Intriguing nonlinear dynamic and stability characteristics of film blowing process and the chronological progress made in its simulation efforts

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Abstract

The successful simulation of the transient behavior of film in film blowing process has proved rather elusive, despite its modeling standards having been laid down more than 30 years ago by Pearson and Petrie in their seminal paper (1970) incorporating a moving material coordinate system. This is mainly because of the highly nonlinear nature of the partial differential equations involved and the difficulties in executing the numerical algorithms. It is thus only recent when Hyun et al. (2004) have successfully obtained for the first time the transient solutions of the dynamics using newly-devised mathematical/numerical schemes which employ a coordinate transformation converting a free-end-point problem to a more amenable fixed-end-point one and an orthogonal collocation on finite elements alleviating the dimensional burden in numerical computation. The transient solutions thus obtained have been found robust even during the severe oscillations of draw resonance instability also revealing a striking dynamic resemblance to experimental observations. In another seminal paper in 1988 by Cain and Denn the dynamics of the system like the multiplicity and stability of the steady states was first investigated systematically. Shin et al. (2007) have recently conducted an extended analysis both experimentally and theoretically with focus on the multiplicity, bifurcation, stability and hysteresis features and also provided physical explanations for the various dynamic features in the system. Quite recently, Lee et al. (2008) have included the flow-induced crystallization (FIC) into the non-isothermal, viscoelastic governing equations to obtain the robust, transient solutions for the whole distance coordinate from the die exit to the nip roll. These solutions are particularly significant in the sense that they have been obtained without assuming the boundary condition of the vanishing gradient of the bubble radius at the freeze-line height (FLH), which had been universally taken by all researchers in hitherto-published simulation models. The same vanishing gradient of the bubble radius at FLH has instead been obtained as part of the solution of the governing equations, not as an assumed boundary condition.

1. Introduction

Film blowing process has been considered an engineering ingenuity in that a simple and robust design enables stretching of film in two directions simultaneously: the axial drawing of the film provided by the nip rolls whereas the circumferential drawing accomplished by the air pressure inside the bubble. Important theoretical and experimental aspects of this process have been studied during the past four decades by many researchers, since the two seminal works laid the foundation for studying this process, i.e., Pearson and Petrie [1,2] establishing the first modeling standards for the film blowing system, and Cain and Denn [3] reporting the first comprehensive stability and multiplicity analysis of the process under isothermal conditions [1-4].

Thanks to those research efforts, the understanding of the film blowing dynamics has been steadily expanded to the level that industrial steady operation for producing the film products with desired properties has been established. However, dynamically complicated phenomena occurring in the process, e.g., (1) limit cycle-type instability (a Hopf bifurcation) called draw resonance manifested by the periodic variations in the state variables including the bubble radius, film thickness, the stress in film, etc., (2) existence of multiple steady states, and (3) even a hysteresis between steady states, provide the researchers with continual challenges and opportunities for better control and optimization of the process.

In this study, a brief history of the fundamental research conducted in recent years by the present author's laboratory is presented focusing on the modeling efforts for simulating the transient and steady behavior of the system. First, the transient solutions of dynamics in the system reported by the group in 2004 [5], the first such success in the history of the film blowing research exhibiting remarkably close resemblance to experimental observations even during draw resonance oscillations, are explained in terms of how the mathematical and numerical schemes were devised and employed to make such transient simulation possible.

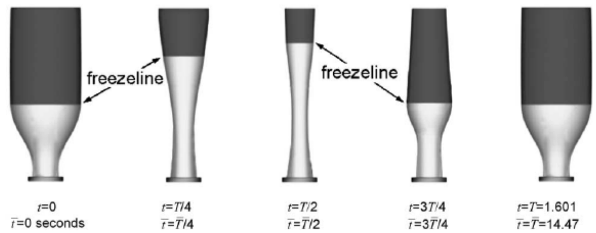
Then, a series of improvements to the governing equations of the simulation model have been made to investigate further into the film blowing dynamics: (1) instead of simulating up to the position of freeze-line height (FLH) only, but rather extending to the nip roll position so that the full range of axial distance for film blowing is covered, (2) the flow-induced crystallization (FIC) is added to include the crystallinity variable, and (3) stress in film can be computed to be compared with the birefringence measurements.

