A CONNECTION THEORY FOR THE ANALYSIS OF LARGE SCALE SYSTEMS

Martin Braae

Department of Electrical Engineering, University of Cape Town

Abstract: A theoretical framework that is based on a generalization of state space systems is proposed. The resulting transfer function connection matrix model describes both subsystem interconnections and dynamics in an integrated mathematical formulation, yet ensures that the dynamic and the connection aspects of the system are separated into two sets of equations that are relatively independent. The set of connection equations are of especial interest when dealing with large scale systems as it allows analysis of system connections without direct reference to its dynamics and without requiring parallel mathematical objects to deal with such interconnections.

Keywords: Control systems, Control theory, Decoupling problems, Distributed models, Interconnection, Large-scale systems, Mathematical systems theory, Subsystems, Systems design, Transfer function matrices.

1. INTRODUCTION

Control engineering theory often focuses on the dynamic (or transient) behaviour of systems, particularly of controlled closed loop systems. In large industrial installations, the interconnections that exist between sub-processes of a plant have a direct impact on the dynamics of the overall controlled system. These connections define the system structure (Maciejowski, 1989) and are particularly significant in large scale systems where complex multivariable problems can result from inappropriate linking of unit processes.

The theory of large scale systems, and distributed control, deals with the effects that subsystem interconnections have on the dynamics of the resulting systems. With the proliferation of networked computer-based controllers in industry, the size of such interconnected systems has increased significantly in recent years and the need for understanding system connections, independently of subsystem dynamics, has become more important.

System structure (i.e. the system configuration that is defined by linkages between its subsystems) has often been dealt with explicitly by defining and analysing various derived mathematical entities that stand in a one-to-one relationship with the original dynamic system. These parallel, equivalent theoretical objects include *flow graphs* (Wilson and Beineke, 1979) that yield results such as Masons Rule, *digraphs* in which the matrices of the state space system are replaced by Boolean equivalents (Bahar and Jantzen, 1995), *interconnection* (or *adjacency*) *matrices* for [**ABC**] state space model formulations with digraph interpretations (Siljak, 1991), and *structured matrices* that specify particular matrix shapes (Siljak, 1991).

A literature search shows that state space models (e.g. Groumpos and Pagalos, 1998; Guan *et al.*, 2002; Guo, *et al.*, 2000; Siljak, 1996) are more widely used than transfer function models (e.g. Hovd *et al.*, 1997; Michel, 1983; Van Antwerp *et al.*, 2001). The trend in algebraic techniques (as opposed to those based on graph theory) is also to use equations in which the dynamics and connections are closely intertwined – Compare Siljak (1996) and Callier *et al.* (1978).

This paper proposes a transfer function connection matrix (TFCM) model by generalizing the state space model. The properties of this new model form are investigated and its use in the analysis of connected systems is illustrated. Finally the connection equation

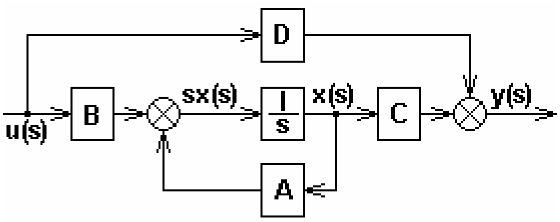

Fig. 1. Block diagram of the state space model.

of the model is applied to an industrial rod milling circuit to demonstrate its use in engineering.

2. THE PROPOSED MODEL

Consider the block diagram in Fig.1 for the linear, time-invariant, continuous state space model:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)\\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)\end{aligned} \tag{1}$$

where $\mathbf{x}$ are the system states, $\mathbf{u}$ the inputs and $\mathbf{y}$ the outputs. Matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ contain the system parameters. (The Laplace variable is "s").

In a state feedback control scheme the system states are connected to its inputs through a constant control law, defined by matrix $\mathbf{K}$, to give:

$$\mathbf{u}(s) = -\mathbf{K}\mathbf{x}(s) + \mathbf{r}(s) \tag{2}$$

where $\mathbf{r}(s)$ is the input to the controlled system.

