OPTIMAL DESIGN OF BATCH-STORAGE NETWORK WITH FINANCIAL TRANSACTIONS AND CASH FLOWS

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ABSTRACT

This paper presents an integrated analysis of production and financing decisions. We construct a model in which a cash storage unit is installed to manage the cash flows associated with production activities such as raw material procurement, process operating setup, inventory holding costs and finished product sales. Temporary financial investments are allowed to increase profit. The production plant is modeled by the Batch-Storage Network model with Recycle Streams developed by Yi and Reklaitis (2003). The objective function of the optimization is minimizing the opportunity costs of annualized capital investment and cash/material inventory minus the benefit to stockholders. The major constraints of the optimization are that the material and cash storage units must not be depleted. A production and inventory analysis formulation, the periodic square wave (PSW) model, provides useful expressions for the upper/lower bounds and average levels of the cash and material inventory holdups. The expressions for the Kuhn-Tucker conditions of the optimization problem are reduced to a subproblem and analytical lot sizing equations. This subproblem is then decomposed into two separable concave minimization network flow problems whose solutions yield the average material and cash flow rates through the networks. The production and financial transaction lot sizes and startup times can be determined by analytical expressions after the average flow rates are already known. We show that, when financial factors are taken into consideration, the optimal production lot and storage sizes are smaller than is the case when such factors are not considered. An illustrative example is presented to demonstrate the potential of this approach.
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Introduction

Most production planning and scheduling models developed to date in process system engineering endeavor to identify a plan or schedule that minimize the overall cost while satisfying production capacity and demand constraints. A key assumption of these models is that an unlimited amount of cash is available. In practice, however, cash is usually the scarcest resource and cash availability is an important factor influencing the feasibility of a production plan or schedule. It is commonplace for a planned production to be unrealizable for a period of time due to a lack of cash to cover the production costs, resulting in other resources being under-utilized during that period. Then, when the cash does become available, overproduction must be conducted to fill backorders. These inefficiencies can cause substantial loss of profit. In fact, every aspect of production involves financial transactions and cash flows. Manufacturers purchase raw materials for production purposes, creating accounts payable owed to suppliers. The actual disbursement of cash occurs when the payment medium used to pay for the purchase, such as a check, is redeemed through the bank system. Raw materials are converted into finished products by consuming operational utilities that incur costs associated with their purchase or production. The finished product inventory is converted into accounts receivable as customers make purchases on credit. Receivables are then collected from customers remitting payment to the company. Cash is received when the payment medium, such as a check, is redeemed through the bank system. In the mean time, the company must pay taxes, salaries, and disburse loans. To prevent temporary cash shortages in some circumstances, new loans must be arranged. If there is excess cash, temporary investment in marketable securities should be considered to increase income. If operating cash flows are not well managed, seemingly profitable firms may experience financial strains that could potentially lead to bankruptcy. For example, if too many resources are tied up in inventory or accounts receivable, then even a profitable company may not be able to pay its bills. Therefore, it is essential to consider cash flow when making production planning and scheduling decisions. A successful firm manages its operations so as to optimize both profit and cash flow.

Yi and Reklaitis (2000) developed a novel production and inventory analysis method called the periodic square wave (PSW) method and used it to determine the optimal design of a parallel batch-storage system. They subsequently extended the PSW formalism to model the more complicated plant structure of a sequential multistage batch-storage network (Yi and Reklaitis, 2002). In another study (Yi and Reklaitis, 2003), the same authors suggested a non-sequential network structure that can deal with recycled material flows in a plant site. In the present study, we extend the batch-storage network model suggested by Yi and Reklaitis (2003) to include both the cash storage and the financial transactions required to support the production activities. In the proposed model, all production activities are accompanied by financial transactions in which the appropriate amount of cash is withdrawn from the cash storage to pay for the costs. Cash is inputted to the storage after delivery of the finished product to consumers. The cash inventory should be managed so as to ensure that it is not depleted. The objective function of the optimization is minimizing the opportunity costs of annualized capital investment and cash/material inventory minus the benefit to stockholders.