2. Transient solutions of the nonisothermal film blowing

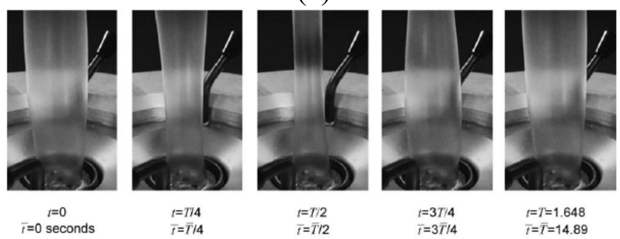
While the basic understanding of the process in terms of steady state operations and linear stability has been greatly advanced with all these efforts [1-4], there still remains the need for transient solutions of the process to reveal its nonlinear dynamics and nonlinear stability, which are acutely warranted for devising systematic strategies for process stabilization and optimization.

As an ongoing research program at the present author's laboratory, the transient behavior and stability of nonisothermal film blowing have been investigated solving the governing equations [5,6]. The problem is basically a moving-boundary one because the nonisothermal nature of the process dynamics makes the freeze-line height move with time. To handle this moving boundary problem on the freeze-line height a coordinate transformation is employed to make time and film temperature as new independent variables in lieu of the original time and distance so that at FLH the new independent variable of film temperature takes on a fixed value: this transformation has thus essentially converted the free-end-point problem into a computationally amenable fixed-end-point one.

To accelerate the computation speed, the orthogonal collocation on finite elements (OCFE) was applied to the axial coordinate. Employing a minimum number of finite elements and a minimum number of collocation points within each element for guaranteeing the accurate transient solutions within the manageable computation time, we have finally succeeded in devising a numerical scheme to generate transient solutions of the process even during the severe instability of draw resonance.



(a)



(b)

Figure 1. Temporal pictures of the draw resonance instability of LDPE blown film. (a) Simulation results at D_R=35, and (b) experimental results under the same operating conditions.

These simulation results provide, for the first time, temporal pictures which are close to those observed experimentally [5], and enable a systematic analysis of the process as regards its stability, multiplicity, sensitivity and stabilization strategies [6]. Fig. 1 shows the comparison of simulation data of draw resonance in a real experimental case. This closeness of the simulation results to real observations is considered a modeling and numerical breakthrough for film blowing process.

3. Crystallization kinetics

The crystallinity in the blown film is considered to be important in determining the desirable attributes of the semi-crystalline polymer film products such as stiffness, mechanical strength, chemical resistance, barrier to gas (e.g., moisture, oxygen) transport, chemical resistance and dimensional stability. Thus it becomes so important and natural to model the so-called flow-induced crystallization (FIC) occurring on the film into the governing equations of film blowing process.

When we have this FIC included in the simulation model as shown Eq. 6 below, we need to expand the range of the axial coordinate as well: instead of up to the FLH, up to the nip roll. This is because the most crystallization ordinarily occurs beyond the FLH, and some deformation in film also occurs in this region.

Regarding the expanded region for the simulation of film blowing process accompanied by FIC, there can be two methods. The first is rather an easy one: the simulation up to the FLH being carried out in the same fashion as before [5] with the vanishing bubble radius assumption at the FLH and for the region between FLH and the nip roll, the deformation of film is computed. It's thus termed the two-region model below. The second method is the fundamentally correct one: the assumption of the vanishing bubble radius gradient is discarded and it is rather obtained as part of the solution of the governing equations. This method is termed the one-region model.

The results by the first method were reported in [9]. From the nonlinear stability analysis, it was concluded that FIC destabilizes the film blowing process just as in the low-speed fiber spinning where the crystallization was found destabilizing because it shortens the distance available for deformation overriding the stabilizing effect of the increased tension in film or fiber.

3.1. Two-region model

The results by this method were presented in [9]. In the region I from the die exit to FLH, the same numerical scheme as in [5] were used, and in the region II from FLH to the nip roll, the assumption of the constant bubble radius was adopted because of r'=0. So in this region II the film deformation occurs on the film thickness only, not on the bubble radius.

