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Stiffness Optimization by Extremum Seeking Control to Realize Energy Saving

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***Abstract*— In recent years, many types of variable stiffness devices are developed. These devices have an ability to change the stiffness of a mechanical system. They can also store energy thanks to elastic elements, and convert it to kinetic energy. By installing these devices in the mechanical system, it is expected that an actuator's energy which is required to achieve a periodic motion will be reduced. In this research, we realize on-line stiffness optimization and on-line energy saving by using variable stiffness device and adopting extremum seeking control. Therefore, even if the target motion and parameters of the plant are subject to change, the system can continue to work with a little energy. It was shown by numerical simulation.**

I. INTRODUCTION

These days, industrial robots are working in various situations. Thanks to these robots, people were delivered from dangerous works and high production efficiency was realized. But in order to operate them, large amount of energy is required. So when it comes to running them for a long time, energy consumption is immeasurable. Considering that more and more industrial robots will be installed in the future, this problem cannot be ignored. Therefore, the research on working them with a little energy, that is "energy saving", has been actively carried out.

To realize energy saving, an energy conversion is considered as the solution. Specifically, if the mechanical system has an ability to store potential energy and convert it into kinetic energy for achieving the target motion, the actuator's energy consumption will be suppressed. For example, pendulums convert gravitational potential energy into kinetic energy and can continue to swing with no external force. Passive dynamic walker [1], [2] is the robot which can walk without actuators, because it owns the ability described above. Furthermore, it is known that living things also move in energy conservation by making use of tendons and muscles.

Therefore, one method to achieve energy saving, using variable stiffness devices is considered effective. Many types of these devices have been developed, such as Antagonistic-Controlled type, Structure-Controlled type, Mechanically-Controlled type, and so on [3]. They are able to store energy because elastic elements are installed in them. By adopting this device, the mechanical system becomes possible to store elastic(potential) energy and convert it into kinetic energy for realizing the desired motion. They also have an ability to change the stiffness of the controlled system. It means that they can regulate the natural frequency of the system. A common movement of industrial robots are repetitive motion, so there exists frequency. When the natural frequency matches the target motion's frequency, the resonance phenomenon is derived. If resonance is realized, the desired motion will be achieved with a little energy of the actuator thanks to the elastic energy [4]. Accordingly, it is expected that the variable stiffness device will dramatically reduce energy consumption of the mechanical system.

In this paper, energy saving realization by utilizing the variable stiffness device is stated. The target motion is periodic and composed of multi frequency components. In the case of a single frequency motion, the optimal stiffness value, which realizes the best energy efficiency, can be calculated easily based on resonance theory. However, in the case of multi frequency motion, the optimal stiffness cannot be computed in a similar way. This time we define an energy cost function, then calculate the stiffness value which minimizes this. With this approach, the optimal stiffness in the case of multi frequency motion can be defined [5].

Moreover, in this study, "extremum-seeking control" [6] is applied to the controlled system. It allows the variable stiffness device to optimize its stiffness on-line. Thus, even when the target motion or parameters of plant are changed depending on time, this controller is able to track the ideal stiffness value and continue to save energy. It was shown by numerical simulation.

The rest of this paper is structured as follows. In section 2, the controlled system is explained. The structure and the theory of extremum seeking control is stated in section 3. An optimal stiffness value based on the theory of extremum-seeking is also calculated in this section. The numerical simulations are presented in section 4 while the conclusion and future work are discussed in section 5.

II. PROBLEM ESTABLISHMENT

A. Dynamics of Controlled System

The controlled system is a mechanical system as shown in Fig. 1.

It has an variable stiffness device and one-degree-of-freedom. Referring to Fig. 1, the motion equation of this system is given as

$$m\ddot{\theta}(t) + d\dot{\theta}(t) + k_h\theta(t) = \tau(t) - k(t)\theta(t). \quad (1)$$

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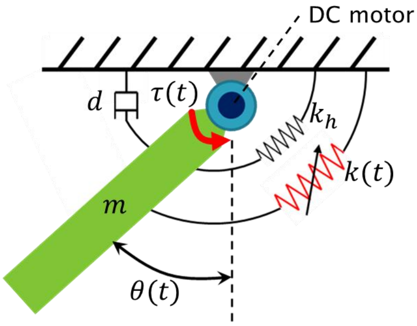

