A Gymnastic Technique-based Control Method for a Three-link Underactuated Robot

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***Abstract*— In this paper, new swing-up controller for a three-link underactuated robot using a technique of horizontal bar gymnast is proposed. This controller is designed based on an equivalent center of mass(ECM) of the gymnast and the robot. The ECM of the gymnasts(ECMG) derived by analyzing a video data and a proposed controller is designed in order to realize that the ECM of the robot imitates the ECMG. The effectiveness of proposed controller is verified by the numeric simulation by using MATLAB/Simulink.**

I. INTRODCUTION

The three-link underactuated robot is one of a underactuated systems and is a simple robot modeled on a horizontal bar gymnast. The first joint of the three-link underactuated robot doesn't have an actuator, i.e., the first joint is imitated a hand of the gymnast. On the other hand the second and third joint which have an actuator are imitated similarly shoulder and waist of the gymnast.

As the control problem of the underactuated robot, a motion controller based on the equivalent center of mass(ECM)[1] has been proposed for the Acrobot which is one of the typical example of underactuated robot[**?**], [3]. This controller [1] designed based on an analysis result of the equivalent center of mass of the gymnasts(ECMG) and can be realized that Acrobot imitate the swing-up motion of the gymnast. An advantage of this method is able to apply to robot with a number of links that is different from the number of links of human, because EMC of robots with a link a different number are formulated a same system, a variable length single pendulum.

In this paper, a motion controller based on the ECMG is applied to the control problem of the three-link underactuated robot so as to prove the controller based on ECMG is applicable to the robot of many links. This controller is designed based on analysis result of the behavior of the ECMG in a swing-up motion and a giant-swing motion which are basic gymnastic motion.

In order to get the behavior of the ECMG of the swing-up and giant-swing motion, it is necessary to analyze the video data which is captured the behavior of the gymnast and to a Body Segment Parameters(BSP) of Japanese athletes[4]. From analysis data, an efficient motion of the gymnast is identified and this motion is formulated in simple approximation equations. For the controller of the three-link underactuated robot, the partial linearization controller[2], [3], [5] which is one of the nonlinear control methods based on a structure theory[5] and is often used as a controller for the underactuated system [2], [3], [6] is designed so as to replicate motion of the three-link underactuated robot as same as the efficient motion of gymnast.

Finally, in order to show the effectiveness of controller, simulations of swing-up and giant-swing control of the three-link underactuated robot are performed by MATLAB/Simulink.

II. THE THREE-LINK UNDERACTUATED ROBOT

A. *Dynamics of the three-link underactuated robot*

Fig. 1 shows the model of the three-link underactuated robot driven by control inputs τ_2 and τ_3 which are the control inputs torque for actuators with the second joint and the third joint. The dynamics of the three-link underactuated robot is shown as

$$\begin{bmatrix} M_{11} M_{12} M_{13} \\ M_{21} M_{22} M_{23} \\ M_{31} M_{32} M_{33} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} + \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} + \begin{bmatrix} \mu_1 \dot{q}_1 \\ \mu_2 \dot{q}_2 \\ \mu_3 \dot{q}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ \tau_2 \\ \tau_3 \end{bmatrix} \quad (1)$$

where, matrix M means the inertial force and vector h, ϕ, μ means the centrifugal and Coriolis force, the gravity, the friction force, respectively. Parameter functions of the dynamics are shown as