Definition of Parameters and Variables

We use the plant structure introduced by Yi and Reklaitis (2003). Suppose that there exists a cash storage unit that, through financial transactions, operates the chemical plant composed of batch process set *I* and material storage unit set *J,* as depicted in Figure 1. Let set *N* with subscript *n* represent the set of temporary financial investments in marketable securities and set *O* with subscript *o* represent the set of stockholders. Corporation income tax is usually proportional to net profit and is thus considered as a payment to a fictitious stockholder without loss of generality. Sales tax, which is usually proportional to sales revenue, is collected from customers when finished products are delivered to them and is paid to the IRS (Internal Revenue Service) yearly. In chemical companies, total labor cost is usually proportional to total sales revenue. We ignore the cash flow of labor costs in the present study because it is treated in the same way as sales tax. Note that the setup cost usually includes the operating labor cost.

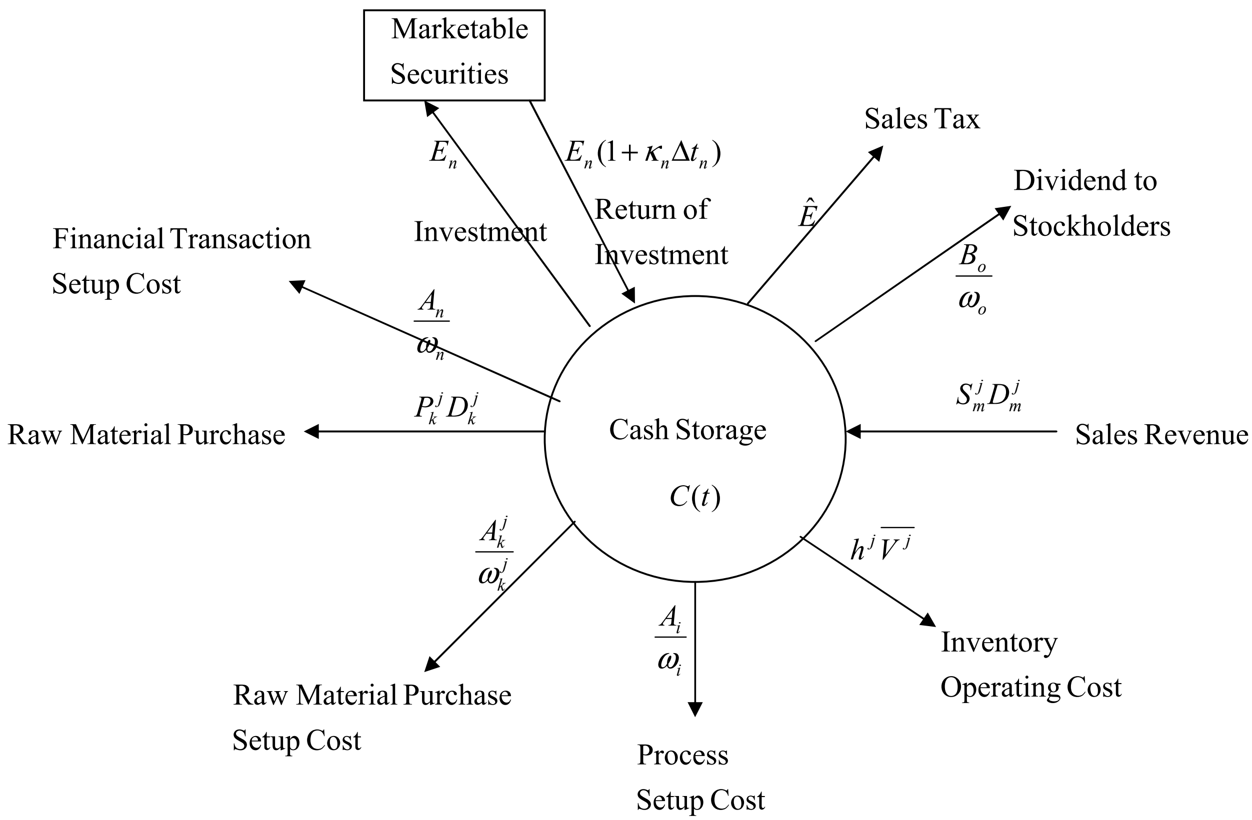

Figure 1. Cash Storage and Financial Transactions

The incoming cash flows into the cash storage unit are:

