Axial Mixing of Binary Mixture in Horizontal Rotating Cylinder

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Keywords: axial, mixing, granular.

Introduction

Granular materials occur quite extensively in nature as well as industry. Nature manifests it from microscopic dust particles to medium-sized sand and food-grains to huge rocks and boulders to massive asteroids. Industrially, it is the second-most manipulated material after water [1]. Considerable volume of granular material is processed in industry using rotating cylinders. These include rotary kilns in cement industry, ball mills for grinding, incinerators for waste disposal and the likes. Depending on the speed of rotation, different modes of motion result which are categorized as slipping, slumping, rolling, cascading, cataracting and centrifuging [2, 3]. The rolling mode is the most employed in industrial processes using rotary equipments such as kilns, drum mixers etc.

In the rolling mode, the granular flow occurs in two regions, a flowing layer, wherein the particles cascade down along the free surface; and a larger fixed bed, wherein the particles are carried up the drum wall. Mixing is predominant within the flowing layer and is the focus of interest.

A number of researchers have conducted studies on axial mixing [4-11]. These mainly focus on axial dispersion and diffusion. Rao *et al.* [9] have studied the effect of hold-up, particle size and speed of rotation on the axial dispersion coefficient and interpreted the results in terms of the transport model of Das Gupta *et al.* [12]. This model of Das Gupta *et al.* [12] is based on particle kinematics of the system derived from random walk theory. Rao *et al.* [9] have also presented a complementary model correlating the axial dispersion coefficient with the surface diffusivity.

The objective of the present work is to study the axial mixing of a binary mixture of granular material in a horizontal rotating cylinder operated in the rolling regime. The mixing is quantified in terms of the individual axial dispersion coefficients obtained from the mono-sized mixtures of particles. The variation of the dispersion coefficients with particle size and rotational speed is studied. The approach is similar to that of Rao *et al.* [9] but is more detailed in that dynamics are studied.

Theory

Mixing in the axial direction is considered to be purely diffusive, and caused by the random collisions of particles in the flowing layer. The axial dispersion model is the most common method of modeling axial mixing in both flowing and non-flowing, horizontal, rotating drums wherein the bed of particles is treated as a continuum. In batch experiments, there is no net axial flow and hence no net axial velocity. Thus, the one-dimensional diffusion equation [7] is,

$$\frac{\partial C}{\partial t} = D_a \frac{\partial^2 C}{\partial z^2} \tag{1}$$

where, D_a is the axial dispersion coefficient, C is the concentration (number fraction) of particles varying with the position, z along the axis and t, the instantaneous time. The solution to the

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above equation 1 is obtained as,

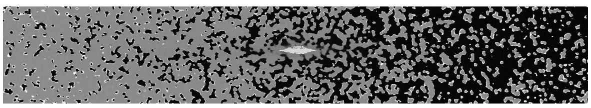
$$C(z,t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1,2...}^{\infty} \frac{1}{2n-1} \exp\Big(-(2n-1)^2 \pi^2 \frac{D_a t}{L^2} \Big) \sin\Big((2n-1)\pi \frac{z}{L} \Big) \tag{2}$$

with L as the cylinder length. At long times, i.e., at large values of $\pi^2 D_a t/L^2$, equation 2 reduces to

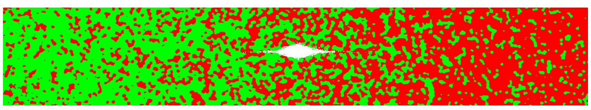
$$C(z,t) = \frac{1}{2} + \frac{2}{\pi} \exp\Big(-\pi^2 \frac{D_a t}{L^2} \Big) \sin\Big(\pi \frac{z}{L} \Big) \tag{3}$$

Experimental and analysis methodology

The granular material used for study comprise of colored glass beads. Experiments consist of tracer studies using a binary mixture of identical glass beads. Equal quantities of the two different-colored tracers are placed in a horizontal cylinder adjacent to each other such that the cylinder is exactly half-filled for all experiments. The cylinder is then rotated at certain rotational speeds and the surface concentration is captured through digital photography at specific intervals of time. The digital images thus obtained are then thresholded using a custom software program to identify the two colors at the pixel level. Figure 1(a) illustrates the original image while figure 1(b) illustrates the thresholded two-color image. The total length of the cylinder along the axis is divided into equal vertical bins. The concentration of light-colored pixels in each bin is determined by averaging the number fraction of all those light-colored pixels in that particular bin. This concentration for each of the bins is plotted against $\sin(\pi z/L)$ which corresponds to the axial position along the cylinder length, as illustrated in Figure 2. The



