Improved relay auto tuning of P/PI controllers for unstable parallel cascade control systems

Simi Santosh [1] and M.Chidambaram[2]

***Abstract*— This paper focuses on simultaneous relay auto tuning of open loop unstable parallel cascade systems. For stable parallel cascade systems,Vivek and Chidambaram[1] had discussed effect of higher order harmonics of symmetrical relay output, which accounts for error in ultimate controller gain estimation. In the present work, this method is extended to parallel cascade controllers for open loop unstable systems. Using improved ultimate controller gains, the conventional relay autotuning method is compared with proposed method for the conventional cascade control configuration (P/PI). The controllers are designed using Zeigler Nichols tuning rules based on the improved ultimate gain.**

Keywords: Parallel cascade, symmetrical relay, P/PI controllers, higher order harmonics.

I. INTRODUCTION

Astrom and Hagglund[2] had introduced the ideal (on-off) relay to generate sustained oscillations. A relay feedback causes a system to oscillate, if the process has a phase lag of π radians and the closed loop response will be a sustained oscillation with a time period, P_u. The ultimate controller gain, k_u and ultimate frequency, ω_u are obtained from the oscillatory response using the following formulae:

$$k_u = 4h/\pi a_0; \omega_u = 2\pi/P_u \quad (1)$$

where h is the relay height and a_0 is the amplitude of the closed loop oscillation. The expression for ultimate controller gain, k_u was obtained by assuming all physical systems to be low pass filters and only principal harmonics of the relay output plays an important role. Thus, all higher order harmonics of the relay output are assumed to be filtered by the physical system and neglected. An error of -18% to +27% in the estimation of k_u using this conventional relay auto tuning is pointed by Li et. al.[3]. An excellent review of relay feedback systems is given by Yu et. al.[4]. Srinivasan and Chidambaram[5] discussed an improved auto tuning method, which considers higher order harmonics of conventional on-off relay for a single loop feedback controller and the method yielded an improved value of k_u, which form the basis of this paper. Vivek and Chidambaram[1] had extended the method proposed by Srinivasan and Chidambaram[5] to tune parallel cascade systems for stable systems. In this paper, the method proposed by Vivek and Chidambaram[1] is extended to parallel cascade systems for open loop unstable systems, which has applications in bioreactors and distillation columns.

[1]Simi Santosh is a PhD scholar at Indian Institute of Technology-Madras, simi.santosh@gmail.com

[2]Dr. M. Chidambaram is Professor, Department of Chemical Engineering, Indian Institute of Technology-Madras, Chennai 600 036, India. chidam@iitm.ac.in

Cascade control consists of two control loops: secondary or inner loop nested within a primary or outer loop. The disturbances entering the inner loop are reduced or eliminated before their effect is felt on the outer loop output variable. There are two control configurations for cascade systems: series cascade and parallel cascade [6,7]. This work discusses design of parallel cascade controllers for open loop unstable systems (a schematic is shown in Fig.1) for conventional cascade control configuration (Proportional controller(P) in the inner loop and Proportional Integral controller,(PI) in the outer loop (P/PI)). In parallel cascade control systems, the manipulated variable(u) affects both controlled variables as seen in Fig.1.