Note in particular that this connection changes the constant matrix $\mathbf{A}$ of the state feedback description to matrix $[\mathbf{A} - \mathbf{BK}]$, while leaving the dynamic block unaltered. This observation led to a generalization of the state space model in which its dynamic block is replaced by a transfer function matrix (TFM) model (Rosenbrock, 1974). The new block diagram becomes the proposed transfer function connection matrix (TFCM) system shown in Fig.2.

The equations, in transfer function notation, for the generalized state space, or TFCM model, are:

$$\begin{aligned}\mathbf{z}(s) &= \mathbf{M}_{zv}(s)\mathbf{v}(s)\\ \mathbf{v}(s) &= -\mathbf{X}_{vz}\mathbf{z}(s) + \mathbf{X}_{vu}\mathbf{u}(s)\\ \mathbf{y}(s) &= \mathbf{X}_{yz}\mathbf{z}(s) + \mathbf{X}_{yu}\mathbf{u}(s)\end{aligned} \tag{3}$$

where $\mathbf{z}$ are the outputs from, and $\mathbf{v}$ the inputs to the dynamic block $\mathbf{M}(s)$. Four connection matrices $\mathbf{X}$,

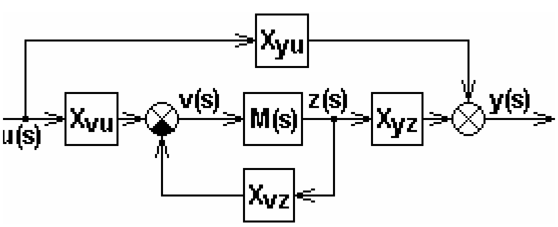

Fig. 2. Block diagram of the proposed TFCM model.

with constant elements, specify the interconnections between signals in the system. The subscripts denote the signals involved in each connection and allow a useful *chain rule check* on subsequent algebraic expressions. (The negative feedback sign convention is used, and subtracted signals are identified in the block diagram by shaded sectors of the comparators.)

The three matrix equations given in Eq.3 define the proposed transfer function connection matrix model and clearly consist of two distinctly different sets of equations. The first equation describes the system dynamics, as a transfer function matrix model $\mathbf{M}_{zv}(s)$, while the second and third equations describe the interconnections that are made within, to, from and around the central dynamic block (respectively matrices $\mathbf{X}_{vz}$, $\mathbf{X}_{vu}$, $\mathbf{X}_{yz}$ and $\mathbf{X}_{yu}$).

The transfer function matrix, $\mathbf{M}_{zv}(s)$, is diagonal or block diagonal or full, depending on whether the detail required from the connection analysis is at the unit-process or the subsystem or the system level.

The equivalent transfer function matrix model, $\mathbf{G}(s)$, for the proposed TFCM model is computed from:

$$\mathbf{X}_{yz}\mathbf{M}_{zv}(s)[\mathbf{I}_{vv} + \mathbf{X}_{vz}\mathbf{M}_{zv}(s)]^{-1}\mathbf{X}_{vu} + \mathbf{X}_{yu} \tag{4}$$

and clearly includes a feedback loop around its dynamic block, $\mathbf{M}_{zv}(s)$. This loop might not exist in all subsystems, in which case the equivalent TFM model reduces to:

$$\mathbf{G}(s) = \mathbf{X}_{yz}\mathbf{M}_{zv}(s)\mathbf{X}_{vu} + \mathbf{X}_{yu} \tag{5}$$

In the simplest subsystems the TFCM model would take the form:

$$\mathbf{G}(s) = \mathbf{X}_{yz}\mathbf{M}_{zv}(s)\mathbf{X}_{vu} \tag{6}$$

and arises when both the connections within and around subsystems do not exist. In this form its *shape* is analogous to the singular value decomposition model used by Van Antwerp *et al.* (2001).

3. STABILITY OF TFCM MODELS

Stability of a TFCM model is determined from its characteristic function. This in turn is computed from the determinant of its return difference matrix at the output (Rosenbrock, 1974):

$$\phi_c(s) = \pm\left|\mathbf{I}_{zz} + \mathbf{M}_{zv}(s)\mathbf{X}_{vz}\right|\phi_o(s) \tag{7}$$

where $\phi_c(s)$ and $\phi_o(s)$ are the characteristic functions of the closed loop and the open loop systems respectively.

