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# An Efficient Procedure to Increase Robustness of Production Plans Embedded in an Integrated Multi-scale Planning and Scheduling Approach

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## Abstract

In these presentation records we propose a procedure to increase robustness of production plans embedded in a modelling approach based on integrated multi-scale optimization models and solution methods for planning and scheduling of a make to order production process under uncertain and varying demand conditions. As an inspiration we have a large real world problem originating from a complex pharmaceutical enterprise. The approach is based on a hierarchically structured moving horizon algorithm. On each level in the algorithm we propose optimization models to provide support for the relevant decisions and the models are solved with decomposition heuristics. The levels are diverse regarding the time scope, aggregation, update rate and availability of data at the time applied. The maximum effective time horizon of the multi-scale approach is one year and we use sales forecasts as input demands instead of actual orders which are usually only available 3 months ahead in time although raw materials need to be procured up to one year in advance. The sale forecasts have historically proven to be rather uncertain and to increase the *robustness* of our long-term plans we use an iterative procedure where the first step is to use a MILP model to obtain a solution based on the sale forecasts. In the next step we generate a number of alternative demand scenarios and run LP models to test the robustness of the MILP solution for each of the demand scenarios. If the long-term production plan is feasible for enough of the demand scenarios, depending on our robustness criteria, then we use the current plan, but if not then we change the demand forecast and run the MILP model again iteratively until the robustness criterion has been met. The demand samples are generated with tailor-made methods based on statistical error analysis.

The approach has been tested and implemented with industrial data from a pharmaceutical enterprise and has proved to be capable of obtaining realistic and profitable solutions within acceptable computational times.

## 1 Introduction

Make to order production within the process industries is generally operated in a very competitive and unpredictable environment where the key factors to succeed are to provide high service levels and flexibility at the same time as offering inexpensive products. To receive business companies must often promise shorter lead-times and the option of adjusting order quantities and changing delivery dates. It is a challenging task to cope with uncertainty and variation in the demand and when combined with the challenge of cutting down the production costs to be able to provide inexpensive products, the proper planning and scheduling of the production becomes very difficult and crucial for success. The pharmaceutical industry is a good example of an industry where planning and scheduling of make to order production is a big challenge. Flexible multi-product production processes have become commonly used as they help companies to respond to changing customer demand and increase plant utilization, but the greater complexity of these processes together with the altered market conditions have rendered the relatively simple planning and scheduling techniques previously used insufficient [1] which emphases the current need for flexible and efficient methods.

Uncertainty is an influential factor when planning and scheduling in reality. Uncertainty is one of the most challenging but very important problems in supply chain management [2]. There are many sources of uncertainty and it depends on the time scale under consideration which ones are most importantly taken into account. The ones most important for planning problems with longer horizons are environmental data such as demand and prices of products and raw materials, whereas process data such as processing times and equipment availabilities are of more importance for scheduling problems. The approaches found in the literature consider uncertainties in either reactive or proactive manner. The reactive approaches do not account for the uncertainties initially when the plan or schedule is made but instead they respond by creating a new plan or schedule when unexpected events occur or parameters changes. The proactive approaches do on the other hand consider the uncertainties and attempt to structure a plan or schedule that is optimal and reliable. The reactive approaches are in fact dealing with deterministic problems whereas the proactive approaches are dealing with stochastic problems which often require more complicated mathematical models and techniques. Among techniques commonly used in the chemical engineering literature is the framework of scenarios [3]. Approaches based on the framework of scenarios attempt to forecast and account for all possible future outcomes through the use of a number of scenarios but the size of the problem increases exponentially with the number of uncertain parameters which make a scenario based approach unsuitable for large real-world problems such as under consideration in this study.

# 2 Problem description

Our study is based on a real world problem originating from a pharmaceutical enterprise and we focus on single plant production planning and scheduling for a secondary production facility with order-driven multistage, multi-product flow-shop production. The plant consists of four main production stages with a large number of multi-purpose production equipment at each stage, operated in batch mode. The production is grouped into campaigns to reduce sequence dependent setup times which are considerable, especially when switching between different product families. Each product has a number of feasible production routes and with over 40 product families and 1000 products the process of planning and scheduling the production in an optimal way becomes extremely complicated.

The overall goal in the problem is to determine a campaign plan and to schedule customer orders within the campaigns. The customers request certain delivery dates for their orders and the plant attempts to meet those requests. The general objective is to meet the quantity and delivery date of customer orders and minimize the unproductive production time while respecting constraints. A further description of the problem environment is given by Stefansson and Shah [4].

# 3 Multi-scale modelling approach

The multi-scale modelling approach has been previously published in [4, 5] and here we just briefly introduce the background and the main functionality of the approach.

## 3.1 General layout

Order driven production planning and scheduling in the pharmaceutical industry is a critical example of a continuous and online decision problem. The production is a dynamic and ongoing process that is affected by several uncertain inputs and the most important one is the demand from customers. To cope with the problem we propose an integrated multi-scale hierarchically structured algorithm. At each level we propose optimisation models to provide support for the relevant decisions, wherein the scope and availability of information at the time of solution differs.

## 3.2 Three hierarchical levels of decisions

At the top level we propose a model to optimise a campaign plan for long term planning purposes. The campaign structure has to be decided earlier than when the orders become available and demand predictions are used instead of the actual orders. The purchasing of raw materials is based on the campaign plan and needs to be performed before the orders become available as the suppliers' lead-times are often greater than the promised lead-time of the production. Also maintenance, shifts and other events must often be planned before orders become available. In these presentation records we will focus at the top level and the procedure we propose to increase the robustness of the long-term plans.

At the middle level we propose a model to reoptimise the campaign plan simultaneously with allocating orders within the campaigns. The plant receives new customer orders each week with a requested delivery date and the orders need to be scheduled and confirmed. When the orders are scheduled within the campaigns the decision maker does not have information about other orders that will be added to the campaigns but have not been received yet. The current orders can change and other unanticipated orders may appear later with earlier due dates, larger quantities or higher priorities which can make it necessary to change the current schedule. However the confirmed delivery dates that have already been promised must be respected if at all possible.

At the lowest level we propose an optimisation model for the detailed scheduling of all production tasks. The optimisation is based on the confirmed customers' orders together with the newest possible real-time information each time it is used.

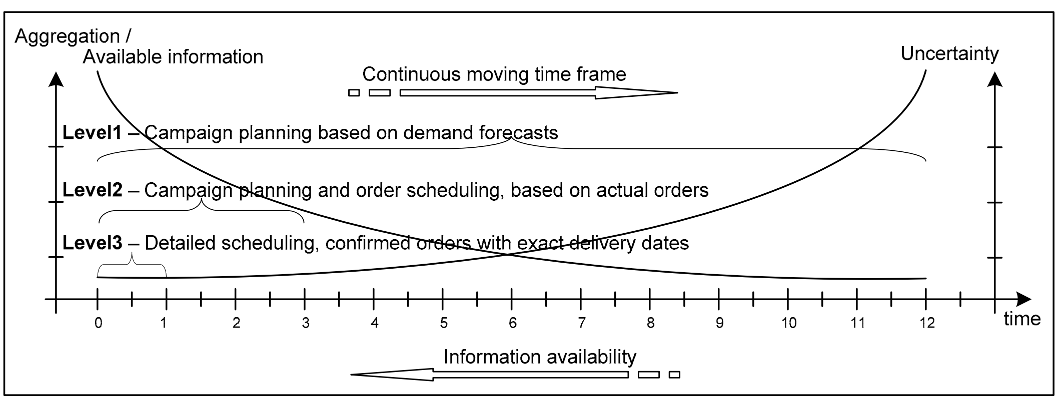


*Figure 2: The hierarchical algorithm or framework drawn together with the representation of uncertainty and availability of information with regard to time.*

The proper integration of the different levels in the algorithm is just as fundamental for its success as the functionality of the individual levels. The levels in the algorithm are integrated with bi-directional flow of information. The decisions created by the upper level models are implemented at the lower levels either as guidelines or/and as strict constraints that must be respected. By selecting carefully which decisions are implemented as guidelines and which as strict constraints it is ensured that feasible solutions are obtained.

