A Model for Axial Voidage Profile in Risers

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Abstract

Particles entering at the bottom of a riser are assumed to aggregate as clusters and accelerate upwards due to drag force exerted by gas along the height in the "Accelerating Region" till they attain a constant final velocity in the "Terminal Region". In the accelerating zone, cluster velocity increases and axial solid hold up decreases with height. In the terminal region, clusters attain a constant velocity and solid hold up along the height of the riser. The model compares well with the literature data.

1. Introduction

Riser reactors are extensively used for fluid catalytic cracking, combustion of coal, calcination of cement raw meal etc. In such reactors, particles are introduced at the bottom of a vertical column to be carried upwards by the drag of a fluid flowing at a sufficiently high velocity. Volume fraction of solids in risers has been extensively studied as conversion in riser reactors depend on it.It has been observed that each gas velocity has a limiting saturation carrying capacity of solids. At solid feed rates smaller than the saturation carrying capacity, particles are accelerated along the height due to fluid drag in the "Accelerating Region" till they attain a constant velocity in the “Terminal Region". It is observed that slip velocities, even in the terminal region, are much higher than terminal velocity of a single particle. This is attributed to the aggregative nature of gas solid fluidization systems and particles are observed to move as aggregates or clusters.

For solid feed rates greater than the saturation carrying capacity, some particles get disengaged from gas flow along the height as gas can not carry them upwards and such disengaged particles reflux along the walls. This leads to accumulation of solids in the column. Such beds are known as choked beds. Present work is focused on non choked dilute phase transport regime.

2. The Model

For solid flow rates smaller than elutriation rates, particles in the form of aggregates, referred to as clusters, are accelerated and transported by gas. A cluster is considered to be a deformable impermeable packet of solids. Gas flowing around the cluster and the wake formed behind the cluster (some call it voids and some call it ghost bubbles) together deform the cluster as an inverted paraboloid as shown in fig.1. This can be a description to explain the observations of Horio and Kuroki (1994) on the shape of clusters in risers.

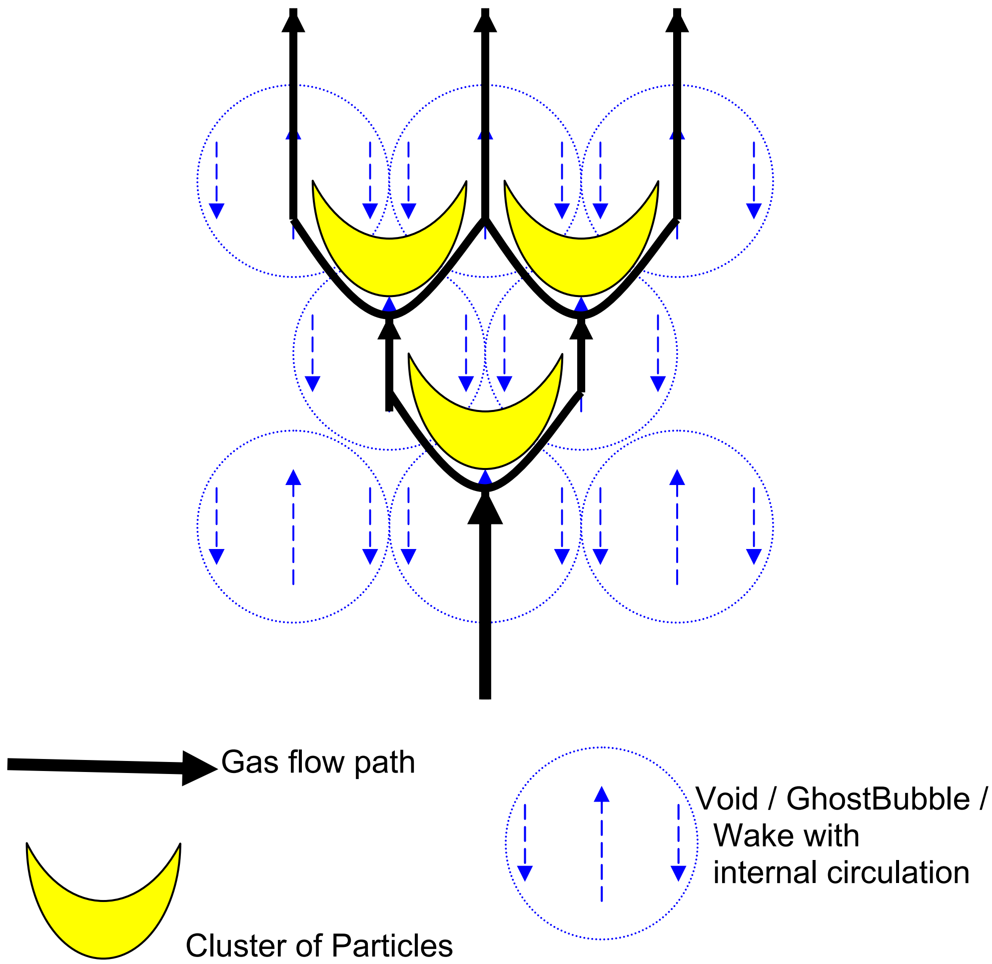

Fig.1 Pictorial visualization of Clusters, Voids and gas flow paths

Subbarao (1986) proposed that ratio of cluster volume to void volume will be in proportion to the ratio of their volumetric flow rates

$$\left(\frac{D_c}{D_v}\right)^3 = \left(\frac{W}{u_{og}\ \rho_c}\right) \tag{1}$$