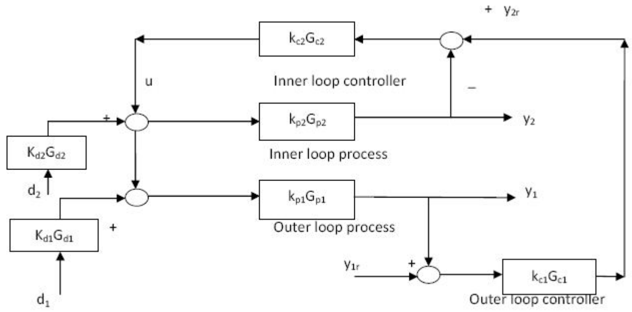

Fig. 1. Parallel Cascade control system

Sequential, one-loop-at-a-time method and simultaneous relay autotuning method[8] are the two available relay auto tuning procedures for parallel cascade controllers. Sequential, one-loop-at-a-time method involves relay autotuning of secondary loop, followed by relay autotuning of primary loop. In simultaneous relay autotuning, the primary loop is tuned first, followed by the secondary loop . For open loop unstable systems, sequential one-loop-at-a-time method cannot be applied, as the secondary loop cannot be stabilized, with primary loop open. In this paper, simultaneous relay auto tuning proposed by Saraf et.al.[8] for open loop unstable series cascade systems is applied to unstable parallel cascade systems . It is a two step procedure, where the first step involves switching both the inner and outer loop controllers to relay, executing the relays to obtain sustained oscillation in both outputs. The primary loop output is analysed to obtain the ultimate controller gain, k_u, and period of oscillation, P_u from which the primary controller parameters are estimated. In the second step, in the primary loop, the relay is switched to controller mode with the parameters obtained from the first step and relay is executed. The secondary loop output is analysed to obtain k_u and P_u of the secondary loop controller, which is utilized to obtain the controller parameters of the secondary loop. Saraf et.al.[8] considered only principal harmonics of the relay output.If relay output is expanded as a fourier series, it was found that higher order harmonics plays an important role. The proposed method includes higher order harmonics of the relay output by estimating a_0, which accounts for the correction in the estimation of k_u, a brief detail of the a_0 estimation is summarized in the Appendix A. Using the corrected ultimate gain values, the controller parameters of inner loop (P) and outer loop(PI) are designed using Zeigler Nichols tuning rules and compared with Saraf et.al.[8] (hereby refered as conventional relay auto tuning method).

II. Methodology: Inclusion of Higher Order harmonics in both loops

Two simulation examples are utilized to explain the proposed method. The first example has a stable FOPTD model in the inner loop and unstable FOPTD model in the outer loop and second example has unstable FOPTD models in the inner and the outer loops.

A. Example-1

Consider a parallel cascade control scheme with inner stable FOPTD process defined by $k_{p2}G_{p2} = 2exp(-2s)/(20s+1)$ and outer unstable FOPTD process defined by $k_{p1}G_{p1} = exp(-4s)/(20s-1)$.In conventional relay auto tuning method, both the secondary and primary controllers are switched to on-off relay. The two relay feedbacks are simultaneously executed for an input relay height, h of 0.1 to obtain sustained oscillatory responses. Thus, a_0 (amplitude of output oscillation=0.022) are obtained for the primary loop first. The ultimate gain $k_u = 4h/\pi a_0 = 5.7875$ and the ultimate time period $P_u = 18$ are thus first obtained for the primary loop. Fig.2 and Fig.3 gives the inner loop and outer loop responses when both the inner and outer loops are under relay. The K_{design} for the primary loop is obtained using the following formulae: $K_{design} = \sqrt{(K_{min}K_{max}} = 2.4057$ where $K_{min} = 1$. 90% of this design value is taken as K_{c1}. The integral time τ_I is obtained using the Zeigler Nichols tuning formulae ($\tau_I = P_u/1.2 = 15$). The relay in the primary loop is then switched to the PI controller with the above parameters. The relay feedback is then executed to find controller parameters for the secondary loop K_{c2}. The ultimate gain $K_u = 3.9603$ and the ultimate time period ($P_u = 13.6$) are obtained for the secondary loop. Using the formulae, $K_{c2} = k_u/(2k_{p2})(= 0.9901)$, the secondary controller parameter is obtained. Fig.4 gives the inner loop relay output when PI controller is used in outer loop. Saraf et. al.[8] has assumed only the principal harmonics for analysis of relay oscillations(conventional relay autotuning).

TABLE I
Estimated ultimate controller gain k_u for the two examples

Eg.	Method	Secondary process		Primary process	
		a^*	k_u	a^*	k_u
Eg-1	N=1	0.022	5.7875	0.0325	3.9603
	N=3	0.0175	7.2757	0.0268	4.7509
Eg-2	N=1	0.0491	2.5932	0.0678	1.8779
	N=3	0.0365	3.4883	0.0537	2.3710
	N=5	0.0355	3.5866	0.0522	2.4392