# 4 Procedure to increase robustness of long-term plans

The demand is the most significant source of uncertainty for the planning at the top level in the hierarchical framework. The demand is based on forecasts which are made in attempt to predict something that will happen in the future. To achieve the best possible plans, perfect knowledge of the future is required which is not possible in practice. Different forecasting methods have been developed that work efficiently for many different forecasting situations. It is however impossible to predict the future accurately and there is always some uncertainty associated with forecasting that needs to be taken into account when decisions and plans are made. The quantities and timing of the demand can change and if the plan do not consider those changes it is very likely that the plan will not be flexible enough for meeting those changes and it will as a result be less robust.

The problem under consideration in this study is quite large and very challenging to solve, even when uncertainties are not considered at all. Stochastic programming is not very suitable for solving the problem because of the well known exponent ional expansion in model size involved with the stochastic modelling techniques. Instead we propose an iterative procedure to test the plans against a host of potential scenarios that cover a high percentage of possible outcomes. The potential scenarios are generated with characteristics that match the characteristics of the real data. The generation of the scenarios is very important for the procedure and it is based on the observation that the error distribution of the demand and the forecast tends to stay the same.

The sale forecasts given in the problem under consideration in this study have historically proven to be rather uncertain and to increase the robustness of our plans at the top level we use an iterative procedure where the first step is to use a deterministic MILP model to obtain a solution based on the forecasted sale without considering uncertainty. In the next step we generate the alternative demand scenarios based on the historical error distribution and the current forecast. The demand scenarios are used as input for a LP model which is used to test the robustness or feasibility of the MILP solution for each of the demand scenario. In the third step we evaluate the overall results and if the long-term production plan is feasible for enough of the demand samples, depending on our robustness criteria, then we use the current plan, but if not then we adjust the demand forecast and run the MILP model again iteratively until the robustness criteria has been met.

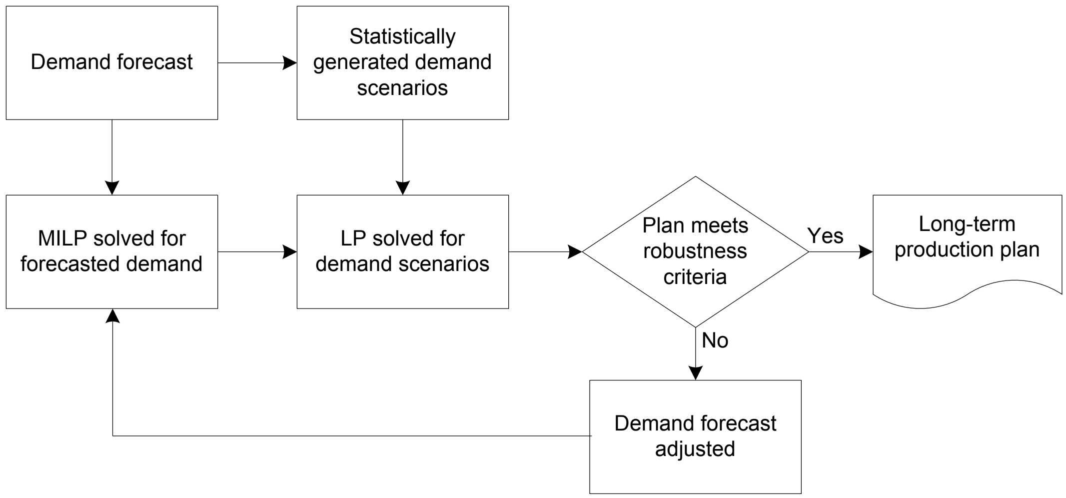


*Figure 1: Flowchart explaining the structure of the iterative procedure. Each component is further explained in the following sections.*

In the following sections we explain each of the components used by the procedure as shown in Figure 1.

## 4.1 Demand forecasts

The demand forecasts that are used as input were generated by the sales department in the pharmaceutical company we receive the data from. The demand forecasts specify how much will be sold of each product family in each month of the year. Sales forecasts were also created with statistical forecasting methods during this study. Those forecasts did in general outperform the forecasts created by the sales department but despite of that we base the experiments and work in the following sections on the forecasts from the company.

## 4.2 MILP optimisation model

### 4.2.1 Nomenclature

**Indices**

| | |
|---|---|
| $j$ | Machines |
| $s$ | Production stages |
| $f, f'$ | Product families |
| $w$ | Weeks |

**Sets**

| | |
|---|---|
| $J$ | Machines |
| $J_s$ | Machines on stage $s$ |
| $J_w$ | Machines that can be used in week $w$ |
| $S$ | Stages |
| $W$ | Weeks |
| $F$ | Product families |
| $F_w$ | Families that can be produced in week $w$ |

$F_j$ Families that can be produced on machine *j*

$F_{j,w}$ Families that can be produced on machine *j* in week *w*

$FS_{f,f'}$ Families that is not allowed to produce at the same time at each stage

$FE_{f,f'}$ Families that are not allowed to produced at the same time in the plant

**Parameters:**

$s_f^{first}, s_f^{last}$ First and last production stage for family *f* , respectively

$D_{f,w}$ Demand for products of family *f* in week *w*

$u_j$ Average setup time for machine *j*

$z_s^{min}, z_s^{max}$ Minimum and maximum number of setups on stage *s*, respectively

$TotCap_w$ Total number of machines that can be used simultaneously in each week w

$A_{fj}$ Maximum amount produced of family f on machine j

$TotInvCapS_s$ Maximum amount stored on inventory at stage s

$InvCapSF_{s,f}$ Maximum amount stored on inventory of each family f at stage s

$\alpha, \beta, \phi$ Parameters to adjust the weight of each criterion in objective function

## 4.2.2 Formulation

Here, we employ a discrete-time MILP model at a weekly level of resolution.

**Variables:**

***Positive continuous variables:***

$I_{f,w,s}$ Inventory of family *f* in week *w* on stage *s*

$\varepsilon_{f,w}$ Amount of family *f* that is delayed in week *w*

$YP_{f,w,s}$ Amount produced in week *w* of family *f* on stage *s*

***Binary variables:***

$Z_{w,j}$ 1 if setup work is needed in week *w* for machine *j*, else 0.

$Y_{f,w,j}$ 1 if product family *f* is produced in week *w* on machine *j*, else 0.