CF1) Collection of account receivable after collection drifting time Δt_m^j from shipping of the finished product to consumer m. (Sales tax is included.)

CF2) Return of temporary financial investment n at interest rate κ_n ($/$/year) after investment period Δt_n.

The outgoing cash flows from the cash storage unit are:

CF3) Disbursement of account payable after disbursement drifting time Δt_k^j for raw material purchase from supplier *k*.
CF4) Temporary financial investment at interest rate κ_n ($/$/year) for investment period Δt_n.
CF5) Financial transaction of purchase setup cost.
CF6) Financial transaction of investment setup cost.
CF7) Financial transaction of processing setup cost.
CF8) Inventory operating cost.
CF9) Dividend to stockholders (can include corporation income tax).
CF10) Sales tax payment to IRS with tax rate ς ($/$). (Labor cost can be treated as the same way.)

In the present work we do not consider the case of the corporation taking bank loans to top up their cash reserves because, once any initial cash shortage has been addressed, it should be unnecessary to take further loans, and such loans consume the benefits to stockholders'. We assume that the temporary financial investment has a setup cost of A_n $/transaction. This transaction cost is withdrawn from the cash storage when the financial investment is made, as is defined in CF6. In addition, we assume that the setup cost transactions of CF5, CF6 and CF7 and the inventory operating cost of CF8 are paid proportionally with material processing. In other words, the cash flows of the setup cost transactions and their material flows have the same cycle time, startup time and storage operation time fraction but different batch sizes. The cash flow of the inventory operating cost is proportional to the inventory level. Each cash flow is represented by the PSW model as follows:

$$PSW(t;D,\omega,t',x) = D\omega\left[\operatorname{int}\left[\frac{t-t'}{\omega}\right] + \min\left\{1, \frac{1}{x} res\left[\frac{t-t'}{\omega}\right]\right\}\right] \quad (1)$$

or

$$PSW'(t;B,\omega,t',x) = B\left[\operatorname{int}\left[\frac{t-t'}{\omega}\right] + \min\left\{1, \frac{1}{x} res\left[\frac{t-t'}{\omega}\right]\right\}\right] \quad (2)$$

where *D* is the average flow rate, *B* is the batch size, ω is the cycle time, t' is the startup time, *x* is the storage operation time fraction and *t* is time (Yi and Reklaitis 2002). Note that $D = \frac{B}{\omega}$. We refer to Eq. (1) as the first type of PSW flow and Eq. (2) as the second type of PSW flow. Note that average flow rate is used in the first type whereas batch size is used in the second type. The two types of PSW flow have different upper/lower bounds and partial derivatives. The average flow rate of sales tax is proportional to that of total sales revenue, that is, $\hat{E} = \varsigma \sum_{j=1}^{|J|} \sum_{m=1}^{|M(j)|} S_m^j D_m^j$, where ς ($/$) is the sales tax rate. The cycle time, startup time and storage operation time fraction of sales tax are given parameters. Note that the sales tax startup time, $\hat{t}_m^j$, is the first taxing date after $t_m^j + \Delta t_m^j$.