(a) Actual recorded photograph



(b) Thresholded image for recorded photograph

Figure 1: Images obtained from experiment

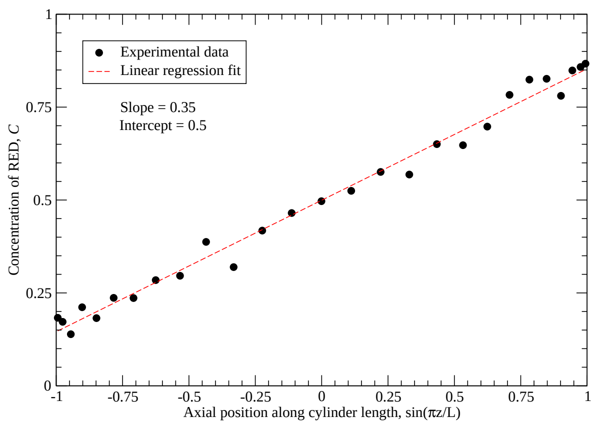


Figure 2: Fraction of red particles plotted against the entire length of the cylinder

diffusivity, in terms of the axial dispersion coefficient is then calculated from the slope of this plot from equation 3. The intercept is found to be close to 0.5.

Results and discussion

Figure 3 illustrates the variation of the concentration profiles with time for 2 mm and 3 mm particles. The concentration when plotted against the position of the bin along the cylinder length as $\sin(\pi z/L)$ yields a family of curves for the different times during the experiment. At initial times this results in a step curve, which then progresses to a straight line with passage of time. The progression of this concentration profile from a step curve to a straight line justifies the application of the theory.