$$M_{11} = C_1 + C_2 + C_3 + 2(C_4 + C_5)\cos q_2 + 2C_6 \cos q_3 + 2C_7 \cos(q_2 + q_3)$$
$$M_{12} = M_{21} = C_2 + C_3 + (C_4 + C_5)\cos q_2 + 2C_6 \cos q_3 + C_7 \cos(q_2 + q_3)$$
$$M_{13} = M_{31} = C_3 + C_6 \cos q_3 + C7 \cos(q_2 + q_3)$$
$$M_{22} = C_2 + C_3 + 2C_6 \cos q_3,\ M_{23} = M_{32} = C_3 + C_6 \cos q_3,\ M_{33} = C3$$
$$h_1 = -(2\dot{q}_1\dot{q}_2 + \dot{q}_2{}^2)(C_4 + C_5)\sin q_2 - C_6(\dot{q}_3{}^2 + 2\dot{q}_1\dot{q}_3 + 2\dot{q}_2\dot{q}_3)\sin q_3$$
$$- C_7(\dot{q}_2{}^2 + \dot{q}_3{}^2 + 2\dot{q}_1\dot{q}_3 + 2\dot{q}_2\dot{q}_3 + 2\dot{q}_1\dot{q}_2)\sin(q_2 + q_3)$$
$$h_2 = \dot{q}_1{}^2((C_4 + C_5)\sin q_2 + C_7 \sin(q_2 + q_3))$$
$$- C_6 \sin q_3(\dot{q}_3{}^2 + 2\dot{q}_1\dot{q}_3 + 2\dot{q}_2\dot{q}_3)$$
$$h_3 = \dot{q}_1{}^2(C_7 \sin(q_2 + q_3) + C_6 \sin(q_3)) + C_6(2\dot{q}_1\dot{q}_2 + \dot{q}_2{}^2)\sin q_3$$
$$\phi_1 = G_1 \sin q_1 + G_2 \sin(q_1 + q_2) + G_3 \sin(q_1 + q_2 + q_3)$$
$$\phi_2 = G_2 \sin(q_1 + q_2) + G_3 \sin(q_1 + q_2 + q_3)$$
$$\phi_3 = G_3 \sin(q_1 + q_2 + q_3)$$
$$C_1 = I_1 + m_1 l_{c1}{}^2 + m_2 l_1{}^2 + m_3 l_1{}^2,\ C_2 = I_2 + m_2 l_{c2}{}^2 + m_3 l_2{}^2$$
$$C_3 = I_3 + m_3 l_{c3}{}^2,\ C_4 = m_2 l_1 l_{c2},\ C_5 = m_3 l_1 l_2$$
$$C_6 = m_3 l_2 l_{c3},\ C_7 = m_3 l_1 l_{c3}$$
$$G_1 = m_3 l_1 g,\ G_2 = m_3 l_2 g,\ G_3 = m_3 l_{c3} g$$

and the parameters of the three-link underactuated robot are defined in Table I.

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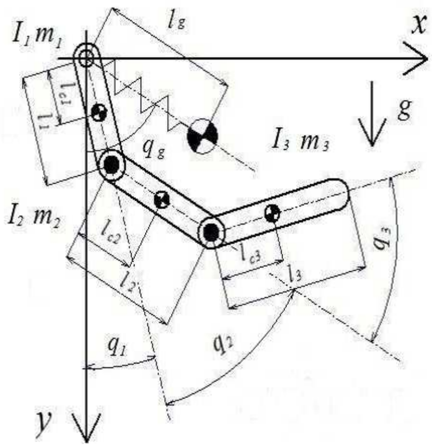

Fig. 1. the three-link underactuated robot

TABLE I
DEFINITIONS OF PARAMETERS (i=1,-,3)

m_i	Mass of the i-th link [kg]
l_i	Length of the i-th link [m]
l_{ci}	Length to the center of mass of the i-th link [m]
I_i	Moment of inertia of the i-th link [kgm^2]
τ_i	Torque which acts on the i-th link [Nm]
q_i	Angle of the i-th link [rad]
$\dot{q}_i$	Angular velocity of the i-th link [rad/s]
μ_i	Coefficient of friction which acts on the i-th joint [Nms/rad]

B. ECM of the three-link underactuated robot

For applying a technique of the gymnast to a controller of the three-link underactuated robot whose number of links are fewer than the gymnast, we used controller which is designed based on ECMG[1], which is a center of the mass of whole system. By using this method, the behavior of the ECM of the three-link underactuated robot(ECMR) and ECMG can be treated as a same system which is a variable length single pendulum, which is shown in Fig. 1, and it is easy to apply a technique of the gymnast to controller of the three-link underactuated robot. Moreover, applying this method to another underactuated serial linkage systems can be expected, because the ECM of all serial linkage systems can be shown by the variable length single pendulum. Therefore, this method can not only apply to Three-links robot, but also apply to more links robot.

Coordinate data (x_g, y_g) of the ECMR shown in Fig. 1 is shown by

$$x_g = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} \quad (2)$$

$$y_g = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} \quad (3)$$

where

$$(x_1, y_1) = (l_{c1} \sin q_1, l_{c1} \cos q_1)$$
$$(x_2, y_2) = (l_1 \sin q_1 + l_{c2} \sin(q_1+q_2), l_1 \cos q_1 + l_{c2} \cos(q_1+q_2))$$
$$(x_3, y_3) = (l_1 \sin q_1 + l_2 \sin(q_1+q_2) + l_{c3} \sin(q_1+q_2+q_3), \quad l_1 \cos q_1 + l_2 \cos(q_1+q_2) + l_{c3} \cos(q_1+q_2+q_3))$$