Nonlinear Optimization Model

We define *C(0)* as the initial cash inventory and *C(t)* as the cash inventory at time *t*. Then, the cash inventory at time *t* is calculated by adding the incoming flows CF1 and CF2 to the initial cash inventory and subtracting the outgoing flows CF3~CF10.

$$
\begin{aligned}
C(t) &= C(0) + \sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}(1+\varsigma)S_m^j PSW(t; D_m^j, \omega_m^j, t_m^j + \Delta t_m^j, x_m^j) \\
&+ \sum_{n=1}^{|N|} PSW(t; E_n(1+\kappa_n \Delta t_n), \omega_n, t_n + \Delta t_n, x_n) \\
&- \sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|} P_k^j PSW(t; D_k^j, \omega_k^j, t_k^j + \Delta t_k^j, x_k^j) - \sum_{n=1}^{|N|} PSW(t; E_n, \omega_n, t_n, x_n) \\
&- \sum_{j=1}^{|J|}\sum_{k\in\{D_k^j\}^+}^{|K(j)|} PSW'(t; A_k^j, \omega_k^j, t_k^j, x_k^j) - \sum_{n\in\{E_n\}^+}^{|N|} PSW'(t; A_n, \omega_n, t_n, x_n) \\
&- \sum_{i\in\{D_i\}^+}^{|I|} PSW'(t; A_i, \omega_i, t_i^{in}, x_i^{in}) - \sum_{j=1}^{|J|} h^j \int_0^t V^j(t)dt - \sum_{o=1}^{|O|} PSW'(t; B_o, \omega_o, t_o, x_o) \\
&- PSW(t; \varsigma \sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|} S_m^j D_m^j, \hat{\omega}, \hat{t}_m^j, \hat{x})
\end{aligned} \tag{3}
$$

where $\{D_k^j\}^+ \equiv \{k \mid D_k^j > 0\}$, $\{E_n\}^+ \equiv \{n \mid E_n > 0\}$ and $\{D_i\}^+ \equiv \{i \mid D_i > 0\}$, that is, the index sets with positive average flow rates. The average level of the cash inventory ($\overline{C}$) and the lower bound of the cash inventory ($\underline{\underline{C}}$) are easily calculated by using the properties of PSW flow model (Yi and Reklaitis, 2003).

$$
\begin{aligned}
\overline{C} &= C(0) + \sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}(1+\varsigma)S_m^j \overline{PSW}(t; D_m^j, \omega_m^j, t_m^j + \Delta t_m^j, x_m^j) \\
&- \sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|} P_k^j \overline{PSW}(t; D_k^j, \omega_k^j, t_k^j + \Delta t_k^j, x_k^j) \\
&- \sum_{i\in\{D_i\}^+}^{|I|} \overline{PSW'}(t; A_i, \omega_i, t_i^{in}, x_i^{in}) - \sum_{j=1}^{|J|}\sum_{k\in\{D_k^j\}^+}^{|K(j)|} \overline{PSW'}(t; A_k^j, \omega_k^j, t_k^j, x_k^j) - \sum_{j=1}^{|J|} h^j \overline{V^j} t \\
&- \sum_{n=1}^{|N|} \overline{PSW}(t; E_n, \omega_n, t_n, x_n) + \sum_{n=1}^{|N|} \overline{PSW}(t; E_n(1+\kappa_n \Delta t_n), \omega_n, t_n + \Delta t_n, x_n) \\
&- \sum_{n\in\{E_n\}^+}^{|N|} \overline{PSW'}(t; A_n, \omega_n, t_n, x_n) - \sum_{o=1}^{|O|} \overline{PSW'}(t; B_o, \omega_o, t_o, x_o) \\
&- \overline{PSW}(t; \varsigma \sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|} S_m^j D_m^j, \hat{\omega}, \hat{t}_m^j, \hat{x})
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
\underline{\underline{C}} &= C(0) + \sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}(1+\varsigma)S_m^j\,\underline{\underline{PSW}}(t;D_m^j,\omega_m^j,t_m^j+\Delta t_m^j,x_m^j) \\
&- \sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}P_k^j\,\overline{\overline{PSW}}(t;D_k^j,\omega_k^j,t_k^j+\Delta t_k^j,x_k^j) \\
&- \sum_{i\in\{D_i\}^+}^{|I|}\overline{\overline{PSW'}}(t;A_i,\omega_i,t_i^{in},x_i^{in}) - \sum_{j=1}^{|J|}\sum_{k\in\{D_k^j\}^+}^{|K(j)|}\overline{\overline{PSW'}}(t;A_k^j,\omega_k^j,t_k^j,x_k^j) - \sum_{j=1}^{|J|}h^j\,\overline{\overline{V^j}}\,t \\
&- \sum_{n=1}^{|N|}\overline{\overline{PSW}}(t;E_n,\omega_n,t_n,x_n) + \sum_{n=1}^{|N|}\underline{\underline{PSW}}(t;E_n(1+\kappa_n\Delta t_n),\omega_n,t_n+\Delta t_n,x_n) \\
&- \sum_{n\in\{E_n\}^+}^{|N|}\overline{\overline{PSW'}}(t;A_n,\omega_n,t_n,x_n) - \sum_{o=1}^{|O|}\overline{\overline{PSW'}}(t;B_o,\omega_o,t_o,x_o) \\
&- \overline{\overline{PSW}}(t;\varsigma\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}S_m^jD_m^j,\hat{\omega},\hat{t}_m^j,\hat{x})
\end{aligned}
\tag{5}
$$