In the case of interconnected subsystems, the latter is the product of the characteristic functions for the individual subsystems. In the case of separable subsystems, the former is also the product of the closed loop characteristic functions of the individual subsystems.

Thus stability of a TFCM model depends on both its dynamic block, $\mathbf{M}_{zv}(s)$, and its characteristic connection matrix, $\mathbf{X}_{vz}$, (that might be zero in some cases). This is analogous to state space models in which system poles are determined by matrix $\mathbf{A}$ alone.

3.1 Decomposition of TFCM Models.

When independent subsystems are interconnected, the dynamic transfer function matrix of the resulting TFCM model is always block diagonal, with one block per subsystem, and some being SISO. In a large interconnected system that has been decomposed into two smaller subsystems, this means that the determinant in Eq.7 takes the form:

$$\begin{vmatrix} \mathbf{I}_{z_1z_1} + \mathbf{M}_{z_1v_1}(s)\mathbf{X}_{v_1z_1} & \mathbf{M}_{z_1v_1}(s)\mathbf{X}_{v_1z_2} \\ \mathbf{M}_{z_2v_2}(s)\mathbf{X}_{v_2z_1} & \mathbf{I}_{z_2z_2} + \mathbf{M}_{z_2v_2}(s)\mathbf{X}_{v_2z_2} \end{vmatrix} \quad (8)$$

Thus a characteristic connection matrix that is block diagonal or block triangular indicates a system that can be decomposed into independent subsystems since the above determinant reduces to:

$$\left|\mathbf{I}_{z_1z_1} + \mathbf{M}_{z_1v_1}(s)\mathbf{X}_{v_1z_1}\right|\left|\mathbf{I}_{z_2z_2} + \mathbf{M}_{z_2v_2}(s)\mathbf{X}_{v_2z_2}\right| \quad (9)$$

and the characteristic equation for the interconnected system becomes:

$$\phi_c(s) = \pm[\frac{\phi_{c1}(s)}{\phi_{o1}(s)}][\frac{\phi_{c2}(s)}{\phi_{o2}(s)}]\phi_o(s) = \phi_{c1}(s)\phi_{c2}(s) \quad (10)$$

where $\phi_{ci}(s)$ are the closed loop characteristic functions for its component subsystems.

If, in addition, a diagonal block of the characteristic connection matrix is zero then that particular subsystem is in open loop.

4. INTERCONNECTION OF TFCM MODELS

Interconnection of subsystems involves the subsystem inputs, $\mathbf{u}(s)$, and outputs, $\mathbf{y}(s)$ as well as any new system inputs (or setpoints), $\mathbf{r}(s)$. Thus the TFCM formulation of the problem effectively isolates the dynamic block from such interconnections and these can be defined by the connection equation:

$$\mathbf{u}(s) = \mathbf{X}^1_{ur}\mathbf{r}(s) - \mathbf{X}^1_{uu}\mathbf{u}(s) - \mathbf{X}^1_{uy}\mathbf{y}(s) \quad (11)$$

This explicitly allows connections between subsystem inputs but (provided the inverse matrix exists) can be simplified to yield the subsystem connection matrix equation:

$$\mathbf{u}(s) = \left[\mathbf{I}_{uu} + \mathbf{X}^1_{uu}\right]^{-1}\mathbf{X}^1_{ur}\mathbf{r}(s) - \left[\mathbf{I}_{uu} + \mathbf{X}^1_{uu}\right]^{-1}\mathbf{X}^1_{uy}\mathbf{y}(s) \quad (12)$$

or

$$\mathbf{u}(s) = \mathbf{X}_{ur}\mathbf{r}(s) - \mathbf{X}_{uy}\mathbf{y}(s) \quad (13)$$