**Constraints**

***Allocation constraints:***

Each machine can only be allocated to one product family each week.

$$\sum_{f\in F_j} Y_{f,w,j} \le 1 \quad \forall\ w\in W, j\in J_w \tag{1}$$

Only one machine can be allocated to each family at each production stage in each week.

$$\sum_{j\in J_{f,s}} Y_{f,w,j} \le 1 \quad \forall\ w\in W, f\in F_w, s\in S_f \tag{2}$$

***Inventory balance and delivery constraints:***

Production activities must be performed according to a certain production route and inventory balance respected. Constraint 4 includes the demand for the final products of each family $D_{f,w}$ and $\varepsilon_{f,w}$ is used for the total amount of family *f* that has not been delivered in week *w*. Constraint 5 limits the amount produced on stage *s* to be less or equal to the amount produced on the previous stage *s-1*.

$$I_{f,w,s} = I_{f,w-1,s} + YP_{f,w,s} - YP_{f,w,s+1} \forall w \in W, f \in F, s \in S_f \setminus S_f^{last} \quad (3)$$

$$I_{f,w,s} = I_{f,w-1,s} + YP_{f,w,s} - D_{f,w} + \varepsilon_{f,w} - \varepsilon_{f,w-1} \forall w \in W, f \in F, s = S_f^{last} \quad (4)$$

$$YP_{f,w,s} \leq I_{f,w-1,s-1} \quad \forall \ \mathrm{f} \in \mathrm{F}, \mathrm{w} \in \mathrm{W}, \mathrm{s} \in \mathrm{S}_f \setminus S_f^{first} \quad (5)$$

A maximum amount of produced goods on inventory of certain family at each stage must be respected as well as the total amount.

$$I_{f,w,s} \leq InvCapSF_{s,f} \quad \forall \ \mathrm{w} \in \mathrm{W} \quad (6)$$

$$\sum_{f \in F} I_{f,w,s} \leq TotInvCapS_s \quad \forall \ \mathrm{w} \in \mathrm{W}, \mathrm{s} \in \mathrm{S} \quad (7)$$

***Capacity constraints:***

Constraint 8 and 9 ensure that the production capacity of machines and the maximum number of machines simultaneously in operation is respected.

$$YP_{f,w,s} \leq \sum_{j \in J_{f,s}} Y_{f,w,j} \cdot A_{f,j} \quad \forall \ \mathrm{f} \in \mathrm{F}, \mathrm{w} \in \mathrm{W}, \mathrm{s} \in \mathrm{S}_f \quad (8)$$

$$\sum_{j \in J} \sum_{f \in F_{j,w}} Y_{f,w,j} \leq TotCap_w \quad \forall \ \mathrm{w} \in \mathrm{W} \quad (9)$$

***Campaign constraints:***

Constraint 10 activates the binary variable $Z_{w,j}$ indicating that a campaign is starting.

$$Z_{w,j} \geq Y_{f,w,j} - Y_{f,w-1,j} \quad \forall \ f \in F_j \ , j \in J_f, \ \ w \in W \setminus w^{first} \quad (10)$$

A maximum and minimum number of campaigns on each production stage should be respected.

$$z^{\min}{}_s \leq \sum_w \sum_{j \in J_s} Z_{w,j} \leq \mathrm{z}^{\max}{}_\mathrm{s} \ \forall \ \ s \in S \quad (11)$$

***Mutual exclusivity constraints:***

It is forbidden to produce certain product families at the same time on the same stage and even at the same time in the entire plant.

$$Y_{f,w,s} + Y_{f',w,s} \leq 1 \quad \forall \ f \in F, f' \in FS_{f,f'}, s \in S_f, w \in W \quad (12)$$

$$\sum_S Y_{f,w,s} + \sum_S X_{f',w,s} \leq 1 \quad \forall \ f \in F, f' \in FE_{f,f'}, w \in W \quad (13)$$

**Objective**

The objective function includes minimization of the setup time, inventory and delays in the corresponding order.

$$\min \quad \alpha \cdot \sum_{w \in W} \sum_{j \in J_w} u_j \cdot Z_{w,j} + \beta \cdot \sum_{f \in F} \sum_{j \in J_f} \sum_{w \in W_{j,f}} I_{f,w,j} + \phi \cdot \sum_w \sum_{f_w} \varepsilon_{f,w} \quad (14)$$

## 4.3 Generation of demand scenarios

The demand scenarios are created by adding a stochastic error term to the forecasted demand. The stochastic error terms are simulated based on the historical distribution of forecasting errors as the error distribution of the demand and the forecast tends to stay the same. The first step of creating the stochastic error terms is to calculate the cumulative distribution function of the error. The cumulative distribution function describes the probability that a random error term is less or equal to a certain value.

$$CDF(E) = P(X \leq E) \quad (15)$$

The cumulative distribution function is structured by calculating all the errors between the historical demand and the historical forecast and then the errors terms are arranged in a cumulative order. The error terms are defined in equation 16.

$$E_t = X_t - F_t \tag{16}$$

Where $E_t$ is the real error term at time $t$. An example of a cumulative distribution function is given in Figure 2.

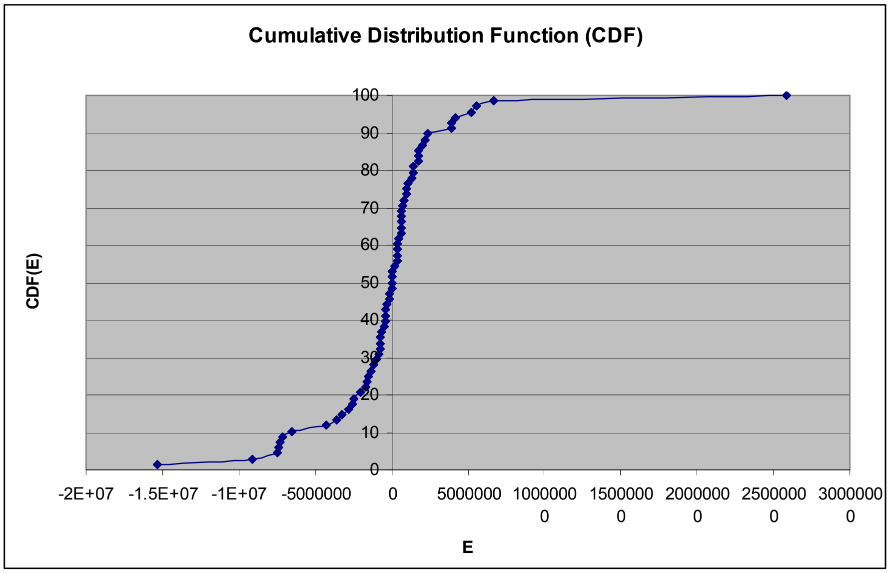


*Figure 2: An example of a Cumulative distribution function (CDF)*

The simulated demand can then be calculated with equation 17.

$$D_t^{sim} = F_t + E_t^{sim} \tag{17}$$

Where $D_t^{sim}$ is the simulated demand at time period t and $E_t^{sim}$ is the simulated real error term at time period $t$. The simulated error terms are sampled from an inverted cumulative distribution function. A random variable between zero and one is first generated for each time period $t$ and then it is used as input for the inverted cumulative distribution function which gives the value of the simulated error term for the corresponding time period:

$$CDF^{-1}(R_t) = E_{tsim} \tag{18}$$

Where $R_t$ is a random variable generated for time period $t$. Figure 3 shows an example of an inverted cumulative distribution function constructed from real error terms.