We assume that the cash in-flows and out-flows are balanced in the long run. The average flow rates of cash flows into and out of the cash storage unit satisfy the following balance equation:

$$
\begin{aligned}
&\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}S_m^jD_m^j + \sum_{n=1}^{|N|}\kappa_n\Delta t_nE_n \\
&= \sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}P_k^jD_k^j + \sum_{i\in\{D_i\}^+}^{|I|}\frac{A_i}{\omega_i} + \sum_{j=1}^{|J|}\sum_{k\in\{D_k^j\}^+}^{|K(j)|}\frac{A_k^j}{\omega_k^j} + \sum_{n\in\{E_n\}^+}^{|N|}\frac{A_n}{\omega_n} + \sum_{j=1}^{|J|}h^j\overline{V^j} + \sum_{o=1}^{|O|}\frac{B_o}{\omega_o}
\end{aligned}
\tag{6}
$$

Suppose η (\$/\$/year) is the rate of opportunity cost of the cash inventory. The objective function of the optimization is to minimize the annualized opportunity costs of capital investment for process/storage units and cash/material inventories minus the dividend to stockholders:

$$
\begin{aligned}
Minimize\ TC = &\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}\left[a_k^jD_k^j\omega_k^j\right] + \sum_{i=1}^{|I|}\left[a_iD_i\omega_i\right] + \sum_{n=1}^{|N|}\left[a_nE_n\omega_n\right] + \sum_{j=1}^{|J|}\left[b^j\,\overline{\overline{V^j}}\right] \\
&+ \eta\overline{C} + \sum_{j=1}^{|J|}\gamma^j\overline{V^j} - \sum_{o=1}^{|O|}\frac{B_o}{\omega_o}
\end{aligned}
\tag{7}
$$

where a_n (\$/\$/year) is the annualized financial investment cost, which is proportional to size of the temporary financial investment. Variable a_n is introduced for the mathematical analogy to a_k^j and a_i. Without loss of generality, the storage size will be determined by the upper bound of the inventory holdup $\overline{\overline{V^j}}$ (Yi and Reklaitis, 2003). The independent variables are selected to be the cycle times (ω_k^j, ω_i and ω_n), start-up times (t_k^j, t_i^{in} and t_n) and average material/cash flow rates (D_k^j, D_i and E_n).