Fig. 1. Controlled System

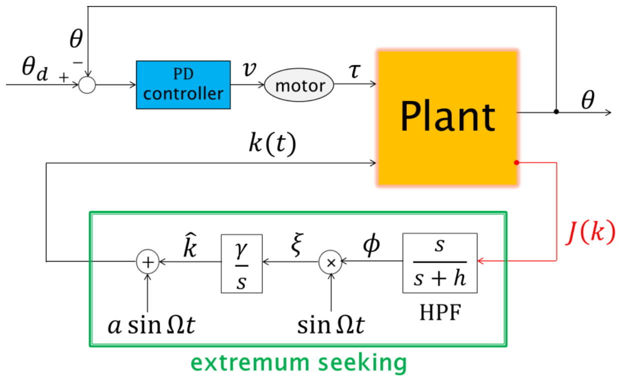

Fig. 2. Schematic Diagram of The Proposed System

In this paper, a desired motion $\theta_d(t)$ is assumed to be a periodic motion with multi frequency components

$$\theta_d(t) = \sum_{i=1}^{n} a_i \sin(i\omega t + \psi_i). \tag{2}$$

Each parameter of (1), (2) is described in Table 1.

TABLE I
DESCRIPTION OF THE PARAMETERS

m	inertia of the mechanical system
d	viscosity of the mechanical system
k_h	stiffness of the mechanical system
$\theta(t)$	position of the link
$\tau(t)$	motor torque
$k(t)$	variable stiffness
n	number of the component
a_i	amplitude of each component
ω	angular frequency
ψ_i	phase of each component

B. Control Objective

The control objective is to achieve the desired motion θ_d with a little motor torque $\tau(t)$ by optimizing the variable stiffness $k(t)$. Which means energy saving. Furthermore, we realize an on-line stiffness optimization by using extremum seeking control.

III. PROPOSED METHOD

A. Extremum Seeking Control

Extremum seeking control has a structure as shown in Fig. 2. It adjusts the parameter to make the cost function $J(k)$ minimum or maximum value continuously. In this case, the parameter is $k(t)$.

B. Proof of Stability

Then, we state the stability of extremum seeking control. At first, we posit the form of the cost function as

$$J(k) = f^* + \frac{f''}{2}(k(t) - k^*)^2. \tag{3}$$

Where f^* and k^* represents the extremum of the cost function J and the optimal value of the parameter $k(t)$. When $f'' > 0 (f'' < 0)$, J is a downwardly(upwardly) convex function.

From Fig. 2,

$$k(t) = a \sin \Omega t + \hat{k}. \tag{4}$$

When substituted into (3), gives

$$\begin{aligned} J(k) &= f^* + \frac{f''}{2}(a \sin \Omega t + \hat{k} - k^*)^2 \\ &= f^* + \frac{f''}{2}(a \sin \Omega t - \tilde{k})^2. \qquad (\tilde{k} = k^* - \hat{k}) \end{aligned} \tag{5}$$

Expanding this expression further, calculated as

$$\begin{aligned} J(k) &= f^* + \frac{f''a^2}{2}\sin^2 \Omega t - f''a\tilde{k}\sin \Omega t + \frac{f''}{2}\tilde{k}^2 \\ &= f^* + \frac{f''a^2}{4} - \frac{f''a^2}{4}\cos 2\Omega t - f''a\tilde{k}\sin \Omega t + \frac{f''}{2}\tilde{k}^2. \end{aligned} \tag{6}$$

The high pass filter removes low frequency components

$$\begin{aligned} \phi &= \frac{s}{s+h}[J(k)] \\ &\approx -\frac{f''a^2}{4}\cos 2\Omega t - f''a\tilde{k}\sin \Omega t + \frac{f''}{2}\tilde{k}^2. \end{aligned} \tag{7}$$

Then, ξ is given by

$$\xi = -\frac{f''a^2}{4}\cos 2\Omega t \sin \Omega t - f''a\tilde{k}\sin^2 \Omega t + \frac{f''}{2}\tilde{k}^2 \sin \Omega t. \tag{8}$$

Applying the identity

$$\cos 2\Omega t \sin \Omega t = \frac{\sin 3\Omega t - \sin \Omega t}{2}.$$

(8) is calculated as

$$\begin{aligned} \xi = &-\frac{f''a^2}{8}(\sin 3\Omega t - \sin \Omega t) - \frac{f''a}{2}\tilde{k} \\ &+ \frac{f''a}{2}\tilde{k}\cos 2\Omega t + \frac{f''}{2}\tilde{k}^2 \sin \Omega t. \end{aligned} \tag{9}$$