By substitution into the TFCM model for the subsystems (Eq.3), the model for the interconnected system becomes:

$$\mathbf{z}(s) = \mathbf{M}_{zv}(s)\mathbf{v}(s) \quad .$$

$$\mathbf{v}(s) = -\{\mathbf{X}_{vz} + \mathbf{X}_{vu}\mathbf{X}_{uy}\left[\mathbf{I}_{yy} + \mathbf{X}_{yu}\mathbf{X}_{uy}\right]^{-1}\mathbf{X}_{yz}\}\mathbf{z}(s) + \{\mathbf{X}_{vu}\mathbf{X}_{ur} + \mathbf{X}_{vu}\mathbf{X}_{uy}\left[\mathbf{I}_{yy} + \mathbf{X}_{yu}\mathbf{X}_{uy}\right]^{-1}\mathbf{X}_{yu}\mathbf{X}_{ur}\}\mathbf{r}(s)$$

$$\mathbf{y}(s) = \{\left[\mathbf{I}_{yy} + \mathbf{X}_{yu}\mathbf{X}_{uy}\right]^{-1}\mathbf{X}_{yz}\}\mathbf{z}(s) + \{\left[\mathbf{I}_{yy} + \mathbf{X}_{yu}\mathbf{X}_{uy}\right]^{-1}\mathbf{X}_{yu}\mathbf{X}_{ur}\}\mathbf{r}(s) \quad (14)$$

The dynamics and connections remain separated, the dynamics of the TFCM model are invariant under connection changes and the connection equations only are altered. These are central features of TFCM models, particularly for the analysis of connections in large-scale systems. (Note: The matrix inversions in the connection matrix equations imply that the elements of connection matrices may start as binary but will in general consist of real rather than integral and binary elements.)

4.1 The Characteristic Connection Matrix.

Linear, time-invariant dynamic systems that result from the interconnection of independent subsystems can be described as a TFCM model. Its dynamic matrix, $\mathbf{M}_{zv}(s)$, will be block diagonal and invariant under connection changes. Its characteristic connection matrix, computed from Eq.14, will identify the effect that interconnections have on the dynamics of the system and, in particular, whether or not the system can be decomposed into component parts for analysis. The characteristic connection matrix also shows which subsystems remain in open loop and which become closed loop problems.

Clearly the dynamic model can always be formulated as a diagonal matrix of transfer functions.

5. ILLUSTRATIONS OF TFCM ANALYSIS

Consider the two dynamic subsystems shown in Fig.3 in which the subsystem transfer functions are:

$$g_1(s) = \frac{e^{-3s}}{1+4s} \quad \text{and} \quad g_2(s) = \frac{1+2s}{1+s} \quad (15)$$

(One subsystem has significant dead-time so it is more amenable to analysis by frequency response methods based on transfer function models than by pole-zero methods based on state space models.)

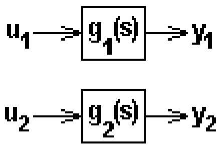

Fig. 3. Two dynamic subsystems.

The TFCM model for the unconnected system is given by:

$$\begin{aligned}
\mathbf{z}(s) &= \begin{bmatrix} g_1(s) & 0 \\ 0 & g_2(s) \end{bmatrix} \mathbf{v}(s) \\
\mathbf{v}(s) &= -\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{z}(s) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{u}(s) \\
\mathbf{y}(s) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{z}(s) - \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{u}(s)
\end{aligned} \quad (16)$$

The corresponding TFM model, $\mathbf{G}(s)$, is identical to the dynamic model, $\mathbf{M}_{zv}(s)$, of the TFCM model.

These subsystems can be connected in many ways to form different structures for the overall system. These structures can result in systems that range from simple open loop SISO systems, as illustrated by Fig.4, through to fully interactive MIMO systems, as illustrated by Fig.7. It is only the connections that are altered, not the dynamics. Since TFCM models allow analysis of the connections independently of the dynamics these models can determine which dynamic models need to be obtained by costly system identification methods (Dougherty and Cooper, 2003).