The wakes behind clusters interact and behave as voids/ ghost bubbles. Such ghost bubbles experience buoyancy force as their density is much lower than average bed density and their rise velocity is a function of their size as observed by Horio and Kuroki (1994). It can be represented by

$$u_{vr} = 0.71\left(g\ D_v\right)^{1/2} \tag{2}$$

The clusters can coalesce to form bigger clusters leading to bigger wakes/ghost bubbles. The bigger wakes develop higher rise velocities and internal circulation. This can lead to break up of the larger ghost bubble due to entrainment of particles if particle terminal velocity is smaller than the internal circulation. Thus, clusters can coalesce and break up as ghost bubbles coalesce and break up leading to an equilibrium. The equilibrium ghost bubble size can be obtained as

$$D_v = \frac{2\,u_t^{\,2}}{g} \tag{3}$$

For coarse particle systems in smaller diameter risers, diameter of the void is limited by column diameter. Subbarao (1986) proposed that void diameter can be estimated as

$$\begin{aligned} D_v &= \frac{2u_t^2}{g} \quad \text{for} \quad D_v < \frac{D_t}{4} \\ &= \frac{D_t}{4} \quad \text{for} \quad D_v > \frac{D_t}{4} \end{aligned} \tag{4}$$

These can be combined to accommodate both the above equations as follows:

$$\mathrm{D_v} = \frac{1}{\dfrac{1}{2\mathrm{u_t^2/g}} + \dfrac{1}{\mathrm{D_t}/4}} \tag{5}$$

Local cluster and gas velocities are

$$\mathrm{u} = \frac{\mathrm{u_o}}{(1-\delta_c)}\,; \qquad \mathrm{v_c} = \frac{W}{\rho_c \delta_c} \tag{6}$$

Considering that all the particles are in the form of clusters only:

$$\delta_c(1-\varepsilon_c) = (1-\varepsilon) \tag{7}$$

Density of clusters can be estimated as

$$\rho_c = \rho_p(1-\varepsilon_c) + \rho_f\varepsilon_c \tag{8}$$

Assuming the cluster size is a constant along the height of riser, cluster velocity in the riser can be estimated by solving force balance on a cluster written as

Net Force = Drag Force - (Gravity force - Buoyancy force)

$$m_c \frac{dv_c}{dt} = \frac{1}{2} C_{Dc} A_c \rho_f (u - v_c)^2 - m_c g \left(1 - \frac{\rho_f}{\rho_c}\right) \quad (9)$$

This equation can be solved for each region easily to obtain simple asymptotic relationships.

2 a. Acceleration region:

In the acceleration region, drag force dominates and Eq.9 can be written as:

$$\frac{dv_c}{dt} = \frac{3C_{Dc}}{4D_c} \frac{\rho_f}{\rho_c} (u - v_c)^2 \quad (10)$$

$$-u \frac{d(u - v_c)}{(u - v_c)^2} = \frac{3C_{Dc}\rho_f}{4D_c\rho_c} dh \quad (11)$$

Assuming inertial conditions to prevail, C_{Dc} is assumed to be a constant.
With the initial condition

$$h = 0, \quad v_c = 0 \quad (12)$$

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this equation is solved to obtain

$$\frac{u}{v_c} - 1 = \frac{4}{3C_{Dc}} \frac{(1 - \varepsilon_c)\rho_p}{\rho_f} \frac{D_c}{h} \quad (13)$$

From Eq.5, 6 and 13

$$\frac{1-\varepsilon}{\varepsilon - \varepsilon_c} - \frac{W}{\rho_p (1 - \varepsilon_c) u_o} = \frac{1}{3C_{Dc}(1 - \varepsilon_c)^{1/3}} \left[\frac{W}{\rho_p u_o}\right]^{4/3} \left[\frac{\rho_p}{\rho_f}\right] \frac{4D_v}{h} \quad (14)$$

2 b. Terminal region :

In this region, clusters move upward at a constant velocity with no acceleration with upward gas drag force balanced by downward gravitational force. From Eq. 9

$$\frac{3}{4} c_D \left(\frac{D_c (u - v_c)_t \rho_f}{\mu_f}\right)^2 = \frac{D_c^3 g \rho_f (\rho_c - \rho_f)}{\mu_f^2} \quad (15)$$

where slip velocity in terminal region $(u\text{-}v_c)_t$ is cluster terminal velocity u_{ct}.