There are two conditional expressions

$$\hat{k} = \frac{\gamma}{s}[\xi] \quad \Leftrightarrow \quad \dot{\hat{k}} = \gamma\xi. \tag{10}$$

$$\dot{\tilde{k}} = \dot{k^*} - \dot{\hat{k}} = -\dot{\hat{k}}. \tag{11}$$

Considering these conditions and that the first, third, and forth term of (9) are attenuated by an integrator, getting

$$\dot{\tilde{k}} = -\gamma\xi = \frac{\gamma f'' a}{2}\tilde{k}. \qquad (\tilde{k} = k^* - \hat{k}) \tag{12}$$

Since $\gamma f'' a < 0$, this is a stable system. Thus, $\tilde{k}$ converges to 0 as time passes. In terms of the original problem, $\hat{k}$ converges to k^*, so $k(t)$ is kept within a small distance of k^*. As a result, the cost function converges to around their extremum f^*.

C. Cost Function

Here, we discuss how to decide the cost function $J(k)$ to achieve energy saving. The mechanical system is powered by DC motor shown in Fig. 3, so we assume the electric power which is consumed by the motor per period. It can be written by

$$W = \int_t^{t+T} v(t)i(t)dt. \tag{13}$$

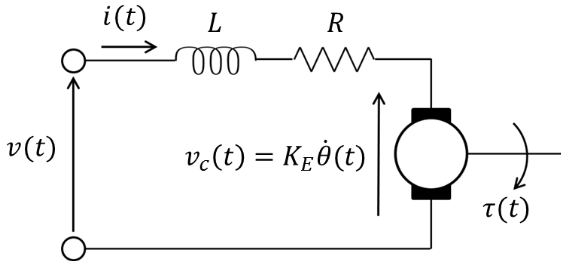

Fig. 3. DC Motor

Also, the following relational expressions are established.

$$v(t) = L\dot{i}(t) + Ri(t) + K_E\dot{\theta}(t). \tag{14}$$

$$\tau(t) = K_T i(t). \tag{15}$$

$$K_T = K_E \tag{16}$$

$$\tau(t) = m\ddot{\theta}(t) + d\dot{\theta}(t) + k_h\theta(t) + k(t)\theta(t). \tag{17}$$

Therefore,

$$\begin{aligned} v(t)i(t) &= L\dot{i}(t)i(t) + Ri(t)^2 + K_T\dot{\theta}(t)i(t) \\ &= L\dot{i}(t)i(t) + Ri(t)^2 + \tau(t)\dot{\theta}(t) \\ &= L\dot{i}(t)i(t) + Ri(t)^2 + m\ddot{\theta}(t)\dot{\theta}(t) \\ &\quad + d\dot{\theta}(t)^2 + (k_h + k(t))\theta(t)\dot{\theta}(t). \end{aligned} \tag{18}$$

Then, substitute a constant stiffness k_c and the desired motion $\theta_d(t)$ into $k(t)$ and $\theta(t)$, (13) is calculated as

$$\begin{aligned} W &= P(t+T) - P(t) + R\int_t^{t+T} i(t)^2 dt + d\int_t^{t+T} \dot{\theta}_d(t)^2 dt \\ &= \frac{R}{{K_T}^2}\int_t^{t+T} \tau(t)^2 dt + d\int_t^{t+T} \dot{\theta}_d(t)^2 dt, \end{aligned} \tag{19}$$

where

$$P(t) = \frac{1}{2}Li(t)^2 + \frac{1}{2}m\dot{\theta}_d(t)^2 + \frac{1}{2}(k_h + k_c)\theta_d(t)^2.$$
$$P(t+T) - P(t) = 0.$$

The second term of (19) depends on the target motion and the parameter of the mechanical system, so minimizing W is equal to minimizing $\int_t^{t+T} \tau(t)^2 dt$. Therefore, we select

$$J(k) = \int_t^{t+T} \tau(t)^2 dt, \tag{20}$$

as a cost function to maximize energy efficiency.

D. Optimal Stiffness

In this part, an optimal stiffness k_{opt} is defined. The optimal stiffness is the stiffness value which minimizes the cost function $J(k)$.