5.1 A Simple SISO Structure.

If the output from the first system is connected to the input of the second the interconnection equation (Eq.11) becomes:

$$\mathbf{u}(s) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \mathbf{r}(s) - \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{u}(s) - \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} \mathbf{y}(s) \quad (17)$$

The TFCM model for the connected system is computed from Eq.14 and becomes:

$$\begin{aligned}
\mathbf{z}(s) &= \begin{bmatrix} g_1(s) & 0 \\ 0 & g_2(s) \end{bmatrix} \mathbf{v}(s) \\
\mathbf{v}(s) &= -\begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} \mathbf{z}(s) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \mathbf{r}(s) \\
\mathbf{y}(s) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{z}(s) - \begin{bmatrix} 0 \\ 0 \end{bmatrix} \mathbf{r}(s)
\end{aligned} \quad (18)$$

Inspection of its characteristic connection matrix indicates (a) that the system can be analysed by considering its component subsystems independently, (b) that both subsystems are in open loop and (c) that the first subsystem will disturb the second.

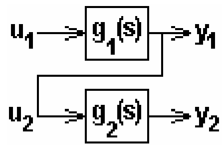

Fig. 4. The first system structure. A simple SISO interconnection.

The corresponding TFM model, $\mathbf{G}(s)$, is computed from Eq.4 and, as expected, found to be:

$$\mathbf{y}(s) = \begin{bmatrix} g_1 \\ g_2 g_1 \end{bmatrix} \mathbf{r}(s) \quad (19)$$

5.2 A More Complex Structure.

The block diagram for a more elaborate interconnection of the same subsystems is shown in Fig.5. The connection equation (Eq.11) for the system is found by inspection of the diagram to be:

$$\mathbf{u}(s) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{r}(s) - \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} \mathbf{u}(s) - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \mathbf{y}(s) \quad (20)$$

or

$$\mathbf{u}(s) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \mathbf{r}(s) - \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \mathbf{y}(s) \quad (21)$$

The resulting TFCM model is computed from Eq.14 and found to be:

$$\begin{aligned}
\mathbf{z}(s) &= \begin{bmatrix} g_1(s) & 0 \\ 0 & g_2(s) \end{bmatrix} \mathbf{v}(s) \\
\mathbf{v}(s) &= -\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \mathbf{z}(s) + \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \mathbf{r}(s) \\
\mathbf{y}(s) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{z}(s) - \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{r}(s)
\end{aligned} \quad (22)$$

Its characteristic connection matrix indicates (a) that the system can be analysed by considering its component subsystems independently, (b) that both subsystems are in closed loop and (c) the first subsystem will disturb the second. The (2,1) element of the characteristic connection matrix clearly illustrates that the final connection matrices are not binary, even though the initial ones were.

The TFM model for the interconnected system is

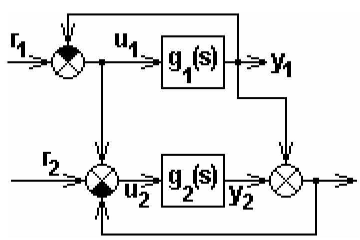

Fig. 5. The second system structure. A more complex interconnection.

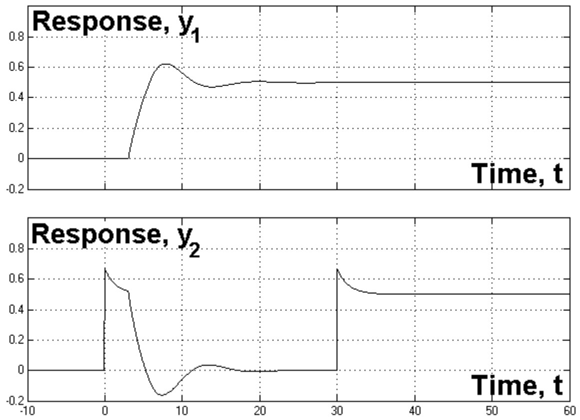

Fig. 6. Closed loop step response for the second system structure.

computed from Eq.4 and given by:

$$\begin{bmatrix} g_1[1+g_1]^{-1} & 0 \\ g_2[1+g_2]^{-1}[1-g_1][1+g_1]^{-1} & g_2[1+g_2]^{-1} \end{bmatrix} \quad (23)$$

This TFM model is verified through digital simulation of the original subsystems, connected as required by Fig.5. The results are shown in Fig.6. Both outputs respond to a unit step change in the first setpoint at time t=0. The final values are $y_1 = 0.5$ and $y_2 = 0.0$. The second output alone responds to a step change in the second setpoint and its final value is $y_2 = 0.5$. These effects are expected from Eq.23.