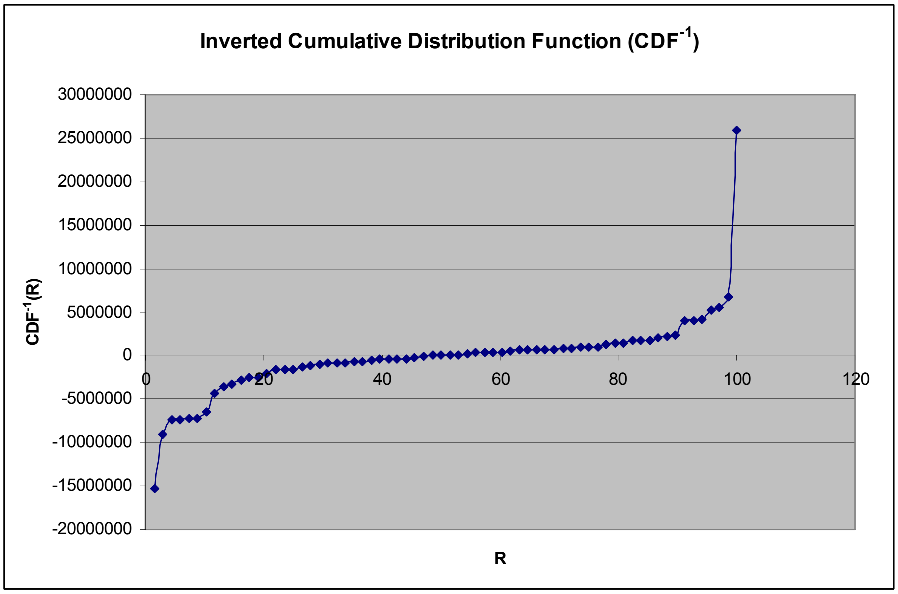


*Figure 3: Inverted Cumulative Distribution Function*

The procedure for generating the simulated demand was implemented in Matlab 7.0.

### 4.3.1 Markov based procedure for generating demand scenarios

Here we attempt to improve the procedure for generating demand scenarios by using a Markov matrix to check and verify the plausibility of the demand scenarios, in particular that the demand profiles must follow certain plausible stages, e.g. a point of very high demand never followed by another point of very high demand. We analyse the demand as a Markov chain which is a sequence of random variables $X_t$ having the property that, given the present, the future is conditionally independent of the past as described in equation 19.

$$P(X_t = j | X_0 = i_0, X_1 = i_1, ..., X_{t-1} = i_{t-1}) = P(X_t = j | X_{t-1} = i_{t-1}) \qquad (19)$$

Where $i_1, ..., i_a$ are the discrete values that the sequence of the random variables takes. Based on historical demand we structure a Markovian transition probability matrix. The matrix describes the probability of transforming from state *s'* in time period *t-1* to stage *s* in time period *t*. Each state represents demand values within certain range.

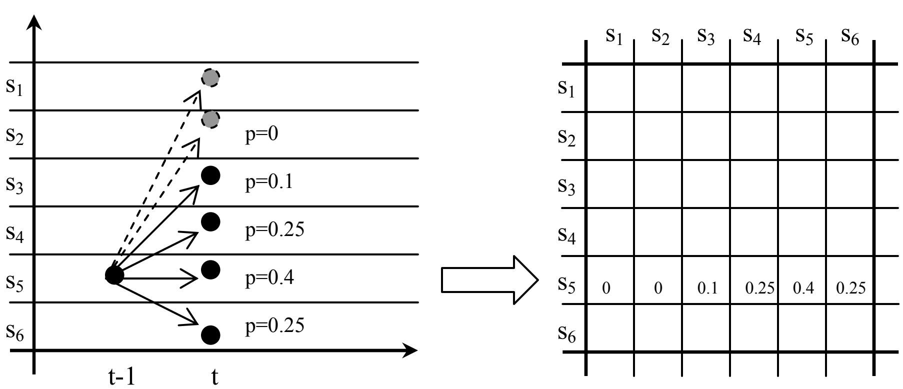

*Figure 4: The figure on the left illustrates the possible values to transfer to from state s based on the historical data. The p values represent the probability of transferring from state s to each of the other stages and the resulting row in the Markov transition matrix can be seen on the right*

After the transition matrix has been structured from the historical demand we generate the demand scenarios in a similar manner as explained in the previous section apart for we use the Markov probabilities to evaluate if the plausibility of the demand values. That is done by comparing each value with the previous value and if the Markovian probability is larger or equal to a predefined value ($\beta$) then we accept the value but if not we reject the value and generate another value until the condition in equation 20 is fulfilled.

$$P\left(X_t = s \middle| X_{t-1} = s'\right) \geq \beta \qquad (20)$$

If the historical demand data for a certain product family is not very rich then the resulting Markov transition matrix will have few values larger than zero and the possible future demand patterns will be very limited. To increase the possible transitions between stages in such cases we have applied an exception rule that allows randomly generated values that are either in the same or neighbouring stages to the forecasted value.

### 4.3.2 Reduced CDF$^{-1}$ procedure

When the inverted cumulative distribution function shown in Figure 2 is analysed it can be seen that there are several values caused by extremely large errors. Some of the values can be categorised as outliers who are caused by some abnormal circumstances or data errors and we may therefore be interested in utilising only a reduced range of the CDF$^{-1}$. Figure 5 explains how we only use a part of the CDF$^{-1}$ for generating demand samples.

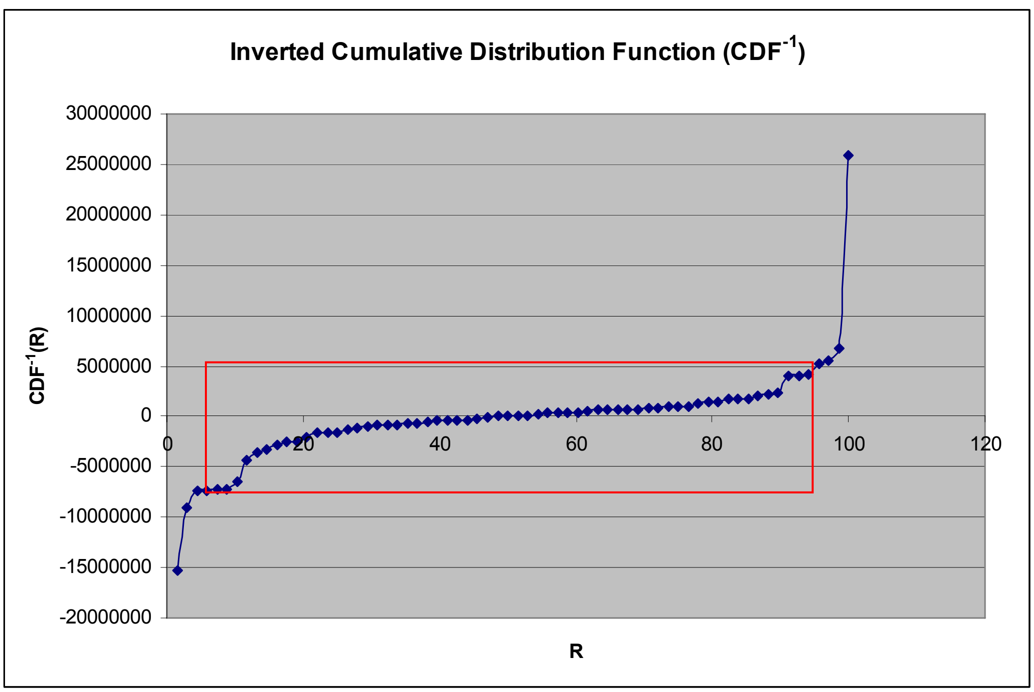


*Figure 5: It can be of interest to exclude the largest errors from the CDF-1 that is used for the generation of the demand scenarios. The figure shows the CDF-1 and the red box shows the part of it that would be used by only using random number within [0.05,0.95].*

We adjust the range of the CDF function by adjusting the range of the random numbers we generate. Instead of randomly generating a number within [0,1] when can e.g. generate a random number within [0.05,0.95] resulting in a reduced CDF as illustrated in Figure 5. This will remove rare outliers which will bias the MILP solution and can be used as a part of the procedures to generate the demand scenarios, anyhow with or without the Markov procedure explained in the previous section.