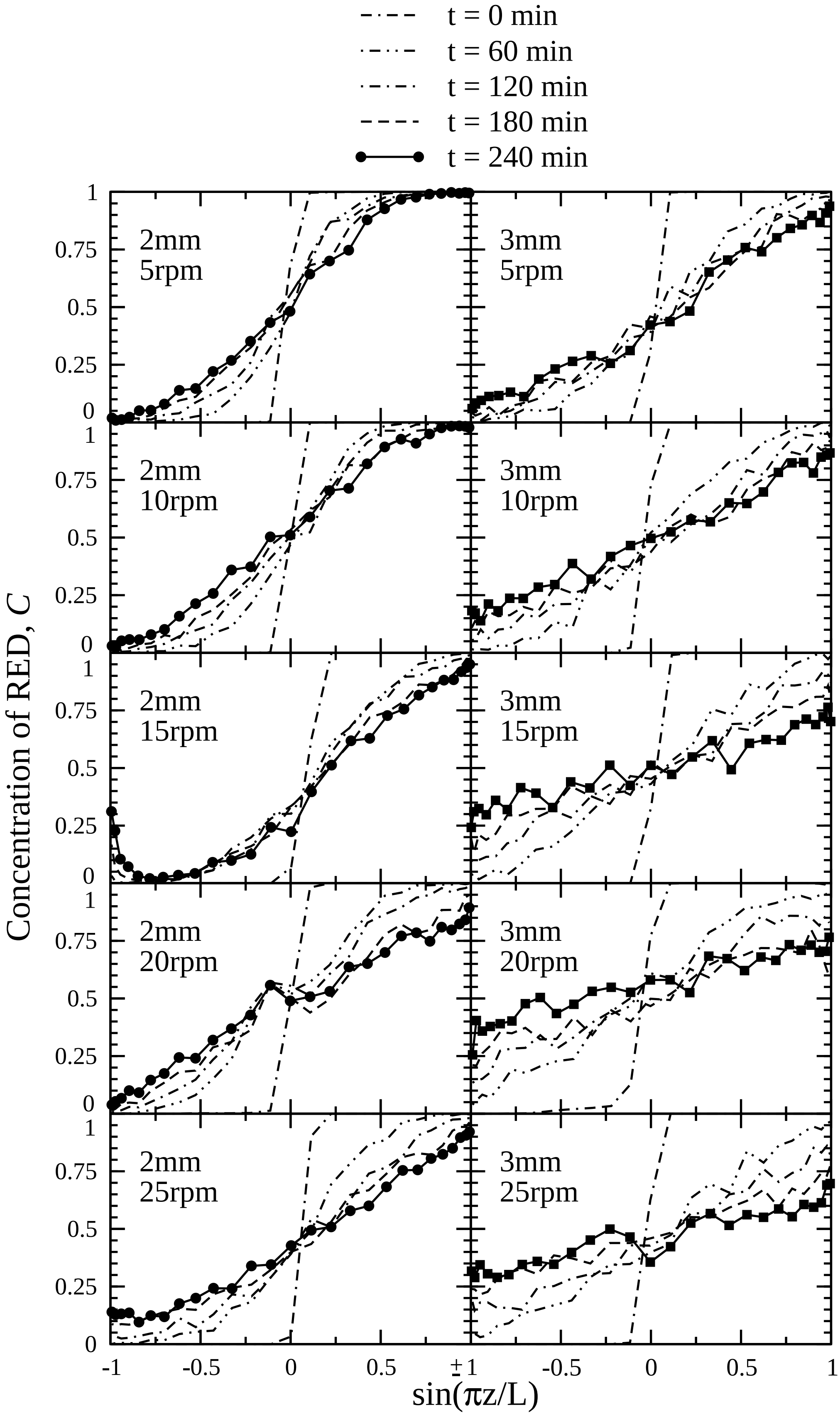


Figure 3: Fraction of red particles plotted against the entire length of the cylinder

The effect of change of rotational speed of the cylinder and particle size on particle dispersion is studied. This variation of the axial dispersion coefficient is illustrated in figure 4. From figure 4 it is evident that the particle dispersion increases with increase in cylinder rotational speed for a given particle size. For a given cylinder rotational speed, the dispersion is greater for larger particles. The dispersion coefficients estimated similarly from actual samples collected at the end of experiment, exhibit this same trend.

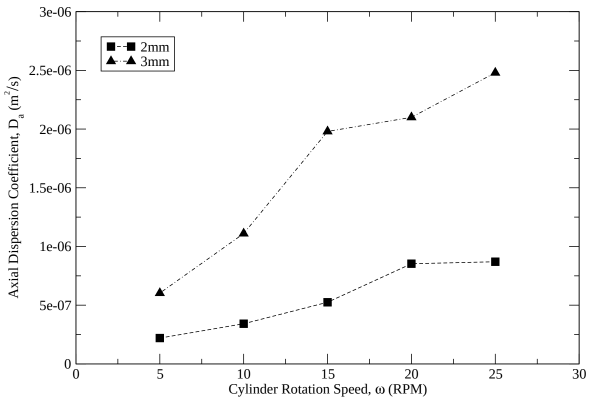


Figure 4: Variation of axial dispersion coefficient

The concentration profile obtained from the experiment for all times is compared with the theory. The theoretical concentration profiles are generated from equation 2. The number of terms in the summation (N) is chosen such that the exponent for the last term is large enough $[(2n-1)^2\pi^2 D_a t/L^2]$. The axial dispersion coefficient is initially calculated from the slope of concentration profile of the last image at the end of each experiment. The theoretical concentration profile is then generated for the entire length of the cylinder using this value of axial dispersion coefficient for N summation terms. The concentration profiles shown in figure 5 have been plotted for each hour including the initial time. The experimental data is in good agreement to the theory even at initial times. It is seen that the theoretical curves are offset from the experimental data near the central region in some cases. This offset could be because of the off-center location of the interface of tracers when placed initially inside the cylinder. This offset has not been accounted while generating the theoretical profiles.

Axial mixing is known to be quite slow compared to radial mixing. An estimate for the time required for complete mixing is attempted. The plot of the concentration against its axial position yields a curve which changes from a step curve to linearity with time. The time at which this profile becomes horizontal is considered as the time for complete mixing. The slope of the concentration profile at z=0 is plotted as the ordinate with the time of operation as abscissa. This results in a line with decreasing slope. The value of the ordinate at its intersection with the abscissa, yields the time for complete mixing. The estimates of the time for complete mixing are given in table 1.