$$(\mathrm{u} - \mathrm{v_c})_t = \frac{u_o(1-\varepsilon_c)}{\varepsilon - \varepsilon_c} - \frac{w}{\rho_p(1-\varepsilon)} = \mathrm{u_{ct}} \qquad (16)$$

Assuming clusters move under inertial control regime and drag coefficient is a constant

$$u_{ct} = \left(\frac{4}{3c_{Dc}} \frac{D_c g(\rho_p - \rho_f)(1-\varepsilon_c)}{\rho_f} \right)^{1/2} \qquad (17)$$

Eq.17 can be used to find out cluster slip velocity (terminal velocity) using Eq.5 for diameter of cluster as .

$$\frac{(1-\varepsilon\ )}{\varepsilon - \varepsilon_c} - \frac{w}{u_o\ \rho_p(1-\varepsilon_c)} = \left[\frac{4(1-\varepsilon_c)^{2/3}}{3c_{Dc}} \right]^{1/2} \left[\frac{\mathrm{W}}{\rho_p \mathrm{u_o}} \right]^{1/6} \left[\frac{\rho_\mathrm{p}}{\rho_\mathrm{f}} \right]^{1/2} \frac{(g\ D_v)^{1/2}}{u_o} \frac{(1-\varepsilon)}{(1-\varepsilon_c)} \qquad (18)$$

Eqs. 14 and 18 can be used to estimate axial solid hold up and higher one of the two will be solid hold up in the riser. Considering that axial solid hold up at the end of accelerating region is equal to the solid hold up at the start of terminal region, height of accelerating region can be estimated from Equation 14 and 18.

3 Results and Discussion

These equations can be presented as

$$\mathrm{Y} = \mathrm{C_{acc}}\,\mathrm{X} + \mathrm{C_{ter}}\,\mathrm{Z} \qquad (19)$$

with

$$\mathrm{Y} = \frac{1-\varepsilon}{\varepsilon - \varepsilon_\mathrm{c}} - \frac{\mathrm{W}}{\rho_\mathrm{p}(1-\varepsilon_\mathrm{c})\mathrm{u_o}} \qquad (20)$$

$$\mathrm{X} = \left[\frac{\mathrm{W}}{\rho_\mathrm{f}\mathrm{u_o}} \right]^{4/3} \left[\frac{\rho_f}{\rho_p} \right]^{1/3} \frac{4\mathrm{D_v}}{\mathrm{h}} \qquad (21)$$

$$Z = \left[\frac{\mathrm{W}}{\rho_\mathrm{f}\mathrm{u_o}} \right]^{1/6} \left[\frac{\rho_\mathrm{p}}{\rho_\mathrm{f}} \right]^{1/3} \left[\mathrm{g}Dv \right]^{1/2} \frac{1}{u_o} \frac{1-\varepsilon}{1-\varepsilon_c} \qquad (22)$$

and

$$C_{acc} = \frac{1}{3C_{Dc}(1-\varepsilon_c)^{1/3}} \qquad (23)$$

$$C_{ter} = \left[\frac{4(1-\varepsilon_c)^{2/3}}{3c_{Dc}}\right]^{1/2} \qquad (24)$$

C_{acc} and C_{ter} needs to be experimentally evaluated as there is no way to predict drag coefficient. Experimental information can be presented as a graph between Y and X to estimate the constant C_{acc} and Y and Z to estimate the constant C_{ter}. Data of Gambhir (1999) is reasonably well explained by assigning C_{acc} a value of 0.2799 and C_{ter} a value of 0.748 by assuming cluster void fraction to be 0.5 and drag coefficient to be 1.5.

These model equations are also compared with experimental observations of other literature data extensively. The trend is well explained by these equations though C_{acc} differed for different investigators (from 0.14 to 0.57).

NOMENCLATURE

C_{acc} Constant for acceleration region
C_{ter} Constant for terminal region
D_c Diameter of Cluster, m
D_p Diameter of particle, m
D_t Column Diameter, m
D_v Diameter of the void, m
g Acceleration due to gravity, m/s^2
h Height of the riser, m
m_c Mass of the cluster, kg
u Actual gas velocity, m/s
u_o Superficial gas velocity, m/s
u_{vr} Void rise velocity
u_t Terminal velocity of a single particle
v_c velocity of clustes, m/s
W Solid feed flux, $kg/m^2/s$
X Prameter
Y Parameter
Z Parameter

Greek Letters

δ_{cl} Cluster fraction
ε_c Cluster voidage
$(1-\varepsilon)$ Average bed particle fraction
μ_f Viscosity of gas, kg/m/s
ρ_c Density of cluster, kg/m^3
ρ_f Density of gas, kg/m^3
ρ_p Particle density, kg/m^3

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