5.3 A Multivariable Structure.

The block diagram for the interconnected systems is shown in Fig.7 and the connections are defined by Eq.11 to be:

$$\mathbf{u}(s) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\mathbf{r}(s) - \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}\mathbf{u}(s) - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\mathbf{y}(s) \quad (24)$$

or

$$\mathbf{u}(s) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\mathbf{r}(s) - \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\mathbf{y}(s) \quad (25)$$

The resulting TFCM model is:

$$\begin{aligned} \mathbf{z}(s) &= \begin{bmatrix} g_1(s) & 0 \\ 0 & g_2(s) \end{bmatrix}\mathbf{v}(s) \\ \mathbf{v}(s) &= -\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\mathbf{z}(s) + \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\mathbf{r}(s) \\ \mathbf{y}(s) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\mathbf{z}(s) - \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\mathbf{r}(s) \end{aligned} \quad (26)$$

The characteristic connection matrix is full so the systems in this structure cannot be decomposed into subsystems for stability analysis. The (2,2) element in the characteristic connection matrix is integral rather than binary. (The connection matrices in a TFCM model will often contain real number elements even though the original connection matrices are binary.)

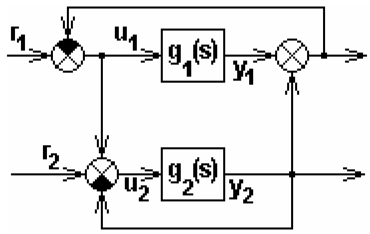

Fig. 7. The third system structure. A multivariable MIMO interconnection.

5.4 Observation.

Clearly it was the connections alone that were changed in the above examples and that resulted in the increasing complexity of the interconnected system structures. The TFCM model allows analysis of the connection matrices independently of the dynamic analysis. Thus it retains an important feature of methods like graph theoretic decomposition techniques (Callier *et al.*, 1978) that have been used in the analysis of system connections.

6. AN INDUSTRIAL APPLICATION

The control of a milling circuit is a historically interesting application of MIMO control engineering techniques in the South African mineral extraction industry. For years the industry had been installing SISO control loops to hold PVs at SPs using appropriate CVs. It was only in the early 1970s that the deployment of an instrument to measure particle size created a situation in which the SISO problems were changed to MIMO by the single additional connection that the new instrumentation allowed. Unsuspecting researchers tackled the problem from various angles until a visit to South Africa by Prof Rosenbrock brought the INA method (Rosenbrock (1974) to their attention. When applied to the milling circuit its MIMO control problem was at last addressed satisfactorily (Hulbert 1983).

The connection equations of the TFCM model predict, without reference to the system dynamics, that the additional system connection from the new

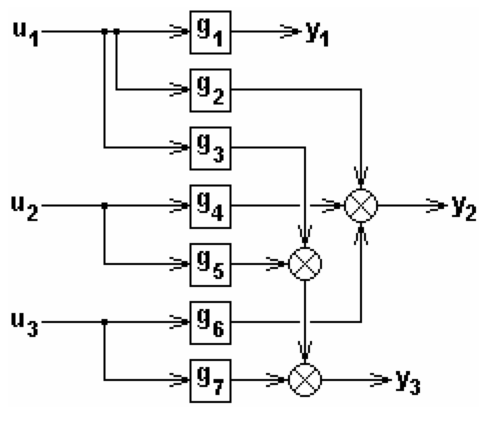

Fig. 8. Simplified Block Diagram of the Controlled Industrial Milling Circuit.