The steady state and transient solutions for all the state variables including crystallinity can be obtained using this model and also conducting the nonlinear stability analysis using it can be found that crystallization destabilizes the film blowing process because just as in the low-speed spinning process.

3.2. One-region model

This one-region model doesn't divide the axial spatial coordinate in film blowing into two regions as in the above, but instead the whole distance variable is treated as the single region. The most important point with this model is that unlike in the previous models, the vanishing of the bubble radius gradient with respect to the axial direction at and beyond FLH is not assumed at all. It is to be obtained as part of the solution instead. Thus the bubble radius can change beyond FLH and the draw ratio is correctly defined at the nip roll position rather than at FLH as in the previous models. There is another minor advantage in this model, i.e., the coordinate transformation from time/distance to time/temperature which was adopted before, is not needed anymore, because the final boundary position of the nip roll is already fixed, i.e., a fixed-end-point-problem, not a free-end-point-problem needing the coordinate transformation.

The same OCFE was employed as before as a spatial acceleration technique for efficient computation.

4. Governing equations

The dimensionless governing equations of the nonisothermal film blowing of Phan-Thien and Tanner (PTT) fluids with crystallization kinetics are as follows. The equation for thermal and flow induced crystallization was developed by Ziabicki [11], and McHugh [12], respectively.

Equation of continuity:

$$\frac{\partial}{\partial t}\left(rw\sqrt{1+A_r^2\left(\partial r/\partial z\right)^2}\right)+\frac{\partial}{\partial z}\left(rvw\right)=0. \tag{1}$$

Axial force balance:

$$\frac{2A_r rw\sigma_{11}}{\sqrt{1+A_r^2\left(\partial r/\partial z\right)^2}}+A_r^2 B\left(r_f^2-r^2\right)=T_z. \tag{2}$$

Circumferential force balance:

$$B=\left(\frac{-wA_r\sigma_{11}\left(\partial^2 r/\partial z^2\right)}{\left[1+A_r^2\left(\partial r/\partial z\right)^2\right]^{3/2}}+\frac{w\sigma_{33}}{A_r r\sqrt{1+A_r^2\left(\partial r/\partial z\right)^2}}\right). \tag{3}$$

Constitutive equation (PTT model):

$$K\boldsymbol{\tau}+De\left\{\frac{\partial\boldsymbol{\tau}}{\partial t}+v\cdot\nabla\boldsymbol{\tau}-\boldsymbol{L}\cdot\boldsymbol{\tau}-\boldsymbol{\tau}\cdot\boldsymbol{L}^T\right\}=\frac{De}{De_0}\boldsymbol{D}. \tag{4}$$

Equation of energy:

$$\frac{\partial\theta}{\partial t}+\frac{v}{\sqrt{1+A_r^2\left(\partial r/\partial z\right)^2}}\frac{\partial\theta}{\partial z}=-\frac{U}{w}\left(\theta-\theta_c\right)-\frac{E_m}{w}\left(\theta^4-\theta_a^4\right)+\Delta H_f\left(\frac{\partial x}{\partial t}+\frac{v}{\sqrt{1+A_r^2\left(\partial r/\partial z\right)^2}}\frac{\partial x}{\partial z}\right). \quad (5)$$

Crystallization kinetics:

$$\frac{\partial x}{\partial t}+\frac{v}{\sqrt{1+A_r^2\left(\partial r/\partial z\right)^2}}\frac{\partial x}{\partial z}=\left(1-x\right)K_{\max}\exp\left[-4\ln 2\left(\frac{\theta-\theta_{\max}}{\theta_{half}}\right)^2+2\kappa De_0\mathrm{tr}\boldsymbol{\tau}\right]. \quad (6)$$

