturbation wave introduced at the interface. The growth rate of this wave is calculated from a linearized version of the Reynolds-averaged Navier–Stokes equations.

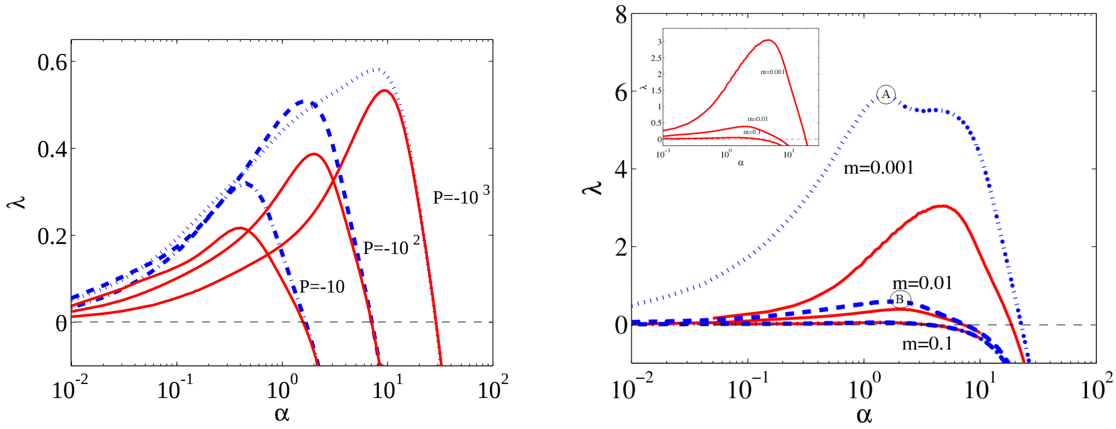
The numerical calculation of the growth rate enables us to highlight the effect of turbulence. Here, we highlight three effects:

1. The growth rate is enhanced by the inclusion of the turbulent shear stress;

2. Far from the air-water regime, growth rates are qualitatively and quantitatively different;

3. Surface roughness introduces mode competition.

In Fig. 4 we plot the growth rate λ as a function of the disturbance wavenumber α. In subfigure (a) we focus on a fluid pair with similar viscosity and density contrasts to air and water. The growth rates when the effects of turbulent shear stresses are included are enhanced relative to the growth rates when this effect is neglected. This effect is quantitative, rather than qualitative, in nature. However, for a system far from the air-water regime, the growth rates are dramatically different: the instability derives its energy not only from a mismatch of viscosities across the interface, but also due to a transfer of energy from the mean flow into the disturbance, which is due to the Reynolds stress in the turbulent flow.



(a) (b)

FIG. 4: (a) Dispersion curve as a function of non-dimensional pressure gradient for $r = \rho_G/\rho_L = 0.001$, $m = \mu_G/\mu_L = 0.01$, and $\delta = d_L/h = 0.1$; (b) Dispersion curve as a function of viscosity contrast for $P = -100$, $r = 0.001$, and $\delta = 0.1$.

Finally, another unstable mode exists, whose destabilizing energy comes from interfacial effects, and the transfer of energy from the mean liquid flow into the perturbation flow. In general, the growth rate of this mode is an order-of-magnitude less than that of the interfacial mode. However, when surface roughness is significant, the interfacial laminar sublayer shrinks, and the growth rate of the interfacial mode is reduced. The surface roughness is included by altering the eddy-viscosity model (1), and is parametrized by a parameter K which measures the level of roughness. The effects of this modification on the growth rate of the instability are shown in Fig. 5. The growth rate of the internal mode is shown to exceed that of the interfacial mode when the nondimensional roughness height $K = \ell_{\mathrm{i}}/d_L$ is comparable to the laminar sublayer height, that is, $\ell_{\mathrm{i}} \approx z_{\mathrm{i}}$.

IV. REVIEW AND OUTLOOK

In this presentation, we have outlined a model to describe the two-phase channel flow, wherein a fully-developed turbulent gas flow blows over a thin laminar liquid layer. The

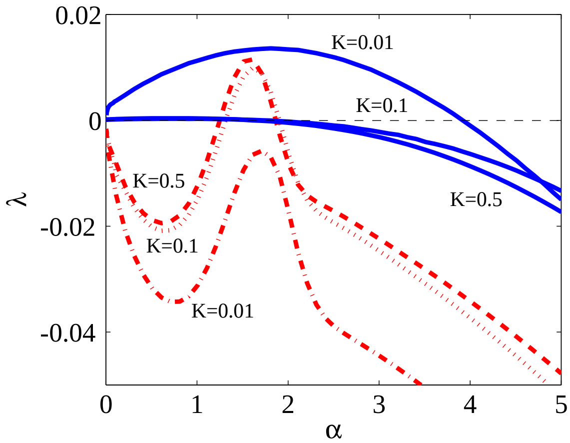


FIG. 5: Effect of roughness: $m = 0.01$, $r = 0.001$, $\delta = 0.1$, and $P = -200$. As roughness increases, the interfacial mode shrinks and the most dangerous mode is internal. This crossover effect takes place at $K \approx 0.1$, that is, for $\ell_\mathrm{i} \approx z_\mathrm{i}$.

model describes in detail the interfacial and wall shear stresses, the Reynolds stresses, and the flow profile. A linear stability analysis determines the stability of this basic state to perturbations in the interface shape. The instability that results is due to a mismatch in the viscosities at the interface, and depends sensitively on the modelling of the turbulent shear stresses in the problem. Mode competition can be induced by enhancing the level of interfacial roughness.

At present, we are extending the very basic approach taken to turbulence modelling in this work by developing a model that describes the near-equilibrium turbulence at the interface, and the far-field rapid distortion of turbulent eddies. This approach will determine to what extent the results of this study depend on the choice of turbulent model used in the stability analysis.

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