the x-axis and y- axis are the horizontal-line and the vertical-line shown in Fig. 1,respectively. From coordinate data of the ECMR, the angular position q_g of the ECMR and the length l_g from the root to the ECMR are obtained as

$$q_g = \arctan \frac{x_g}{y_g} \quad (4)$$

$$l_g = \sqrt{x_g^2 + y_g^2} = \sqrt{f_1 + f_2 \cos q_2 + f_3 \cos(q_2+q_3) + f_4 \cos q_3} \quad (5)$$

where

$$\begin{cases} f_1 = \frac{2m_1 l_{c1} m_2 l_1 + {m_2}^2 {l_1}^2 + m_3 l_2 (2m_2 l_{c2} + m_3 l_2)}{(m_1+m_2+m_3)^2} + f_5 \\ f_2 = \frac{m_3 l_1 (2m_1 l_{c1} + 2m_2 l_1 + m_3 l_1) + m_3 l_2 (2m_2 l_{c2} + m_3 l_2) + {m_3}^2 {l_{c3}}^2}{(m_1+m_2+m_3)^2} \\ f_3 = \frac{2m_3 l_{c3} (m_1 l_{c1} + m_2 l_1 + m_3 l_1)}{(m_1+m_2+m_3)^2} \\ f_4 = \frac{2m_3 l_2 l_{c3} (m_2+m_3)}{(m_1+m_2+m_3)^2} \\ f_5 = \frac{m_1^2 l_{c1}^2 + m_2^2 l_{c2}^2 + m_3^2 l_{c3}^2 + m_3 l_1 (2m_1 l_{c1} + 2m_2 l_1 + m_3 l_1)}{(m_1+m_2+m_3)^2} \end{cases}$$

III. ANALYSIS OF THE ECM OF THE GYMNAST

This section discusses numerical analysis of the technique of the gymnast. In this study, a motion of a KIP that is typical basic technique of swing-up and a Giant-swing of the gymnast are analyzed by using the motion capture technique.

A. Getting a motion data of the ECM of the gymnast

As a experimental technique data of the gymnast, the motion of the KIP and the giant-swing are recorded by a video camera(frame rate is 26.1[frame/sec]) and analyzed by using a motion capture technique. The personal data are following.

- Age : 13 [years old]
- Gender : Male
- Body height : l_h=1.456 [m]
- Body weight : m_h=38.2 [kg]
- Gymnastic career : 6 [years]

For analyzing behavior of the gymnast, at first, coordinate data of body of the gymnast for each flames are derived by using motion capture software (PV Studio 2D, OA SCIENCE Co.,Ltd.) and coordinate data of the ECNG are computed by coordinate and physical data of the gymnast. Coordinate data (x_{hg}, y_{hg}) of the ECMG can be obtained as follow.

1. Separating a human body in 6 segments (forearm, upper arm, head, body, thigh and lower legs)
2. Estimating a mass and a length from joint to the center of mass of each segment by using Body Segment Parameters(BSP) of Japanese athletes[4].
3. Coordinate data of the ECMG can be derived by combining estimated mass, length from joint to the center of mass and angular position of each segment.

Moreover, using coordinate data of the ECMG, an angular position q_{hg} of the ECMG , a length l_{hg} , a potential energy U_h, a kinetic energy T_h from the root to the ECMG are obtained as following equations.

$$q_{hg} = \arctan \frac{x_{hg}}{y_{hg}} \tag{6}$$

$$l_{hg} = \sqrt{x_{hg}^2 + y_{hg}^2} \tag{7}$$

$$U_h = m_h g l_{hg}(1 - \cos q_{hg}) \tag{8}$$

$$T_h = \frac{1}{2} m_h (\dot{l}_{hg}^2 + l_{hg}^2 \dot{q}_{hg}^2) \tag{9}$$

B. Analyzing the behaviors of the ECMG of the kip motion

The behaviors of the ECMG of the swing-up motion are shown in Fig. 2 which illustrates the behaviors of q_{hg}, l_{hg}, U_h and T_h.