## 4.4 LP model to evaluate robustness

The demand scenarios are used as an input for a LP model which is used to evaluate the robustness of the MILP solution for each of the demand scenario. The campaign structure decided by the MILP model is fixed and used as an input for the LP model which attempts to fulfil each of the alternative demand scenarios as well as possible with the campaign structure intended for the forecasted demand. The LP model is quite similar to the MILP model apart from the $Y_{f,w,j}$ and $Z_{w,j}$ variables which are now defined as a parameters.

**Variables:**

***Positive continuous variables:***

| | |
|---|---|
| $I_{f,w,s}$ | Inventory of family *f* in week *w* on stage *s* |
| $\varepsilon_{f,w}$ | Amount of family *f* that is delayed in week *w* |
| $YP_{f,w,s}$ | Amount produced in week *w* of family *f* on stage *s* |

***Parameters:***

| | |
|---|---|
| $Z_{w,j}$ | Based on MILP results, 1 if setup work is needed in week *w* for machine *j*, else 0. |
| $Y_{f,w,j}$ | Based on MILP results, 1 if product family *f* is produced in week *w* on machine *j*, else 0. |
| $D_{f,w}$ | Demand for products of family *f* in week *w*, this vector is now based on the alternative demand scenarios instead of the forecasted demand. |

### Constraints

### *Inventory balance and delivery constraints:*

Production activities must be performed according to a certain production route and inventory balance respected. Constraint 22 includes the demand for the final products of each family $D_{f,w}$ and $\varepsilon_{f,w}$ is used for the total amount of family *f* that has not been delivered in week *w*. Constraint 23 limits the amount produced on stage *s* to be less or equal to the amount produced on the previous stage *s-1*.

$$I_{f,w,s} = I_{f,w-1,s} + YP_{f,w,s} - YP_{f,w,s+1} \forall w \in W, f \in F, s \in S_f \setminus S_f^{last} \quad (21)$$

$$I_{f,w,s} = I_{f,w-1,s} + YP_{f,w,s} - D_{f,w} + \varepsilon_{f,w} - \varepsilon_{f,w-1} \forall w \in W, f \in F, s = S_f^{last} \quad (22)$$

$$YP_{f,w,s} \le I_{f,w-1,s-1} \quad \forall \ \mathrm{f} \in \mathrm{F}, \mathrm{w} \in \mathrm{W}, \mathrm{s} \in \mathrm{S}_f \setminus S_f^{first} \quad (23)$$

A maximum amount of produced goods on inventory of certain family at each stage must be respected as well as the total amount.

$$I_{f,w,s} \le InvCapSF_{s,f} \quad \forall \ \mathrm{w} \in \mathrm{W} \quad (24)$$

$$\sum_{f \in F} I_{f,w,s} \le TotInvCapS_s \quad \forall \ \mathrm{w} \in \mathrm{W}, \mathrm{s} \in \mathrm{S} \quad (25)$$

### *Capacity constraints:*

Constraint 26 ensures that the production capacity of machines and the maximum number of machines simultaneously in operation is respected.

$$YP_{f,w,s} \leq \sum_{j\in J_{f,s}} Y_{f,w,j} \cdot A_{f,j} \qquad \forall\ f \in F, w \in W, s \in S_f \tag{26}$$

## Objective

The objective function includes minimization of the setup time, inventory and delays in the corresponding order (the setup time is a constant here as it has been fixed).

$$\min \quad \alpha \cdot \sum_{w\in W} \sum_{j\in J_w} u_j \cdot Z_{w,j} + \beta \cdot \sum_{f\in F} \sum_{j\in J_f} \sum_{w\in W_{j,f}} I_{f,w,j} + \phi \cdot \sum_{w} \sum_{f_w} \varepsilon_{f,w} \tag{27}$$

## 4.5 Robustness criteria and demand forecast adjustments

The aim of the procedure is to estimate more realistically the performance of the production plans as there is always some uncertainty associated with the future demand and then use the estimation to improve the robustness of the production plan. The main goal of the production plan is to fulfil the demand with the minimum cost of production (inventory and setup). If there is a large deviation between the forecasted sale and the alternative demand scenarios (or the actual sale upon the time of realisation) then it will be hard to fulfil the demand and the deliveries of the products will be delayed. To estimate the performance of the production plan we use the On Time In Full (OTIF) measure for each product family $f$ in each time period $t$. OTIF is a measure of the ability of the business to supply its products at the time agreed with the customer at the quantity agreed and of the right quality and can be defined by equation 28.

$$OTIF_{f,t} = 1 - \frac{ND_{f,t}}{NS} \tag{28}$$

Where $ND_{f,t}$ is the number of delayed delivery instances and $NS$ is the number of simulations performed for each product family. We also use an *OTIF* measure to evaluate the overall performance of the entire plan.

$$OTIF = 1 - \frac{\sum_{f,t} ND_{f,t}}{\sum_{f,t} NS} \tag{29}$$

By reducing the value of the *OTIF* measure we increase the overall robustness of the production plan and as a result we use the *OTIF* measure as a stopping criterion for the procedure.

**Robustness criteria:**
*OTIF = 0* or *OTIF* has increased from last iteration (30)

We use the $OTIF_{f,t}$ measure as an indication of which values of the forecasted demand we should modify for the next iteration of the procedure.

**Demand adjustment criteria:**
If any of the product families has an $OTIF_{f,t}$ value larger than 0.3, then select the first instance and increase the forecasted demand for the certain family and period by 10% or one minimal unit if the smallest unit is less than 10% of the demand. (31)

This was a successful criterion for when and how to modify the demand but it can truly be specified in a different manner and depends on the characteristics of the problem and data under consideration.

# 5 Results

## 5.1 Demand scenarios

In this study we are working with an industrial problem and we have based all our experiments on data from the actual problem. The simulated demand scenarios are created by adding stochastic error terms to the forecasted demand where the stochastic error terms are simulated based on the historical distribution of forecasting errors as previously described. The demand is quite unpredictable and the data displays a great deviation between the forecasted and actual sales and the demand scenarios are consequently composed of wide range of values for each demand point. The more inaccurate the forecast is according to the historical data, the greater will the distribution of demand scenarios be and the demand scenarios also reflect forecasting errors such as a trend to either over or underestimate the demand of certain product. The following plots give an example of the range and the distribution of simulated demand values for one of the product families.

