Nonequilibrium Compression Effects in the Dewatering of Fibrous Suspensions: Analysis Using Dual Porosity Dual Permeability Approach

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ABSTRACT

Cake filtration is an important unit operation to separate particulate phases from a fluid phase. In this operation, the suspension is forced through a screen causing the build up of a solid cake or a fibrous mat. The pressure applied for dewatering is transmitted in part to the solid phase in the cake which deforms in response. In many filtration and expression situations, the solid phase deforms elastically. However, when filter cakes are formed from flocculated suspensions or suspensions containing nanoporous and compressible particles, the solid phase shows complex plastic, visco-elastic or even visco-plastic behavior. Papermaking pulp fibers are an example of deformable nanoporous structures which can express water from their internal structure when subjected to compression during the papermaking operation.

We have developed a new approach to analyze the dewatering process where the cake solids are themselves porous and compressible using a dual compressibility/permeability concept. In the new approach, we postulate that the cake structure can be considered as a medium of dual permeability and compressibility. Thus, the solid particles within the cake are separated by a porous region we refer to as the 'macropore' region. Furthermore, the solid particles themselves are porous and compressible. The pore structure within the particles is referred to as the 'micropore' region. It is expected that the sizes of the pores between the solid particles are larger than the pores within the solid particles, at least during the initial stages of the consolidation process.

Our model yields two linked diffusion type equations which can be solved simultaneously to yield the water fluxes and concentration profiles throughout the system. We discuss applications of this approach and compare its predictions with other conventional single medium approaches. The predictions are also compared with experimental data and the validity of the dual medium approach is established.

In the present work we analyze the cake filtration operation at the microscopic level with particular reference to dewatering of the solid phase and its impact on the overall consolidation. We first develop a mathematical description incorporating the effect of dewatering based on the non-linear diffusion equation. We modify this model by including a <u>double porosity</u> description resulting in <u>two coupled diffusion equations</u> describing interchange of water due to local squeezing of the fiber cell walls in conjunction with consolidation due to decreasing macro porosity. Our composite model explicitly considers dewatering due to three phenomena namely: inter-aggregate or macro-pore space reduction, intra-aggregate or micro-pore consolidation & squeezing water into the macro-pore space and the expression from the particles without traversing the macro-pore space. Dimensional analysis of this model shows the relative significance of each mechanism. An analysis of the microscopic deformation mechanics ties into the global model by using the representative elementary volume concept (REV). The interaction between the network and the individual aggregates (fibers) is thus analyzed.

1 MATHEMATICAL MODEL

The filter cake is assumed to contain pores of two different types: the inter-aggregate or macro-pores whose volume fraction is represented by ε_1 and that of the intra-aggregate or micro-pores represented by ε_2. Defining two void ratios, e_1 and e_2, we can obtain the following mathematical description of consolidation of this dual porosity medium. Two non-linear functions are defined below as 'generalized diffusivities'[1].

$$\frac{\partial e_1}{\partial t} = \frac{\partial}{\partial m_1}\left[\frac{K_1(e_1)}{\mu \frac{dp_{s1}}{de_1}} \frac{\partial e_1}{\partial m_1}\right] + k_i\{p_{s1}(e_1) - p_{s2}(e_2)\} \qquad \text{Eq. 1}$$

$$\frac{\partial e_2}{\partial t} = \frac{\partial}{\partial m_2}\left[\frac{K_2(e_2)}{\mu \frac{dp_{s2}}{de_2}} \frac{\partial e_2}{\partial m_2}\right] - k_i\{p_{s1}(e_1) - p_{s2}(e_2)\} \qquad \text{Eq. 2}$$

The permeabilities and the phase compressibilities are given by standard equations. In this model, the change in the void ratio, e_1 represents the water expelled from the inter-aggregate or inter-fiber void space whereas the change in e_2 represents water expelled from the aggregates themselves. The diffusivity D_1 describes the ease with which water negotiates the macro-pore space and D_2 is proportional to the mobility of the water in the micropore region.

2 NUMERICAL CALCULATIONS & EXPERIMENTAL METHOD

The above model was solved using the Finite Element Method. Bilinear elements were chosen for representing the solution curves and the Galerkin method was used to solve the resulting equation set for the coefficients. The numerical implementation provided the inter- and the porosities (e_1, e_2), the hydraulic pressure (p_h) and structural pressure distributions (p_s), time evolution of these two pressures, water outflow flux, j_w and total solid consistency development. We measured the water expressed under different loading conditions in order to investigate the effect of mciropore dewatering in more detail. For this purpose, a computer controlled Instron machine was fitted with a solid piston operating inside a cylindrical cell. A pulp mat saturated with water was placed inside the cell with a screen provided at the bottom for water flow. The piston position was recorded and the mass of water expressed from the pulp pad was recorded using a balance. Various kinds of load pulses could be applied to the

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[1] $D_1(e_1) = \frac{K_1(e_1)}{\mu}\frac{de_1}{dp_{s1}}$; $D_2(e_2) = \frac{K_2(e_2)}{\mu}\frac{de_2}{dp_{s2}}$; $\mathrm{P} = \frac{\omega \mathrm{M}^2}{\overline{\overline{\mathrm{D}_1}}}$, $\kappa = \frac{\mathrm{k_i M}^2}{\overline{\overline{\mathrm{D}_1}}}$, $\gamma = \frac{\overline{\mathrm{D}_1}}{\overline{\overline{\mathrm{D}_2}}}$

piston and the piston could also be moved at defined speeds. The recorded weight of water and piston position were used to determine the average solid concentration.