The objective function Eq. (7) is convex and the constraints are linear with respect to $\omega_k^j, \omega_i, \omega_n$, t_k^j, t_i^{in} and t_n if D_k^j, D_i and E_n are considered as parameters. However, the convexity with respect to $D_k^j, D_i \text{ and } E_n$ is not clear. First, we obtain the solution for Kuhn-Tucker conditions with respect to $\omega_k^j, \omega_i, \omega_n$, $t_k^j, t_i^{in} \text{ and } t_n$ when $D_k^j, D_i \text{ and } E_n$ are considered as parameters, and then, we further solve the problem with respect to $D_k^j, D_i \text{ and } E_n$ (Yi and Reklaitis, 2003).

Solution of Kuhn-Tucker Conditions

The solution of the Kuhn-Tucker conditions of the first level optimization problem, which entails minimizing the objective function Eq. (7) subject to the constraints $\underline{\underline{V}}^j \geq 0$ and $\underline{\underline{C}} \geq 0$ with fixed values of $D_k^j, D_i \text{ and } E_n$, is obtained by the algebraic manipulation summarized in Yi and Reklaitis (2003). Optimal cycle times are:

$$\omega_k^j = \sqrt{\frac{A_k^j}{D_k^j \Psi_k^j}} \tag{8}$$

$$\omega_i = \sqrt{\frac{A_i}{D_i \Psi_i}} \tag{9}$$

$$\omega_n = \sqrt{\frac{A_n}{E_n \Psi_n}} \tag{10}$$

where

$$\Psi_k^j = \left(\frac{h^j + \gamma^j + \eta h^j + \eta P_k^j}{2} + b^j \right)(1 - x_k^j) + a_k^j \tag{11}$$

$$\Psi_i = a_i + \left(1 - x_i^{in}\right) \sum_{j=1}^{|J|} \left(\frac{h^j + \gamma^j + \eta h^j}{2} + b^j \right) f_i^j + \left(1 - x_i^{out}\right) \sum_{j=1}^{|J|} \left(\frac{h^j + \gamma^j + \eta h^j}{2} + b^j \right) g_i^j \tag{12}$$

$$\Psi_n = a_n + \eta(1 - x_n)(1 + 0.5\kappa_n \Delta t_n) \tag{13}$$

Note that, due to the financial cost factors, the optimal lot sizes of Eqs. (8) and (9) are smaller than those derived previously using approaches such as the classical EOQ model. Because the values of the multipliers are positive, we obtain the following expressions from $\underline{\underline{V}}^j = 0$ and $\underline{\underline{C}} = 0$:

$$\sum_{k=1}^{|K(j)|} D_k^j t_k^j + \sum_{i=1}^{|I|} \left(g_i^j - f_i^j \right) D_i t_i^{in} = V^j(0) - \sum_{m=1}^{|M(j)|} (1 - x_m^j) D_m^j \omega_m^j + \sum_{m=1}^{|M(j)|} D_m^j t_m^j - \sum_{i=1}^{|I|} \left[(1 - x_i^{in}) f_i^j + (1 - x_i^{out}) g_i^j \right] D_i \omega_i \tag{14}$$

$$
\begin{aligned}
&\sum_{n=1}^{|N|}\left\{(t_{\bar{o}}-t_n)+\frac{(1-x_n)}{\Psi_n}\right\}\sqrt{A_n\Psi_n E_n}+\sum_{n=1}^{|N|}\left[\Delta t_n(1+\kappa_n\Delta t_n)-\kappa_n\Delta t_n(t_{\bar{o}}-t_n)\right]E_n \\
&\quad = C(0)+\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}S_m^j D_m^j\left[t_{\bar{o}}-(1+\varsigma)\left(t_m^j+\Delta t_m^j\right)-\varsigma\left\{(1-\hat{x})\hat{\omega}-\hat{t}_m^j\right\}\right] \\
&\quad -\sum_{i\in\{D_i\}^+}^{|I|}\left[A_i(1-x_i^{in})+\sqrt{A_i\Psi_i D_i}\,(t_{\bar{o}}-t_i^{in})\right] \\
&\quad -\sum_{j=1}^{|J|}\sum_{k\in\{D_k^j\}^+}^{|K(j)|}\left[P_k^j D_k^j\left\{(1-x_k^j)\omega_k^j+\left(t_{\bar{o}}-t_k^j-\Delta t_k^j\right)\right\}+A_k^j(1-x_k^j)+\sqrt{A_k^j\Psi_k^j D_k^j}\,(t_{\bar{o}}-t_k^j)\right] \\
&\quad -\sum_{j=1}^{|J|}(1+t_{\bar{o}})h^j\,{}^*\overline{V^j}-\sum_{o=1}^{|O|}B_o(1-x_o)-\sum_{n\in\{E_n\}^+}^{|N|}A_n(1-x_n)
\end{aligned}
\tag{15}
$$