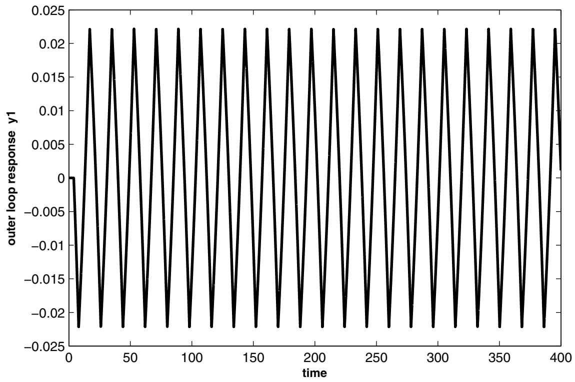

Fig. 2. Response in y_1 for symmetric relay in both loops for example-1

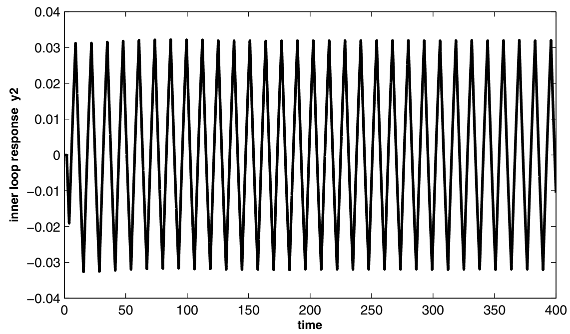

Fig. 3. Response in y_2 for symmetric relay in both loops for example-1

Vivek and Chidambaram [1] has extended the method of Srinivasan and Chidambaram[5] to tune parallel cascade controllers for stable FOPTD systems, wherein, the higher order harmonics of the output relay are considered to find the ultimate gain k_u. Here the value of a_0 is found using procedure briefed in Appendix A and depends on the deviation from the sine wave. Depending on the deviation from the sine wave, the number of higher order terms considered for higher order harmonics(N) varies and can have value of 3, 5 or 7. Since the output wave is a triangular waveform, (A.5) is used for higher harmonic study. In this example, $N = 3$ and $N = 5$ higher order harmonics were studied. An improvement is seen in the responses, when higher order harmonics is considered. $N = 3$ and $N = 5$ curves are studied and it was found that there is not much improvement between the two. So in this example,$N = 3$ higher order

TABLE II

P/PI CONTROLLER SETTINGS FOR TWO EXAMPLES

Eg.	Method	Secondary process	Primary process	
		k_{c2}	k_{c1}	τ_I
Example-1	N=1	0.9901	2.1651	15
	N=3	1.1877	2.4277	15
Example-2	N=1	0.4695	1.4493	18.05
	N=3	0.5928	1.6809	18.05
	N=5	0.6098	1.7044	18.05

harmonics is considered.

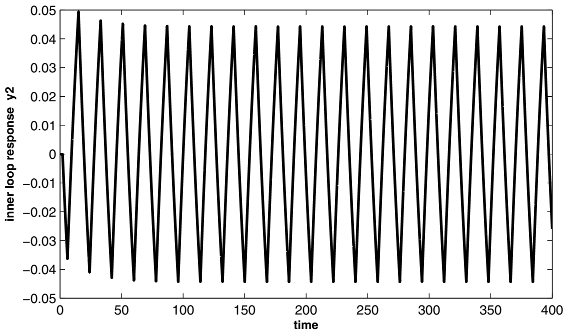

Fig. 4. Response in y_2 for symmetric relay in secondary loop for example-1

The value of a_0 is not the amplitude of the closed loop oscillation as in the conventional relay tuning. Depending on the value of N(number of terms to be considered for higher order harmonics), a_0 changes and denoted as a^* when higher order harmonics is considered . With a symmetrical relay $(h = 0.1)$ in both the loops, output oscillations are recorded for the outer loop . At time,t^*, $(t^* = 0.5\pi/\omega_u = 4.5125)$, it is possible to calculate the value of $y(t^*)$ as 0.0202 from the output oscillations (refer to Appendix A). Since the observed oscillations are close to triangular waveform, (A.5) is used to find the value of $a^*(= 0.0175)$. Using the value of a^*, $k_u = 4h/\pi a^*(= 7.2757)$ is obtained. The a^* and ultimate gain k_u values are tabulated in Table 1. The controller settings $(K_{c1}$ and $\tau_I)$ are found using Zeigler Nichols tuning rules and tabulated in Table 2. The relay in the outer loop is replaced by PI controller and the oscillations of the secondary output are analyzed and P controller settings are found out as before. The P/PI controller settings of the two methods are tabulated in Table 2.

The closed loop responses are evaluated for $N = 1$ and $N = 3$ and response curves for the servo and regulatory cases are shown in Fig.5 and Fig.6. From the figures, it can be seen for $N = 1$(considering fundamental harmonics only), the response is oscillatory in nature, with overshoot and settling time also very large. In the case of $N = 3$, the response is less oscillatory and stabilizes faster. The overshoot and the settling time are less on comparison with $N = 1$ curve. Fig.7 shows the manipulated variable versus time and the error indices for the comparison are given in Table 3.