3 RESULTS

Fig 1 shows the total water removed from the mat as a function of $t^{1/2}$. The $t^{1/2}$ scale demonstrates the diffusive nature of consolidation. The water outflow curve (labeled 1) is linear for most of the nip (in terms of $t^{1/2}$) when the micropore dewatering is small (case 1, $\kappa >> 1$). On the other hand, when the cell-wall's compressibility and permeability increase by larger amounts, the fibers begin to deform and expel water. This disturbs the linear trend and curves 2 and 3 indicate significant cell-wall compression. When the initial consolidation process was followed by a second consolidation in sequence, the result is shown as curve 3. . Fig 2 shows a comparison of the predictions of the model incorporating the effect of cell wall dewatering with experimental data on bleached kraft pulp pads. A good correlation can be observed between the predictions and the experimental data. We also note that the water expression is non-linear indicating that secondary consolidation is a significant phenomenon during these experimental conditions. This effect originates in the compression of the cell walls and resulting water expression.

Micro-pore Dewatering This model uses a fitted constant in the coupling term to describe water exchange between the two void spaces. This constant can be obtained only by fitting the results of water fluxes with experimental dewatering data. However, a microscopic picture of deformation can identify the macroscopic dewatering and solids content development. For this purpose, we propose a representative elementary volume consisting of a single fiber that undergoes local consolidation. Fig 4 shows the water expressed from a REV as function of time indicating the magnitude of cell wall dewatering using a fiber-in-cell model.

4 CONCLUSIONS

Cake filtration is a complex consolidation operation with many phenomena competing on a variety of time scales. Classical models are based on the assumption that the solid phase adjusts instantaneously to the pressure conditions. Newer models use phenomenological descriptions to extend this assumption and assume specific forms of linear or non-linear visco-elasticity and plasticity in order to obtain better description of the mechanics of consolidation. In this paper, we show that a dual porosity approach, conceptualizing the wet cake as a composite medium containing solid phase and pore space varying on two different scales can be more physically meaningful. The dual porosity approach provides a suitable description of secondary consolidation which matches with experimental observations. We also showed that under appropriate limiting conditions, the dual porosity model reduces to primary consolidation. Therefore, by means of a simple scaling analysis we are able to estimate the extent of cell-wall dewatering in relation to the overall dewatering phenomenon. Therefore, this approach can be a quite powerful predictive and optimizing tool for papermaking.

The dual porosity approach is particularly attractive since it can be easily extended to consider the microscale physics. For this purpose, a simple fiber-in-cell REV is used for cell-wall consolidation mechanics. This provides us a means to analyze the mechanics of other critical phenomena such as pressure non-uniformities and relate them to fiber properties. This modeling effort can develop further understanding of how the wet-pressing operation effects sheet structure and strength property developments.

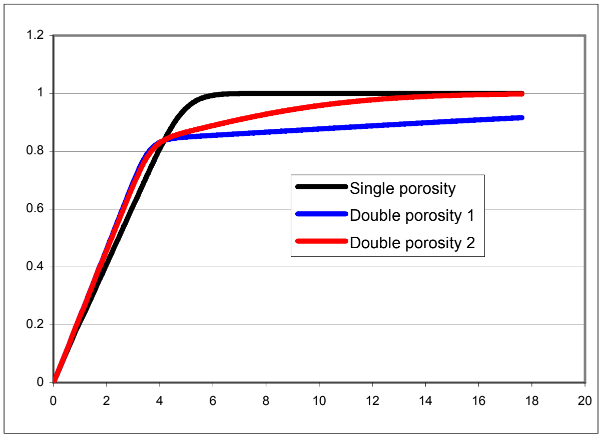

Figure 1. Solid content development with √t.

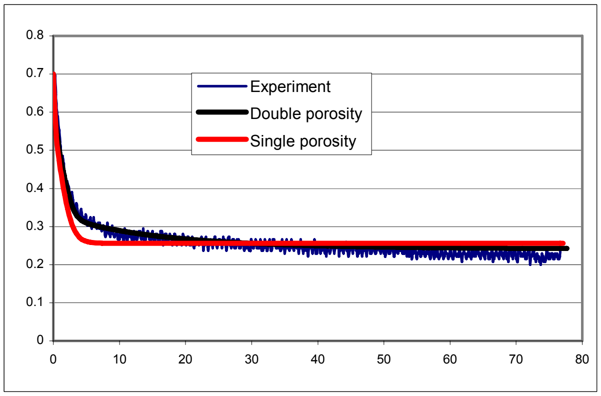

Figure 2. Experimental and calculated cake thickness. Single and double porosity model predictions shown.

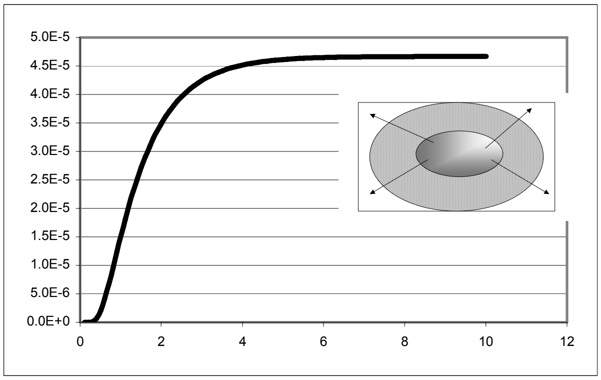

Figure 3. Water expression flux from fiber or aggregate as function of time. REV is shown in inset.