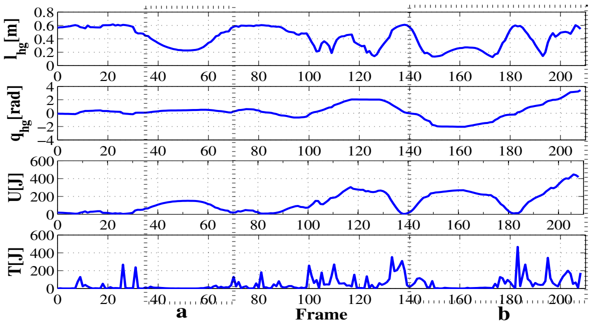

Fig. 2. Behavior of the ECMG of the swing-up motion

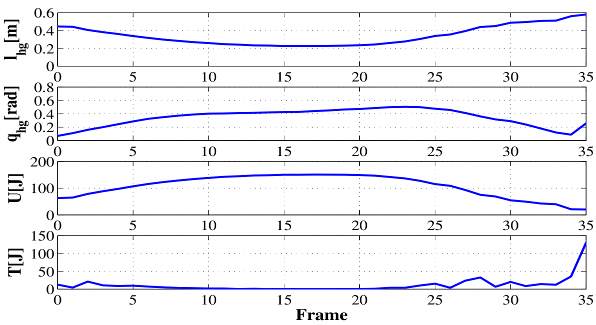

Fig. 3. Behavior of the ECMG of the swing-up motion(enlarged figure of range enclosed by the square (a) in Fig.2)

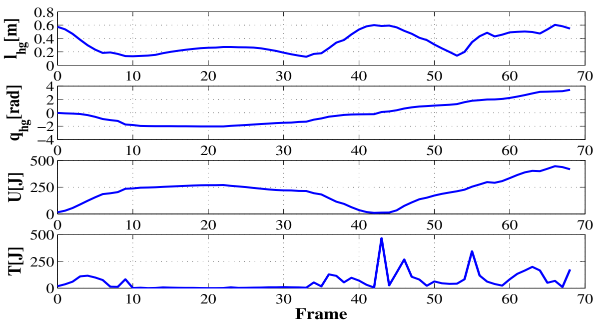

Fig. 4. Behavior of the ECMG of the swing-up motion(enlarged figure of range enclosed by the square (b)in Fig.2)

For analyzing the behaviors of the ECMG in the swing-up motion, we focus on a motion of a first swing-up action (Frame=35-70 shown by square (a) in Fig.2) and a motion (Frame=140-210 shown by the square (b) in Fig.2) when q_{hg} is large values.

Fig.3 and 4 show enlarged figures of range enclosed by the square (a) and (b) in Fig.2, respectively.

From Fig.3 and 4, it shows that kinetic energy of the gymnast is sharply increasing after swinging l_{hg} slowly, that is, in the motion of KIP, the gymnast can get a large swing by repeating simply the motion of moving the ECM close to the horizontal bar. Thus, we regard these simple motions as a most efficient motion for swing-up and a relationship between q_{hg} and l_{hg} in these motions are formulated in order to apply these efficient motions to the swing-up control of the three-link underactuated robot.

An approximative length l_{hg}^s of the ECM, which corresponds to angular position q_{hg} of ECM in swing-up motion, can be obtained as simple low order polynomial equations as following equations.

$$l_{hg}^s(q_{hg}) = \begin{cases} l_{hg1} & (|q_{hg}| < 0.451 \ \cap \ \dot{q}_{hg} \geq 0) \\ l_{hg2} & (|q_{hg}| < 0.451 \ \cap \ \dot{q}_{hg} < 0) \\ l_{hg3} & (0.451 \leq |q_{hg}|, 0 \leq q_{hg} \ \cap \ \dot{q}_{hg} \geq 0) \\ l_{hg4} & (0.451 \leq |q_{hg}|, 0 > q_{hg} \ \cap \ \dot{q}_{hg} \geq 0) \\ l_{hg5} & (0.451 \leq |q_{hg}|, \dot{q}_{hg} < 0) \end{cases} \tag{10}$$

where, l_{hg1} and l_{hg2} replicate the motion of Fig.3, l_{hg3}, l_{hg4} and l_{hg5} replicate the motion of Fig.4.

$$\begin{cases} l_{hg1} = 0.0008975 t_1^2 - 1.3996 t_1 + 545.89 \\ \quad \text{where } t_1 = \frac{1.6784 - \sqrt{1.6784^2 - (0.0042884(656.42 + q_{hg}))}}{0.0021442} \\ l_{hg2} = 0.0008975 t_2^2 - 1.3996 t_2 + 545.89 \\ \quad \text{where } t_2 = \frac{1.6784 + \sqrt{1.6784^2 - (0.0042884(656.42 + q_{hg}))}}{0.0021442} \\ l_{hg3} = 0.0038465 t_3^2 - 0.010063 t_3 + 0.14741 \\ \quad \text{where } t_3 = \frac{q_{hg} - 1.1499}{0.12712} \\ l_{hg4} = -0.0047824 t_4^2 + 0.0022503 t_4 + 0.60712 \\ \quad \text{where } t_4 = \frac{q_{hg} - 0.13846}{0.78684} \\ l_{hg5} = -0.0034581 t_5^2 - 0.0048116 t_5 + 0.61030 \\ \quad \text{where } t_5 = \frac{q_{hg} - 0.34275}{-0.16418} \end{cases}$$

C. Analyzing the behaviors of the ECMG of the giant-swing motion

The behaviors of the ECMG in the giant-swing motion are shown in Fig. 5 which illustrates the behaviors of q_{hg}, l_{hg}, U_h and T_h.