TABLE III

COMPARISON OF ERRORS FOR TWO EXAMPLES

Eg.	Method	Servo			Regulatory		
		IAE	ISE	ITAE	IAE	ISE	ITAE
Eg. 1	N=1	109.8	62.41	12110	33.21	6.288	3522
	N=3	60.35	29.9	42.48	15.16	2.267	1020
Eg. 2	N=1	25.61	13.27	729.1	38.86	31.05	1346
	N=5	11.75	7.305	128.1	17.37	9.478	380

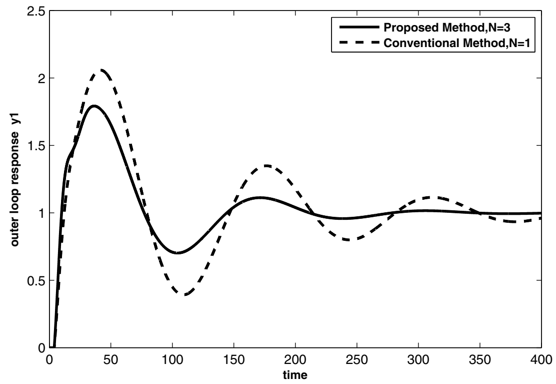

Fig. 5. Servo response for example-1

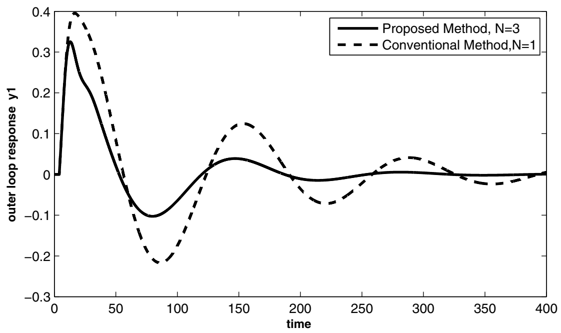

Fig. 6. Regulatory response for example-1

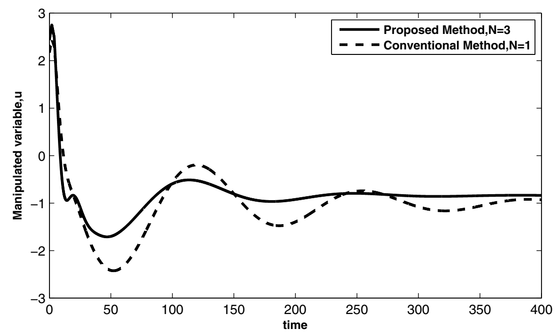

Fig. 7. Manipulated variable versus time for example 1 under servo conditions

B. Example-2

Consider a parallel cascade control scheme with inner unstable FOPTD process defined by $k_{p2}G_{p2} = 2.0exp(-2s)/(10s-1)$ and outer unstable FOPTD process defined by $k_{p1}G_{p1} = exp(-4s)/(10s-1)$. Using a symmetrical relay of height 0.1, the outer loop oscillations are recorded and amplitude and frequency of oscillations noted for N=1,N=3 and N=5. Here, the observed relay oscillations are close to a triangular waveform and (A.5) is used for further analysis of higher order harmonics as in the first example. Estimated a^* and ultimate gain k_u values are tabulated in Table 1. Using the Zeigler Nichols tuning rules, as mentioned in the first example, the PI settings for the outer loop can be calculated. Using PI settings in the outer loop and symmetrical relay in the inner loop, the inner loop sustained oscillations are recorded and the P controller settings are estimated. The same procedure is followed for $N=3$ and $N=5$ higher harmonics. The responses of the inner and outer loop when both relays are executed is shown in Fig.8 and Fig.9. The controller parameters are tabulated in Table 2. Error comparison of two methods show a huge variation and is tabulated in Table 3.