where

$$
\begin{aligned}
{}^*\overline{V^j} &= \sum_{k=1}^{|K(j)|}\frac{(1-x_k^j)}{2}D_k^j\omega_k^j+\sum_{m=1}^{|M(j)|}\frac{(1-x_m^j)}{2}D_m^j\omega_m^j \\
&\quad +\sum_{i=1}^{|I|}\frac{(1-x_i^{out})}{2}g_i^j D_i\omega_i+\sum_{i=1}^{|I|}\frac{(1-x_i^{in})}{2}f_i^j D_i\omega_i
\end{aligned}
\tag{16}
$$

We find that the optimal material storage size is ${}^*\overline{\overline{V^j}}=2\,{}^*\overline{V^j}$. Then, the optimal objective value is as follows:

$$
\begin{aligned}
{}^*TC(D_k^j,D_i,E_n) &= 2\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}\sqrt{A_k^j\Psi_k^j D_k^j}+\sum_{j=1}^{|J|}\sum_{k=1}^{|K(j)|}P_k^j D_k^j+2\sum_{i=1}^{|I|}\sqrt{A_i\Psi_i D_i} \\
&\quad +2\sum_{n=1}^{|N|}\sqrt{A_n\Psi_n E_n}-\sum_{n=1}^{|N|}\kappa_n\Delta t_n E_n \\
&\quad +\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}\left[\left(\frac{h^j+\gamma^j+\eta h^j+\eta(1+\varsigma)S_m^j}{2}+b^j\right)(1-x_m^j)\omega_m^j-S_m^j\right]D_m^j \\
&\quad +\eta\sum_{i\in\{D_i\}^+}^{|I|}0.5(1-x_i^{in})A_i+\eta\sum_{j=1}^{|J|}\sum_{k\in\{D_k^j\}^+}^{|K(j)|}0.5(1-x_k^j)A_k^j \\
&\quad +\eta\sum_{n\in\{E_n\}^+}^{|N|}0.5(1-x_n)A_n+\eta\sum_{o=1}^{|O|}0.5B_o(1-x_o)+\eta\sum_{j=1}^{|J|}\sum_{m=1}^{|M(j)|}0.5S_m^j D_m^j\varsigma(1-\hat{x})\hat{\omega}
\end{aligned}
\tag{17}
$$

The second level optimization problem entails minimizing the objective function of Eq. (17) under the constraints of Eqs. (14), (15) and material balance around storage with respect to the design variables D_k^j, D_i, E_n and t_n. The second level optimization problem is a nonconvex nonlinear programming with bilinear terms, $D_k^j t_k^j$ and $D_i t_i^{in}$, as well as separable concave terms (square roots). Because some of the average flow rates will be zero at the optimum, it is not easy to compute the derivatives of their square roots at the optimum. Moreover, A_i, A_k^j and A_n should be zero if their corresponding average flow rates go to zero at the optimum. To address this issue, the objective function Eq. (17) should include binary variables to exclude the setup costs whose average flow rates become zero. We can introduce a suboptimal approach to reduce the computational complexity. The second level optimization problem can be replaced with another model (e.g., ordinary linear programming) to compute the average rates of material and cash flows without damaging the optimality of the lot sizing equations derived from the first level optimization problem.