Since the giant-swing motion of the gymnast repeats the same motions, we analyzed only one period of the giant-swing motion of the gymnast. This motions are divided into 4 actions so called 'Afuri', 'Nuki', 'Otoshi' and 'Hiraki'[7]. For applying these motions to the giant-swing control of the three-link underactuated robot, these motions are formulated in simple approximation equations.

By analyzing using Fig.5, the relationship between the length and angular position of the ECM in the giant-swing motion can formulated approximately by simple second order polynomial expression. Formularization of Fig.5 results in

$$l_{hg}^{g}(q_{hg}) = \begin{cases} l_{hg6} & (-\pi \leq_{hg} < -2.50) \\ l_{hg7} & (-2.50 \leq q_{hg} < -1.57) \\ l_{hg8} & (-1.57 \leq q_{hg} < -0.909) \\ l_{hg9} & (-0.909 \leq q_{hg} < 0.629) \\ l_{hg10} & (0.629 \leq q_{hg} < 2.36) \\ l_{hg11} & (2.36 \leq q_{hg} < \pi) \end{cases} \quad (11)$$

where

$$\begin{cases} l_{hg6} = -0.0006297t_6^2 + 0.022405t_6 + 0.39586 \\ \quad \text{where } t_6 = \frac{-0.010172+\sqrt{0.00010172+0.0047(3.509+q_{hg})}}{0.0024} \\ l_{hg7} = -0.0011847t_7^2 + 0.069355t_7 - 0.42656 \\ \quad \text{where } t_7 = \frac{0.26307+\sqrt{0.0692+0.0250(-0.14709+q_{hg})}}{0.0125} \\ l_{hg8} = -0.011664t_8^2 + 0.82802t_8 - 14.116 \\ \quad \text{where } t_8 = \frac{4.1971+\sqrt{17.6156+0.2492(-69.104+q_{hg})}}{0.1246} \\ l_{hg9} = -0.0034048t_9^2 + 0.27973t_9 - 5.1322 \\ \quad \text{where } t_9 = \frac{1.3045+\sqrt{1.7017+0.0737(-22.107+q_{hg})}}{0.0368} \\ l_{hg10} = 0.0028537t_{10}^2 - 0.29233t_{10} + 7.9386 \\ \quad \text{where } t_{10} = \frac{q_{hg}+5.8746}{0.14893} \\ l_{hg11} = -0.00034387t_{11}^2 + 0.0056854t_{11} + 0.56311 \\ \quad \text{where } t_{11} = \frac{q_{hg}-2.5417}{0.051675} \end{cases}$$

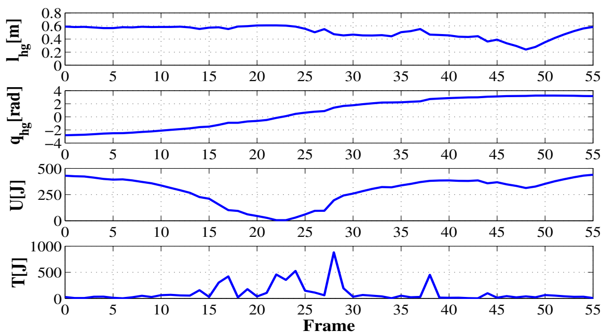

Fig. 5. Behavior of the ECMG of the giant-swing motion

IV. CONTROLLER BASED ON TECHNIQUE OF THE GYMNAST

In this section, detailed structure of proposed controller is discussed. Controller (depicted in Fig.6) is designed focusing on the ECMR and the ECMG and has three sections.

1. Deriving a relationship of the ECM by connecting the ECM of gymnast with the ECM of robot.
2. Transform l_g^r based on an analysis result in section III to desired angular position q_2^r of second link and angular position q_3^r of third link.
3. Tracking controller using partial linearization method in order to track q_2 to q_2^r and q_3 to q_3^r[5].