The needed torque to realize target motion is calculated by substituting (2) and k_c into (17)

$$\begin{aligned} \tau(t) = \sum_{i=1}^{n} a_i\{&-(m\omega^2 i^2 - k_h - k_c)\sin(i\omega t + \psi_i) \\ &+ d\omega i\cos(i\omega t + \psi_i)\}. \end{aligned} \tag{21}$$

Then, the cost function $J(k_c)$ is given by

$$\begin{aligned} J(k_c) &= \int_t^{t+T} \Bigg[\sum_{i=1}^{n} a_i\{-(m\omega^2 i^2 - k_h - k_c)\sin(i\omega t + \psi_i) \\ &\qquad + d\omega i\cos(i\omega t + \psi_i)\}\Bigg]^2 dt. \\ &= \frac{T}{2}\sum_{i=1}^{n} a_i^2\{(m\omega^2 i^2 - k_h - k_c)^2 + d^2\omega^2 i^2\}. \end{aligned} \tag{22}$$

k_{opt} is equal to k_c which satisfies the following equation

$$\begin{aligned} \frac{\partial J(k_c)}{\partial k_c} &= -T\sum_{i=1}^{n} a_i^2(m\omega^2 i^2 - k_h) + k_c T\sum_{i=1}^{n} a_i^2 \\ &= 0. \end{aligned} \tag{23}$$

Accordingly, the optimal stiffness k_{opt} is calculated as

$$k_{opt} = \frac{\sum_{i=1}^{n} i^2 a_i^2}{\sum_{i=1}^{n} a_i^2} m\omega^2 - k_h. \tag{24}$$

IV. SIMULATION

A. Condition

The simulation conditions are shown in Table 2.

TABLE II

SIMULATION CONDITIONS

	time [s]	desired motion θ_d [rad]	m [kgm^2]
①	$0 \sim 80$	$2.0\sin(2\pi t) + 1.2\sin(4\pi t + 1.2\pi)$	1.0
②	$80 \sim 160$	$2.0\sin(2\pi t) + 1.2\sin(4\pi t + 1.2\pi)$	1.5
③	$160 \sim 240$	$1.2\sin(2\pi t) + 0.5\sin(4\pi t + 0.7\pi)$	1.5

The other parameters used in the simulation are indicated in Table 3.

TABLE III
VALUE OF THE PARAMETERS

d	$0.1[\mathrm{Nms/rad}]$
k_h	$1.0[\mathrm{Nm/rad}]$
a	3
γ	-0.22
h	14
Ω	$15[\mathrm{rad/s}]$

B. Results

The simulation results are shown in Figs. 4 to 8. Fig. 4 shows the actual stiffness value k obtained through the proposed method and the optimal stiffness k_d calculated from (9). From this figure, it can be seen that k converged to k_d by using extremum seeking control. Also, the link trajectory θ almost converged to the desired motion θ_d as seen in Figs. 5 to 7. Fig. 8 shows the motor torque to achieve the target motion. τ_d is the needed torque in the original system and τ is the one in the proposed system, respectively. As you can see, τ is smaller than τ_d.

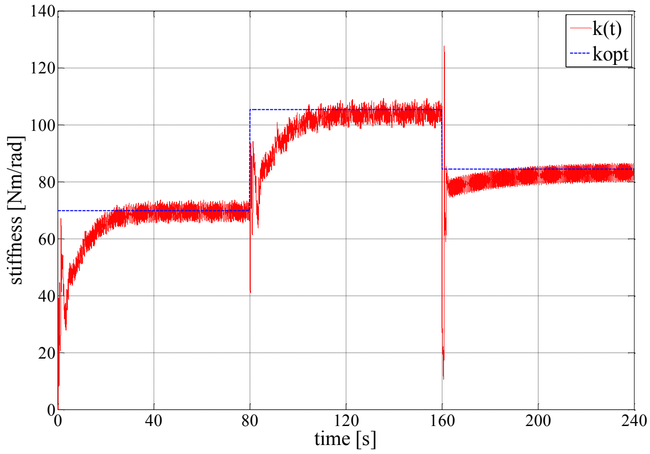

Fig. 4. k_{opt}(blue) and k(red)