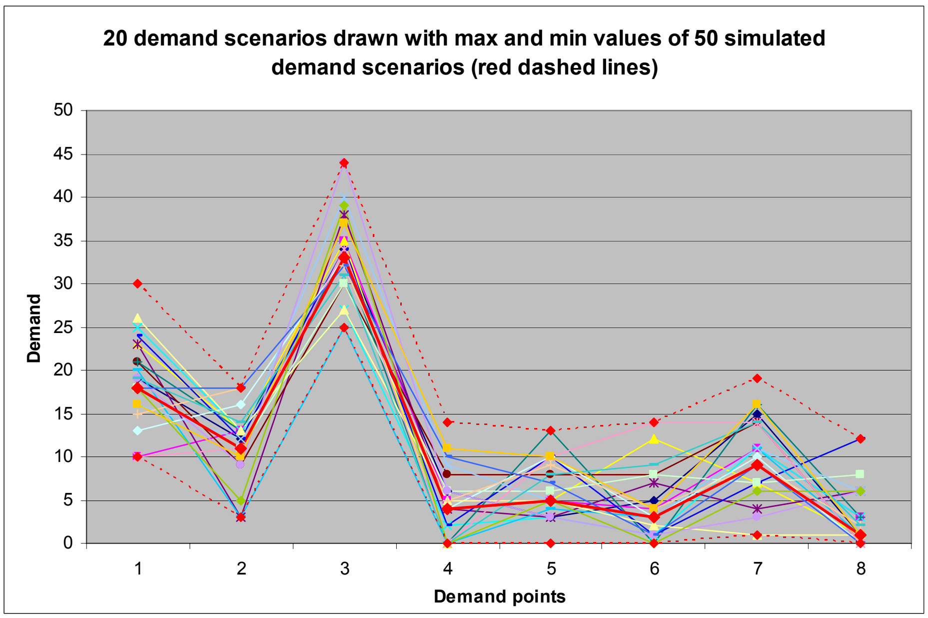


*Figure 6: The figure shows examples of the values of the simulated demand scenarios obtained for a certain product family. The dashed lines represent the max and min values of each demand point and the bold line represents the actual forecasted demand value.*

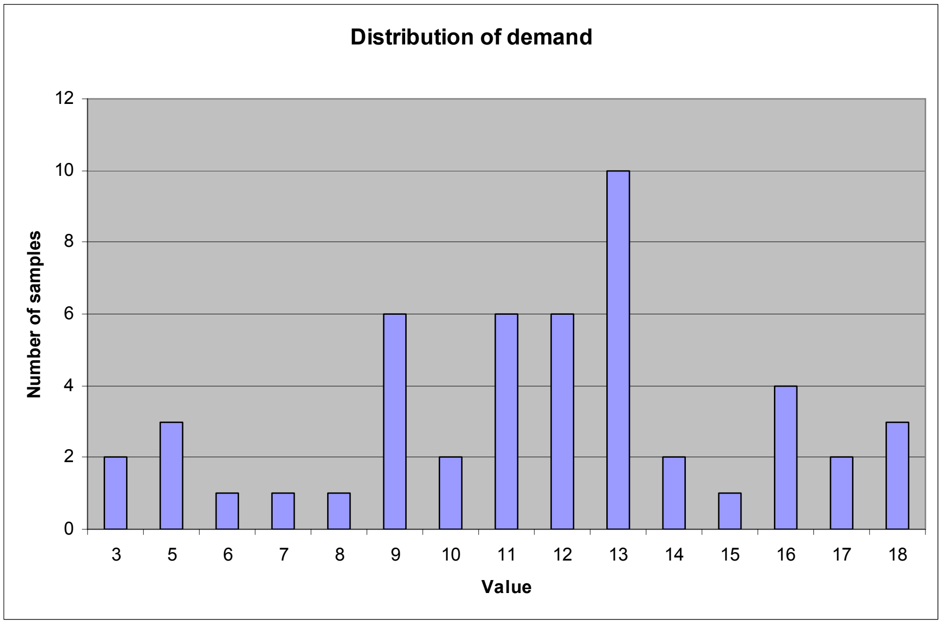


*Figure 7: The figure shows the distribution of values in certain time period for certain product family.*

We also generated demand scenarios by using a reduced range of the invert cumulative distribution function as we are interested in creating production plans and schedules that are robust for the majority of possible demand values but not all, i.e. we are less interested in uncommon extreme values. We experienced with different ranges or random numbers for simulating the error terms and Figure 8 illustrates some examples of the outcome of using random numbers from different ranges.

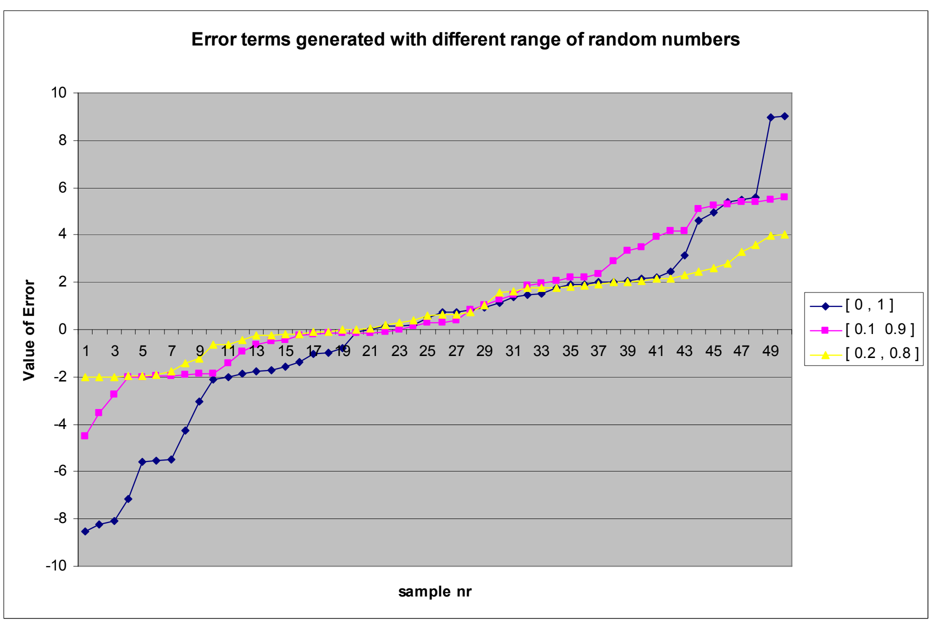


*Figure 8: The figure shows the simulated error terms obtained when different ranges of random numbers were used as input.*

We preferred to use random numbers form the range [0.05, 0.95] as it covers 90% of the cumulative distribution function and excludes the extreme error values that do no often occur (and are perhaps caused by data errors) but greatly affect the feasibly of the plans and schedules under evaluation. The distribution of demand scenarios simulated with random numbers from that range can be seen in Figure 9.

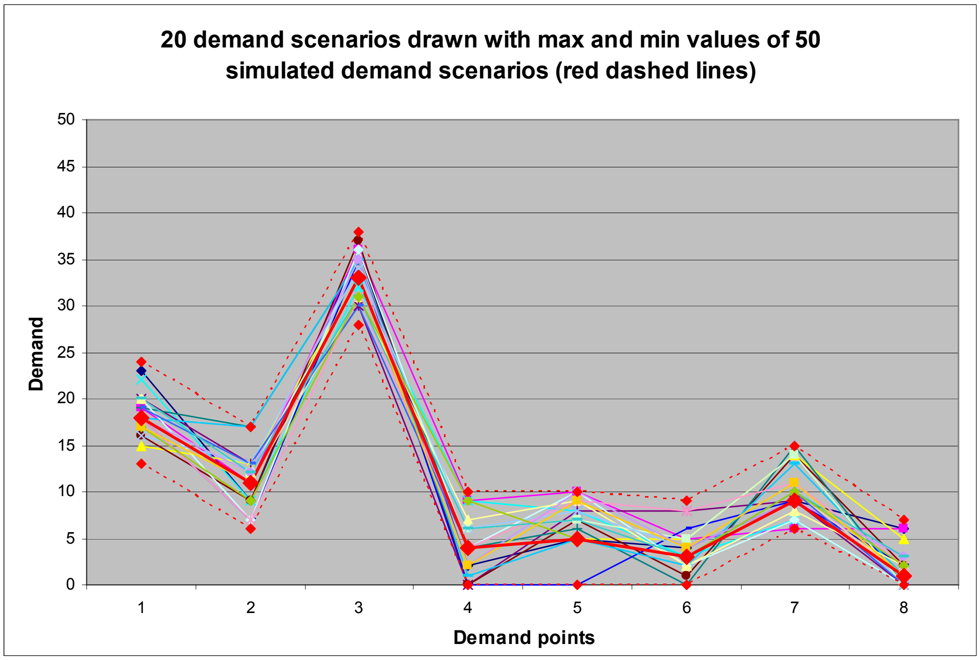


*Figure 9: The figure shows an examples of the values of the simulated demand scenarios obtained for a certain product family when random numbers within [0.05,0.95] were used to simulate the error terms. The dashed lines represent the max and min values of each demand point and the bold line represents the actual forecasted demand value.*