Table 1: Time for complete mixing, by interpolation

Rotational speed (RPM)	Time for complete mixing (hr)	
	2mm	3mm
5	37.68	13.68
10	25.76	8.89
15	15.98	6.22
20	11.69	5.59
25	11.56	5.2

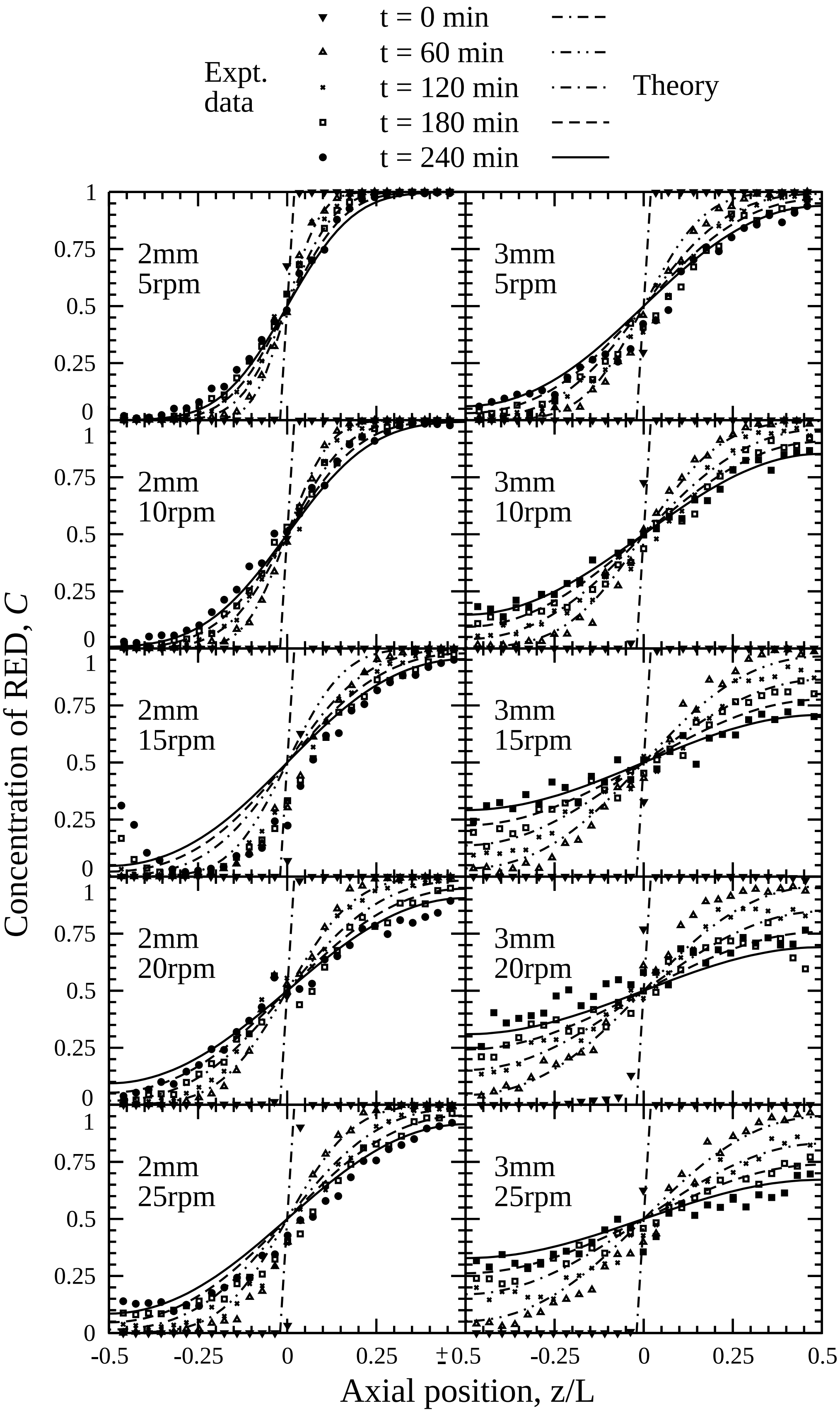


Figure 5: Comparison of theory with experiment

Conclusion

The axial dispersion of particles increases with speed of cylinder rotation for a given size. It also increases with particle size for a given speed of rotation. The experimental concentration profiles and the theoretical concentration profiles show a good match even for the initial times. It is proposed to correlate the results to the flow and diffusivity in the layer by Hajra and Khakhar [13]. The results shall also be compared with those of Rao *et al.* [9] and Sherritt *et al.* [10].

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