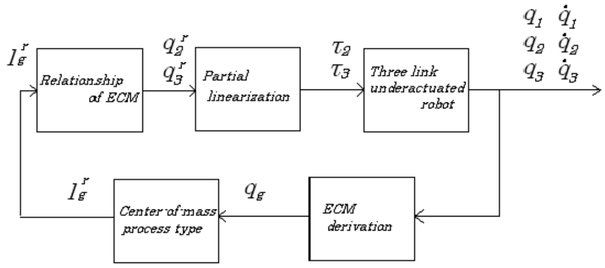

Fig. 6. Block diagram of proposed controller

A. Deriving a relationship of ECM

A desired length of the ECMR, which can reproduce a KIP and giant swing motion, is decided based on analysis results l_g^s and l_g^g shown by eq.10 and 11, respectively.

Since l_g^s and l_g^g are the desired lengths of the ECMG corresponding to q_g and since the size of human and the three-link underactuated robot are not the same, l_g^s and l_g^g are transformed to l_g^r so as to match the size of the Acrobot by using following transform equation.

$$l_g^r = l_g^{min} + (l_{hg}^i(q_g) - l_{hg}^{min})\frac{l_g^{max} - l_g^{min}}{l_{hg}^{max} - l_{hg}^{min}}, \quad (i = s, g) \quad (12)$$

where $l_g^{max}(l_g^{min})$ means maximum(minimum) length of the ECMR and $l_{hg}^{max}(l_{hg}^{min})$ means maximum(minimum) length of the ECMG, respectively.

B. Transform l_g^r to q_2^r and q_3^r

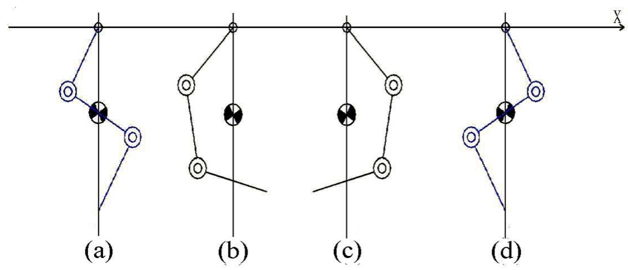

Fig. 7. The pattern of the link which attains the one ECM

A desired angular positions of the second link and the third link realizing desired length ${l_g}^r$ of the ECM is derived by using eq. (5). However, in the three-link underactuated robot, multiple patterns of a link achieving the center of gravity position of the robot exist as shown in Fig.6. Thus, in this study, we assume a desired angular position $q_2^r = q_3^r$ to decide the ECMR. By substituting l_g^r to l_g in eq. (5), desired angular position of the second link q_2^r and the third link q_2^r can be obtained by

$$q_2^r = q_3^r = \arccos\left(\frac{-(f_2+f_4)+f}{4f_3}\right) \quad (13)$$
$$f = \sqrt{(f_2+f_4)^2 - 8f_3(f_1 - f_3 - lg^2)}$$

C. Tracking controller of q_2 for q_2^r and q_3 for q_3^r

The controller to track the angular positions of the second link q_2 and the third link q_3 to the desired angular positions ${q_2}^r$ and ${q_3}^r$ is used partial linearization method[5].

Now, following nonlinear feedback is designed as control input.

$$\tau_2 = \frac{M_c(\phi_1+h_1)+(M'_{22}M'_{33}-M'_{23}M'_{32})(\phi_2+h_2)+M_a}{M'_{22}M'_{33}-M'_{23}M'_{32}} \quad (14)$$

$$\tau_3 = \frac{M_d(\phi_1+h_1)+(M'_{22}M'_{33}-M'_{23}M'_{32})(\phi_3+h_3)+M_b}{M'_{22}M'_{33}-M'_{23}M'_{32}} \quad (15)$$

$$\begin{aligned} M_a &= M'_{33}|M|\upsilon_2 - M'_{22}|M|\upsilon_3 \\ M_b &= -M'_{32}|M|\upsilon_2 + M'_{22}|M|\upsilon_3 \\ M_c &= (M'_{21}M'_{33} - M'_{23}M'_{31}) \\ M_d &= (M'_{22}M'_{31} - M'_{32}M'_{21}) \end{aligned}$$

where, $|M|$ is a determinant of the inertial matrix M in eq. (1), M'_{ij} are elements of the inverse inertial matrix M^{-1}. Moreover, υ_2,υ_3 are the equivalent control input which can be set in nonlinear feedback eq. (14), eq. (15) and it can be achieved

$$\ddot{q}_2 = \upsilon_2 \quad (16)$$

$$\ddot{q}_3 = \upsilon_3. \quad (17)$$

In order to track q_2 to q_2^r and q_3 to q_3^r, let equivalent control $\upsilon_i (i = 2, 3)$ are designed as

$$\upsilon_i = \ddot{q}_i = -k_{pi}(q_i - q_i^r) - k_{di}\dot{q}_i, \quad (k_{pi} > 0, k_{di} > 0) \quad (18)$$

then linearized subsystem is

$$\ddot{q}_i + k_{pi}(q_i - q_i^r) + k_{di}(\dot{q}_i - \dot{q}_i^r) = 0. \quad (i = 2, 3) \quad (19)$$