Discussion with an Example Plant Design

We used the same plant design example in Yi and Reklaitis(2003) in which cash availability had been implicitly assumed unlimited. A schematic diagram of the plant structure is depicted in Yi and Reklaitis (2003). In this study, we included cash flows and financial transactions in the model by means of installing a cash storage unit. Figure 2 shows the optimal cash inventory profile as calculated using Eq. (3).

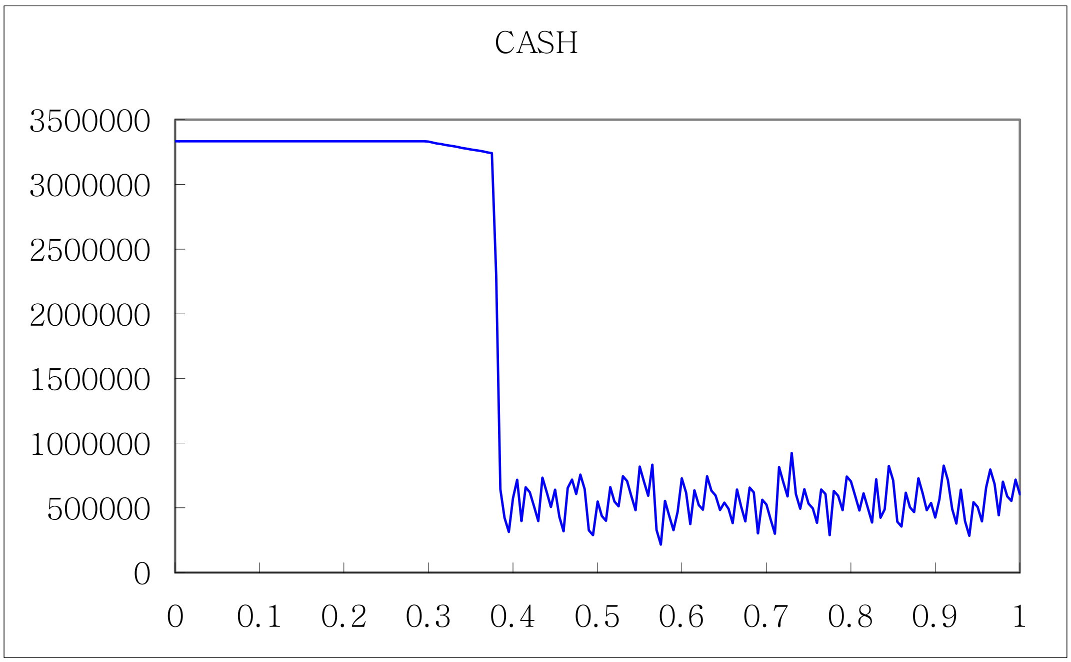

Figure 2. Cash Inventory Profile

Conclusion

In this paper we have extended the common production optimization model to include simultaneous decision-making on production and financing. The optimal production plan in the presence of binding financial constraints differed from the plan generated under the assumption of unlimited cash availability. Inclusion of financial factors in the model led to a decrease in the optimal production lot and storage sizes. The objective function of the optimization was minimizing the opportunity costs of annualized capital investment and cash/material inventory minus stockholder benefits. Backlogging costs of the cash and material inventories and sequence dependent production setup costs were not considered in this study. The average flow rates of material and cash flows were calculated by solving separable concave minimization problems by using a piecewise linearization technique (Tsiakis et. al., 2001). Lot sizes and startup times were determined by analytical equations.

In spite of the enlarged scope of the problem, the computational burden was light due to the use of mostly analytical results and the numerically easy subproblem structure such as separable concave minimization.
The batch-storage network used in this study is very general to cover most business activities such as raw material procurement, transportation, labor, tax as well as production and financial transaction. This study will contribute to preventing even a profitable company from being bankrupt because of bad management of operating cash flows and lead to genuine enterprise optimization.

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