PV, y_3, would result in a MIMO problem. The relevant connection matrix equation is:

$$\mathbf{u}(s) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{r}(s) - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{y}(s) \qquad (27)$$

The characteristic connection matrix for the milling circuit is readily computed from Eq.14 to be:

$$\mathbf{v} = -\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \mathbf{z} + \mathbf{X}_{vr}\mathbf{r} \qquad (28)$$

This $\mathbf{X}_{vz}$ matrix can be partitioned to the lower block triangular matrix:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{bmatrix} = -\left[\begin{array}{ccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right] \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \\ z_7 \end{bmatrix} + \mathbf{X}_{vr}\mathbf{r} \qquad (29)$$

The lower block diagonal is not reducible (Finney and Heck, 1996), so the new instrumentation introduced a MIMO problem for loops 4 to 7. Loop 1 is a SISO problem and models 2 and 3 are in open loop.

Disconnecting the loop from y_3 results in the simpler characteristic connection matrix:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{bmatrix} = -\left[\begin{array}{ccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \\ z_7 \end{bmatrix} + \mathbf{X}_{vr}\mathbf{r} \qquad (30)$$

This shows models 1 and 4 are SISO closed loops while the rest are in open loop.

7. CONCLUSION

A transfer function connection matrix model has been proposed for defining dynamic systems and for analysing connections within an interconnected system. This algebraic model separates the dynamics and the connections into two independent sets of equations in one integral model of the entire system. This allows the effects of its interconnections to be analysed without direct reference to its dynamics thereby reducing, or possibly eliminating, the need for parallel mathematical objects, such as digraphs and adjacency matrices. In industrial applications it postpones and minimizes the need for costly dynamic modelling. The TFCM model is very flexible and can always be formulated in such a way that its dynamics form a diagonal transfer function matrix. In such cases the model can be used to decompose large scale systems at unit process level.

The TFCM model has been shown to predict the rapid change in system complexity, from SISO to MIMO, that was encountered in the design of a control scheme for a large industrial plant.

REFERENCES

Bahar, M. and J. Jantzen. (1996). Digraph programs for multivariable control. *IFAC Conf. Large Scale Systems*, London, p263-268.

Callier, F.M., W.S. Chan and C.A. Desoer. (1978) Input-output stability of interconnected systems using decompositions. *IEEE Trans. Automatic Control*, **AC-23** (2), 150-162.

Dougherty, D and D. Cooper (2003). A practical multiple model adaptive strategy for single-loop MPC, *Control Engineering Practice*, **11**(2), p141-159.

Finney, J.D. and B.S. Heck. (1996). Matrix scaling for large-scale system decomposition. *Automatica*, **32** (8), 1177-1181.

Groumpos, P.P and A.V. Pagalos. (1998) A two-level structural model for large scale systems. *Computers in Industry*, **36**, 147-154.

Guan, Z-H, G. Chen, X. Yu and Y. Qin. (2002). Robust decentralized stabilization for a class of large-scale time-delay uncertain impulsive dynamical systems. *Automatica*, **38**, p2075-2084.

Guo Y, D.J. Hill and Y. Wang. (2000). Nonlinear decentralized control of large-scale power systems. *Automatica*, **36**, 1275-1289

Hovd, M, R.D. Braatz and S. Skogestad (1997). SVD controllers for H_2-, H_∞- and μ- optimal control. *Automatica,* **33** (3), 433-439.

Hulbert, D.G. (1983). Multivariable control and optimisation of the operation of a milling circuit at East Driefontein. Randburg, Council for Mineral Technology, *Report M98*. 20pp.

Maciejowski, J.M. (1989). *Multivariable feedback design*, Addison-Wesley, Wokingham.

Michel, A.N. (1983). On the status of stability of interconnected systems. *IEEE Trans. Automatic Control*, **AC-28** (6), June 1983, p639-652.

Rosenbrock, H.H. (1974). *Computer-aided control system design*. Academic Press, London.

Siljak, D.D. (1991). *Decentralized control of complex systems*. Academic Press, Boston.

Siljak, D.D. (1996). Decentralized control and computations: Status and prospects. *A. Rev. Control*, **20**, 131-141.

VanAntwerp, J.G., A.P. Featherstone, R.D. Braatz. (2001). Robust cross-directional control of large scale sheet and film processes. *J. Process Control*, **11**, 149-177.

Wilson, R.J. and L.W. Beineke. (1979). *Application of graph theory*. Academic Press, Boston.