When Figure 6 and Figure 9 are compared it can be seen that the variation of the demand scenarios has reduced and a difference in the robustness measurements was also noticed when the production plans were evaluated as explained in the following sections.

As mentioned previously we applied a Markov based procedure to improve the generation of demand scenarios. A Markov matrix is used to check and verify the plausibility of the demand scenarios. An example of demand and the resulting Markov matrix is given in Figure 10 and Table 1.

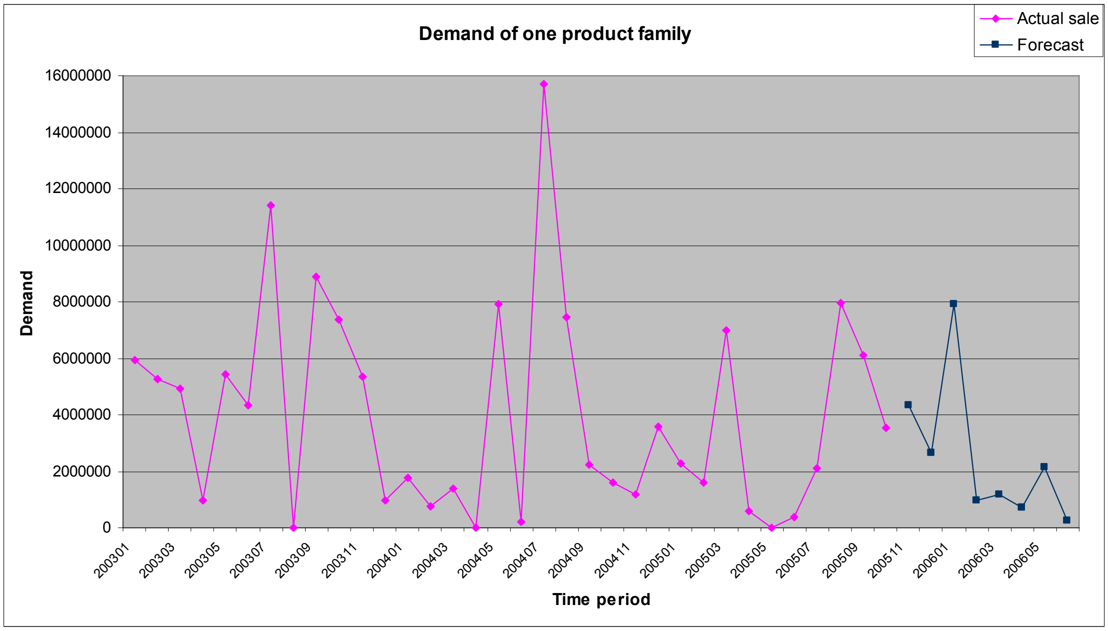


*Figure 10: An example of demand and forecast for a certain product family. The resulting Markov matrix can be seen in table 1.*

*Table 1: The Markov matrix describes the probability of moving from the states in the column on the left to the states in the row at the top. If the demand is within certain range (state) then the probability of the demand being in any of the other ranges (stages) in the next time period can be read from the matrix.*

| | **0** | **2000000** | **4000000** | **6000000** | **8000000** | **10000000** | **12000000** | **14000000** |
|---|---|---|---|---|---|---|---|---|
| **0** | 0.50 | 0.14 | 0.07 | 0.14 | 0.07 | 0.00 | 0.00 | 0.07 |
| **2000000** | 0.50 | 0.25 | 0.00 | 0.25 | 0.00 | 0.00 | 0.00 | 0.00 |
| **4000000** | 0.33 | 0.00 | 0.50 | 0.00 | 0.00 | 0.17 | 0.00 | 0.00 |
| **6000000** | 0.33 | 0.33 | 0.17 | 0.17 | 0.00 | 0.00 | 0.00 | 0.00 |
| **8000000** | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| **10000000** | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| **12000000** | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| **14000000** | 0.00 | 0.00 | 0.00 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 |

We believe the Markov matrix increased the plausibility of the demand scenarios and we believe the procedure is useful for generating them. The exception rule turned out to be important for the product families with short or sparse demand history and it must be concluded that checking the demand scenarios with the Markov transitions matrixes is not as useful for those time series although we believe it still increased the plausibility.

## 5.2 Computational times

The motivation for not solving the problem with a holistic stochastic model was the huge computational time involved and the objective with the procedure used here was to obtain more robust plans within acceptable solution times. The computational time is therefore of great importance for the usefulness of the proposed procedure. The computational time for solving each LP model is very low and solving the LP models for all the demand scenarios requires much less time than needed for solving the MILP model.

*Table 2: Computational times of the different components in the procedure to increase robustness*

| Model | Time [CPU seconds] |
|---|---|
| MILP | 152.95 |
| LP | 11.12 |
| Other | 5.11 |
| Total | 169.18 |

The overall solution time of the procedure depends mainly on the solution time of the MILP model and the number of iterations required until the procedure concludes.

## 5.3 Robustness

In the following section we give examples of results illustrated with graphs showing the $OTIF_{f,t}$ values of the plans obtained and how the robustness of the plan is increasing as the overall *OTIF* value decreases.

### 5.3.1 Example I - created with full $CDF^{-1}$ function

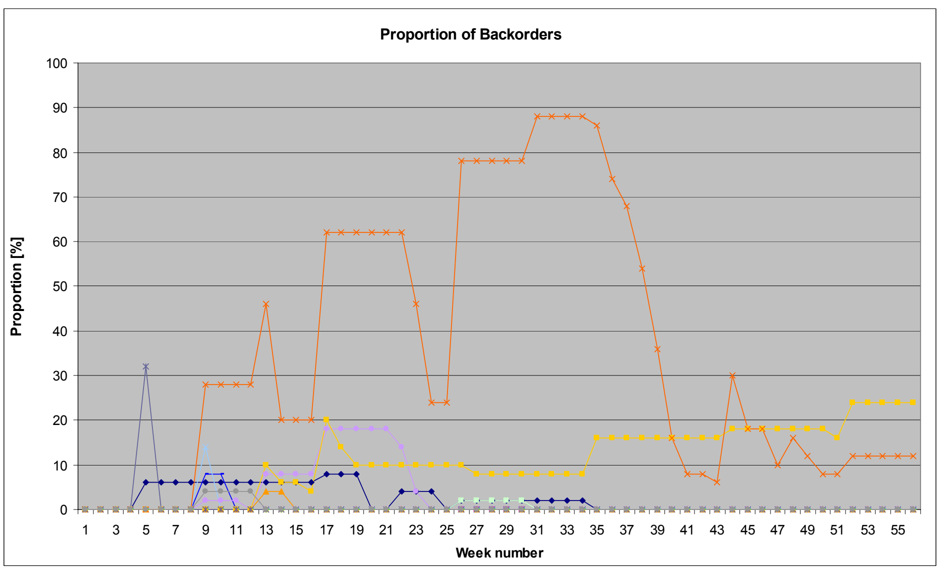


*Figure 11: The graph shows the proportions of backorders at each timeperiod of the planning horizon. OTIF = 89.70%*