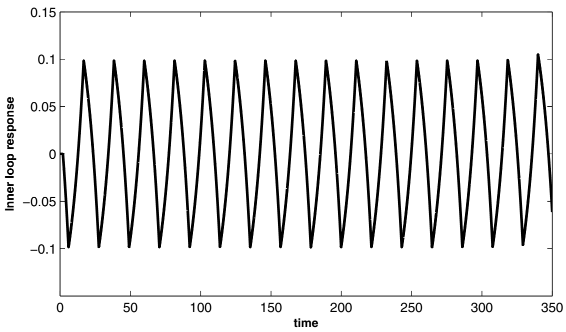

Fig. 8. Secondary loop response, y_2 for symmetric relay in both loops for example-2

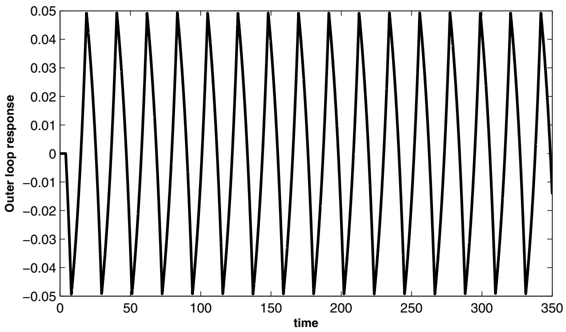

Fig. 9. Primary loop response, y_1 for symmetric relay in both loops for example-2

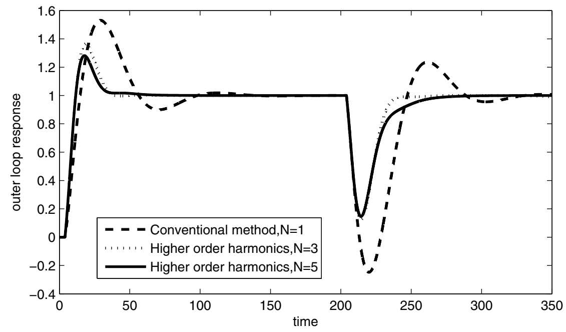

Fig. 10. Response curves for example -2 under perfect conditions

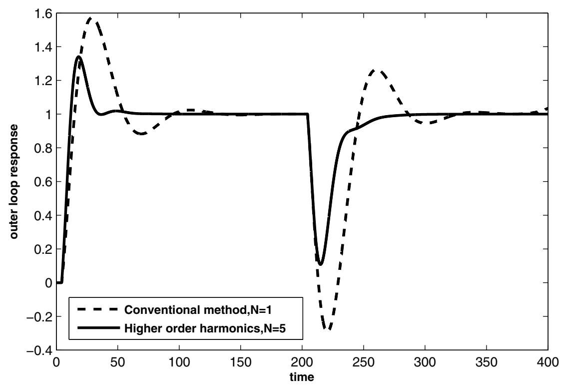

Fig. 11. Response curves for example -2 with 10% increase in time delay of the outer loop

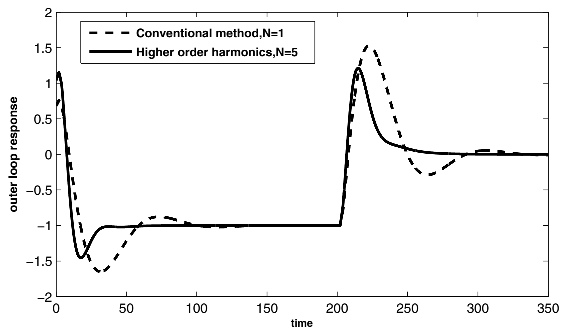

Fig. 12. Manipulated variable versus time for example-2

The closed loop responses are evaluated for $N = 1$(Conventional method), $N=3$ and $N=5$ and response curves under perfect condition and 10% uncertainity increase in time delay in the outer loop is shown in Fig.10 and Fig.11. From the figures, it can be seen that proposed method ($N=3$ and $N=5$) gives improved performance than conventional method ($N=1$) curve. Comparison for $N=3$ and $N=5$ shows that performance for $N=5$ is better than $N=3$. Fig.12 shows the manipulated variable versus time response curve. The effect of measurement noise is studied by adding a random noise (standard deviation of 0.01) in the outer loop and the response curve is shown in Fig. 13. From the figures, it is seen that a robust performance is obtained.

For an unstable single loop FOPTD system, ratio of process time delay to time constant should be less than 1 to be stabilised by a proportional controller,which is also the required criteria for sustained oscillation by relay autotuning. For a parallel cascade system, this ratio is further reduced. In the present simulation studies, numerical values of time delay and time constant are chosen so as to satisfy the above criteria.