Boundary conditions:

$$v=1,\ r=1,\ w=1,\ \theta=1,\ \boldsymbol{\tau}=\boldsymbol{\tau}_0,\ x=0.\ \text{ at } z=0, \text{ and } t\geq 0 \quad (7a)$$

$$\frac{v}{\sqrt{1+A_r^2(\partial r/\partial z)^2}}=D_R,\ \frac{d^2r}{dz^2}=0. \qquad \text{at } z=z_L=1, \text{ and } t=0 \quad (7b)$$

$$\frac{v}{\sqrt{1+A_r^2(\partial r/\partial z)^2}}=D_R\left(1+\delta\right),\ \frac{d^2r}{dz^2}=0. \qquad \text{at } z=z_L=1, \text{ and } t>0 \quad (7c)$$

where, where r denotes the dimensionless bubble radius, w the dimensionless film thickness, v the dimensionless fluid velocity, t the dimensionless time, z the dimensionless distance coordinate, B the dimensionless air pressure difference between inside and outside the bubble, T_z the dimensionless axial tension, θ the dimensionless film temperature, $\boldsymbol{\tau}$ the dimensionless extra stress tensor, $\boldsymbol{D}$ the dimensionless strain rate tensor, ε and ξ the PTT model parameters, De the Deborah number, U the dimensionless heat transfer coefficient, E_m the dimensionless radiation coefficient, θ_c the dimensionless cooling air temperature, θ_a the dimensionless ambient temperature, x the dimensionless crystallinity, $K_{\max}$ the maximum crystallization rate constant, θ_{max} the maximum crystallization rate temperature, θ_{half} the half width of crystallization rate curve along the temperature, κ the dimensionless FIC enhancement factor, D_R the draw ratio, δ the step disturbance at t=0, and z_L the dimensionless distance between the die exit and the nip rolls. The aspect ratio ($A_r \equiv r_0/L$) is newly introduced to improve the numerical stability of the system.

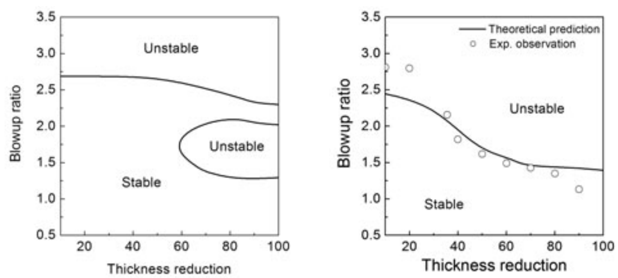
The boundary condition of the radius of the bubble at freeze-line height having the zero slope with respect to the z-coordinate can make not only no further deformation beyond freeze-line height, but also severe unphysical wiggle at the boundary of the nip roll [13].

5. Results and discussion

The new model with FIC in a single region from die exit to nip roll presented in Eqs.1-5 indeed yielded much better results than the previous model. First, it produces neither unreasonable overshoot of stress component near FLH as exhibited by the previous models, nor unphysical wiggle at the boundary of nip rolls reported by [13]. The birefringence data turned out to be in good agreement with stress predictions. Second, the bubble radius during draw resonance predicted by the present model more accurately agrees with experimental data, especially the skewed shape of the bubble radius curves. Third, not only the temporal pictures, but the stability windows show the better agreement with experiments. Especially, the fictitious secondary stable region at the high thickness reduction regime predicted by the previous model (Fig. 2(a)) disappeared in the new result by the present study as shown in Fig. 2(b). Moreover, the theoretically predicted operating windows well coincide with the experimentally determined critical blowup ratios.

6. Conclusions

The transient behavior and stability analysis of the nonisothermal film blowing process with crystallization kinetics have been investigated using new model. Unlike the previous model for film blowing, new model doesn't assume the boundary condition of the radius of the bubble at freeze-line height having the zero slope with respect to the axial coordinate. Instead, the governing equations of the system yield this important result as part of the solution of the set of the partial differential equations which are defined from the die exit all the way to the nip roll. By adopting new model, the stress profile does not make unreasonable overshoot near freeze-line in the steady state solutions, and the bubble radius curves with time during draw resonance exhibit skewness, agreeing with experiments again and agreement in the draw resonance severity with experiments is again better. On the stability maps, the fictitious secondary stable region predicted by the previous models disappears in the new results. More detailed nonlinear dynamical characteristics of the film blowing process including multiplicity, bifurcation and hysteresis will be presented at the conference.



(a) (b)

Figure 2. (a) Stability map of the film blowing process by linear stability analysis based on the previous model [5], and (b) experimental data on bubble stability with theoretical operating windows by the new model

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