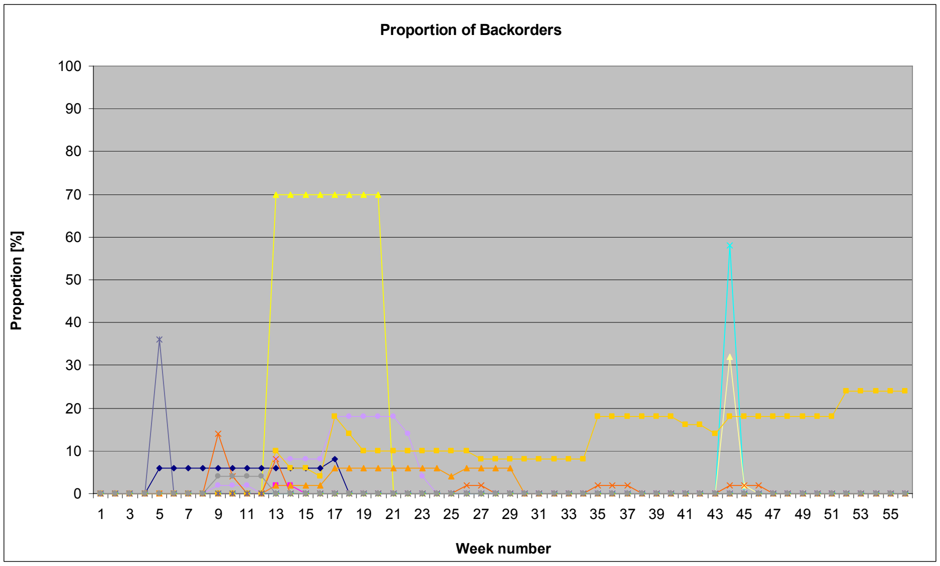


*Figure 12: OTIF = 93.88%*

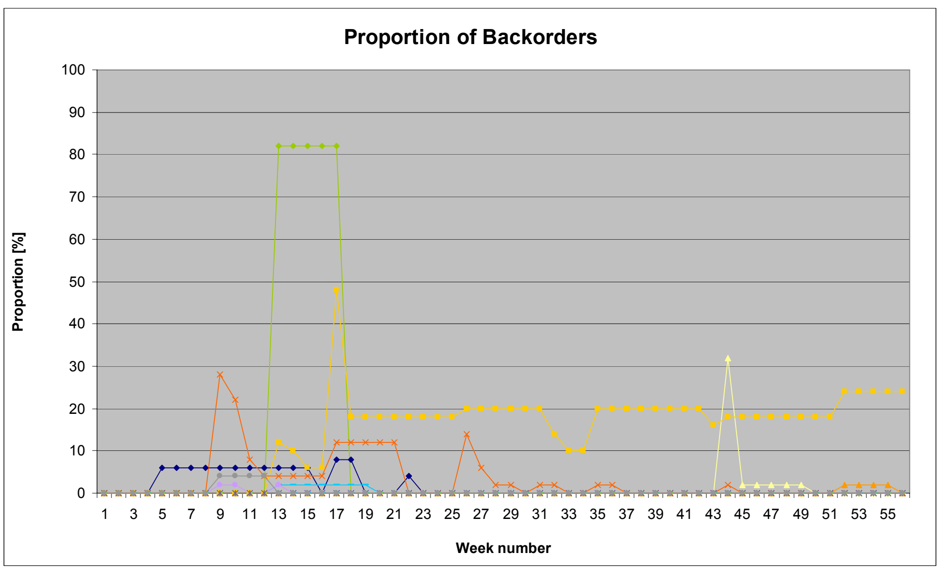


*Figure 13: OTIF = 95.12%*

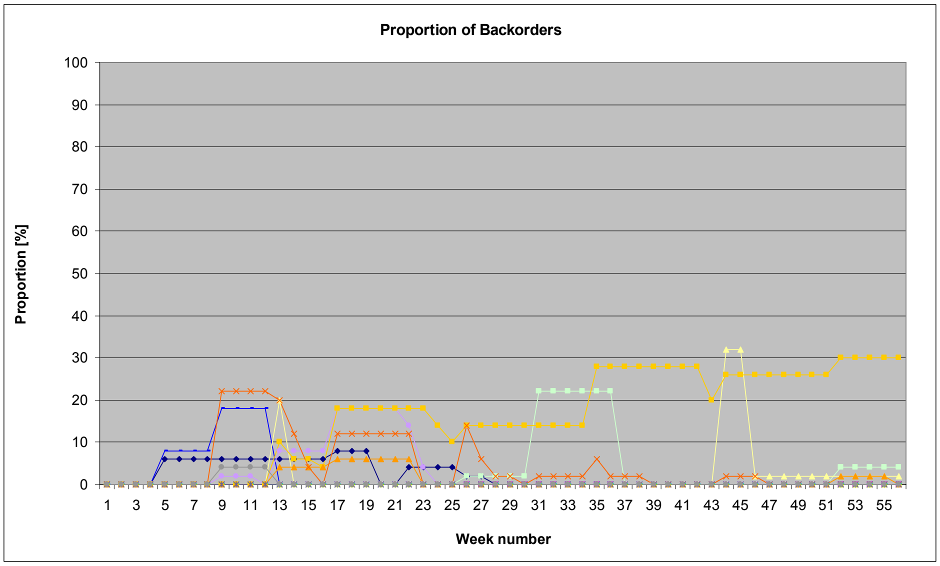


*Figure 14: OTIF = 94.00%*

The procedure terminates here because the *OTIF* value is higher than the one obtained in the previous iteration. The procedure to increase the robustness of the long-term plans changed the overall *OTIF* measure from 89.70% to 95.12%.

# 6 Conclusions and further work

Production companies, especially those operating in competitive make to order environments have to cope with uncertain and varying demand from customers which is a very challenging task and in particular when combined with the challenge of cutting down production costs at the same time. We have proposed an efficient procedure to increase robustness of production plans embedded in an integrated multi-scale planning and scheduling approach designed for planning and scheduling of a make to order production process under such conditions. The approach has been tested with industrial data and the results obtained so far indicate that it is capably of obtaining realistic and profitable solutions within acceptable computational times. We believe the generation of various demand scenarios that match the characteristics of the original data is useful for testing the robustness of the production plans in an efficient manner and the overall iterative procedure to increase the robustness was able to obtain significant improvements. The procedure to increase the robustness is rather greedy and it does not guaranty the most robust plans although he has obtained significant improvements for the test cases we have used. We believe our approach is a computationally efficient alternative to approaches based on stochastic programming methods and this kind of approach can be usefull when improvements are required for large real-world problems.

A significant amount of work remains on improving the general approach and carrying out further testing. It would also be of interest to do further comparison with other approaches existing in the literature to obtain improved understanding of the benefits and the competitiveness of our approach.

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