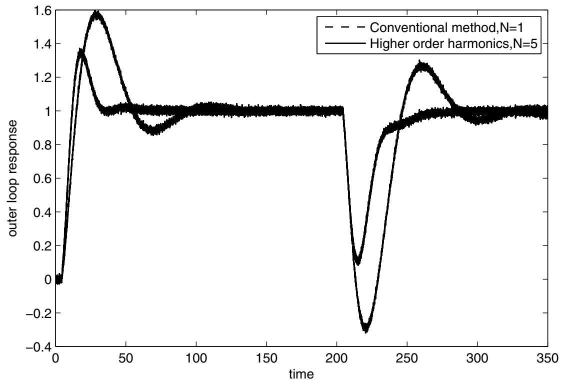

Fig. 13. Response curves with measurement noise in outer loop with a standard deviation of 0.01 for example-2

III. CONCLUSIONS

Modified relay autotuning of parallel cascade control of stable systems proposed by Vivek and Chidambaram[1] is extended to tune open loop unstable parallel cascade systems. The performance of P/PI controllers are compared with conventional relay method. Results show that inclusion of higher order harmonics of the relay output gave a good improvement in the closed loop response due to improved values of the estimated ultimate controller gain, k_u.

APPENDIX A

Astrom and Hagglund[2] had reported expressions for ultimate gain and ultimate time period as

$$k_u = 4h/\pi a; \omega_u = 2\pi/P_u \quad (A.1)$$

The input to any process under relay consists of many sine waves and for a FOPTD system, output wave is also sinusoidal in nature,but with different amplitude and frequency. By assuming that all physical systems are low pass filters and thus exclusion of higher order dynamics of the relay output leads to large error in estimation of k_u . But in literature, for many systems, it has been reported that the output wave deviates from a pure sine wave. To account for the k_u correction , the value a_0 is not the amplitude of the process output under relay as mentioned before and is found using the procedure by Srinivasan et. al.[5] from the output oscillations and depends on the number of terms to be considered for higher order harmonics (N) . Srinivasan and Chidambaram[5] had included higher order harmonics for calculation of the ultimate controller gain k_u, by estimating the value of a_0 from the output oscillatory graph itself and depends on the deviation from pure sine wave. A brief summary of this method applied to parallel cascade cascade control is what follows. With symmetrical relays in both loops, for the outer loop, sustained oscillations are analyzed. Consider the time t^* ,where

$$t^* = 0.5\pi/\omega_u \quad (A.2)$$

where, ω_u is the frequency of the observed output oscillations. Using this t^*, $y(t^*)$ can be calculated form the oscillatory graph.It has been observed that output responses can be either of triangular or rectangular waveform. If the observed oscillations are close to a rectangular waveform, then the new amplitude,a^* is calculated by the following expression and depend on the number of terms considered for higher order harmonics.

$$y(t^*) = a^*(1 - (1/3) + (1/5) - (1/7) + (1/9) + ...) \quad (A.3)$$

Let N be the number of terms considered in the above equation for higher order harmonics. Usually the value of N can be 3 or 5 or 7 and depends on the deviation from pure sine wave. For the summation term,if limiting value is taken as 0.25π ,the expression is

$$a^* = 1.273y(t^*) \quad (A.4)$$

If the observed oscillations are close to triangular waveform, the amplitude a^* can be calculated as

$$y(t^*) = a^*(1 + (1/9) + (1/25) + (1/49) + (1/81) + ...) \quad (A.5)$$

Using the limiting value of summation term as $(0.125\pi^2)$,

$$a^* = 0.810y(t^*) \quad (A.6)$$

Using the new value of amplitude, a^*, the value of k_u is given by the following expression

$$k_u = 4h/\pi a^* \quad (A.7)$$

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Nomenclature

s	Laplace variable
d_1	disturbance entering outer loop
d_2	disturbance entering inner loop
k_{c1}, τ_I	outer loop controller settings
k_{c2}	inner loop controller setting
$k_{p1}G_{p1}$	outer loop transfer function
$k_{p2}G_{p2}$	inner loop transfer function
$k_{L1}G_{L1}$	transfer function for disturbance in outer loop
$k_{L2}G_{L2}$	transfer function for disturbance in inner loop
a_0	amplitude of oscillation corresponding to fundamental harmonic
a^*	amplitude of oscillation considering higher order harmonics
k_u	Ultimate controller gain
h	relay height
ω_u	ultimate frequency of oscillations