Since eq. (19) are stable polynominal equations, it is possible to track the angular positions of the second link q_2 and the third link q_3 to the desired angular positions ${q_g}^r$, that is,

$$\lim_{t\to\infty} q_2 = q_2^r, \lim_{t\to\infty} q_3 = q_3^r.$$

As a result, tracking the length of the ECMR l_g to the desired length ${l_g}^r$ can be achieved and the ECMR operates similarly most efficient motion for swing-up and giant-swing to the of the ECMG.

V. NUMERICAL SIMULATIONS

In this Section, in order to verify an effectiveness of proposed controller, a numerical simulation of swing-up and giant-swing control of the three-link underactuated robot is performed by using MATLAB/Simulink.

A. Condition of simulation

The mechanical parameters of the three-link underactuated robot and several conditions used in the simulation are shown in Table II,and in Table III. Simulations started the three-link underactuated robot with initial values as $[q_1, q_2, q_3, \dot{q}_1, \dot{q}_2, \dot{q}_3] = [0, 0, 0, 0, 0, 0]$.

After swing-up the three-link underactuated robot to upright position, controller is switched to giant-swing controller which based on the ECMG.

TABLE II

PARAMETERS OF THE THREE-LINK UNDERACTUATED ROBOT

	1^{st} link	2^{nd} link	3^{rd} link
m_i[kg]	0.100	0.0750	0.130
l_i[m]	0.200	0.150	0.260
l_{ci}[m]	0.100	0.075	0.130
I_i[kgm^2]	3.333×10^{-4}	1.406×10^{-5}	7.323×10^{-4}
μ_i[Nm]	0.006	0.006	0.006

TABLE III

PARAMETERS OF CONDITIONS USED IN THE SIMULATION

Swing-up controller of the 2^{nd}link	k_p=1200/1500 k_d=60/60
Swing-up controller of the 3^{rd} link	k_p=1200/1500, k_d=60/60
Giant-swing controller of the 2^{nd}link	k_p=4000 k_d=100
Giant-swing controller of the 3^{rd} link	k_p=4000, k_d=100
Length of the ECMR	l_g^{max}=0.4080[m] l_g^{min}=0.0983[m]
Length of the ECMG	l_{hg}^{max}=0.6156[m] l_{hg}^{min}=0.1269[m]
Sampling time	T_s=0.005[s]
swing-up and giant-swing time	T_{end}=20[s]

B. The numerical simulation result and consideration

Fig. 8 shows the simulation results of the swing-up and the giant-swing control using the method based on the ECMG and illustrates responses of the angular positions q_2, q_3, the control inputs τ_2,τ_3, the length of the ECMR l_g, the angular position of the ECMR q_g, the potential energy T_g and the kinetic energy T_g of the ECMR, respectively.

Moreove, Fig. 8 illustrates the desired values q_2^r, q_3^r, and l_g^r, respectively, as broken lines so as to show a comparison between controlled values and desired values. This comparison result shows realizing the tracking the controlled values to the desired values, and proves an effectiveness of the proposed tracking controller based on the partial linearization method.

From Fig. 8, it shows that the robot can swing-up to upright position from the pendant position by a simple motion, and, at $Time = 15.5$, the controller is switched to the giant-swing controller around upright position.

As a result, by applying the behavior of the ECMG to the three-link underactuated robot, the kinetic energy increases, the robot can achieve the swing-up and giant-swing motions based on the ECMG.

VI. CONCLUSION

We applied the controller based on the ECMG to the three-link under underactuated robot to inspect possibility of the controller based on the ECMG. The controller based on the ECMG focuses on the equivalent center of mass(ECM) of the gymnast(ECMG) and the three-link under underactuated robot(ECMR), because dynamics of the ECMR and ECNG can be regarded as same dynamics and this idea can be expected to applying the another underactuated serial linkage systems. The Analysis of the EMCG is used motion capturing technique and coordinate data of EMCG is obtained by using Body Segment Parameters(BSP) of Japanese athletes. From numerical analysis data of behavior of the ECMG, the efficient motion which can be approximated in simple low-order polynomial equations can be identified for swing-up and giant-swing. The desired angular positions of the second link and the third link are computed in order to realize the motion of ECMR as same as the efficient motion of ECMG. And tracking controller, which can track the second link and the third link to desired angular positions, is designed by partial linearization method. In order to verify this method, numerical simulations of swing-up and giant-swing control for the three-link underactuated robot are performed by using proposed method. As a result, not only the Acrobot but also the three-link underactuated robot is controllable by the controller based on the ECMG.It is thought that there is the possibility of the expansion to another underactuated serial linkage systems in the controller based on the ECMG.

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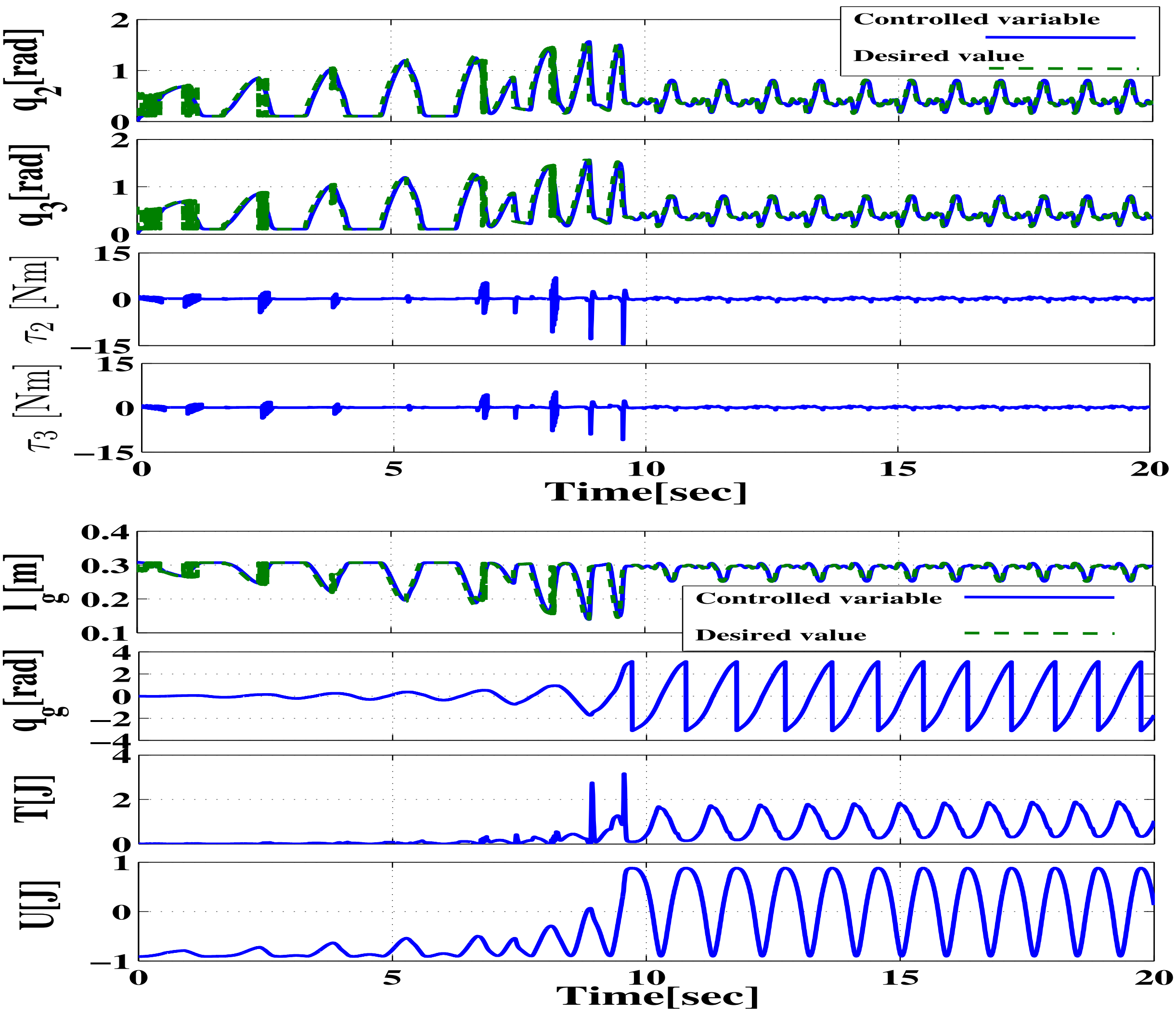

Fig. 8. Behavior of the swing-up and giant-swing control of the robot