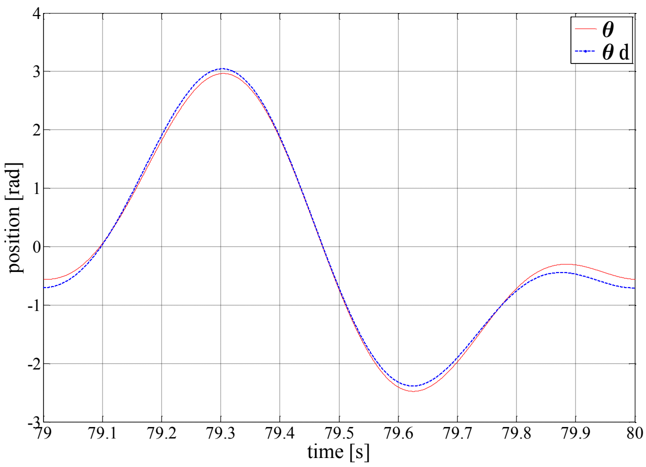

Fig. 5. θ_d(blue) and θ(red) in condition ①

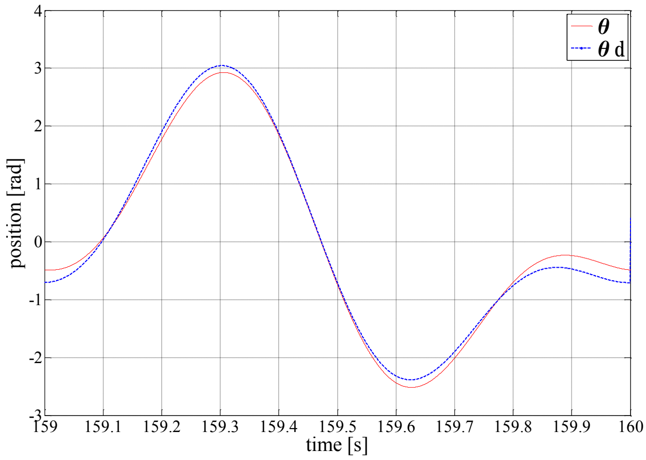

Fig. 6. θ_d(blue) and θ(red) in condition ②

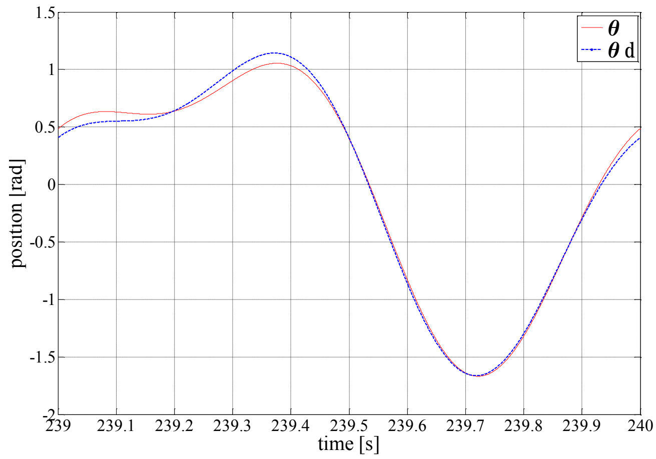

Fig. 7. θ_d(blue) and θ(red) in condition ③

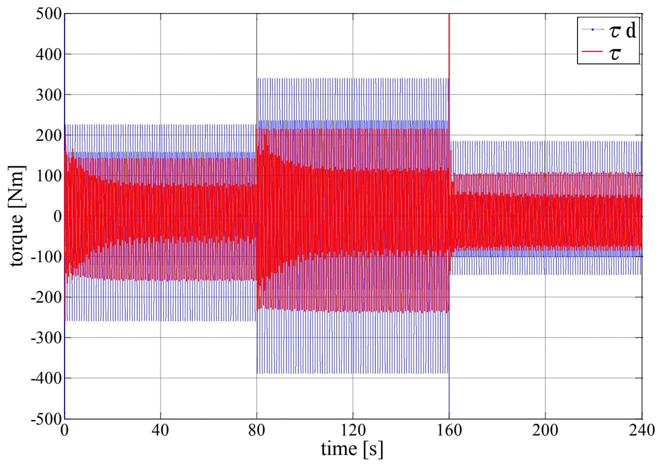

Fig. 8. τ_d(blue) and τ(red)

The following table summarizes the energy consumption of the DC motor in steady-state calculated from (13).

TABLE IV
ENERGY CONSUMPTION

condition	original system [W]	proposed system [W]
①	7.942×10^5	2.808×10^5
②	18.03×10^5	6.314×10^5
③	3.626×10^5	1.285×10^5

According to this, we can understand that the power consumption of the actuator is suppressed to about 36% by optimizing the stiffness of the mechanical system.

V. CONCLUSIONS

In this paper, we realized energy saving of the mechanical system by using the variable stiffness device. The device have an ability to store energy and change the stiffness of the controlled system. When the desired motion is a periodic, the needed motor torque to achieve the motion is suppressed by optimizing the stiffness of the system. It was shown by numerical simulation.

In addition, we introduced "extremum seeking control" into the system and realized on-line stiffness optimization. So, even if the target motion or the parameters of the plant are subject to change, the proposed system can continue to save energy.

In future work, we will extend this method to the multi-degree-of-freedom system which most industrial robots have. Furthermore, we will improve extremum seeking control because this system has a defect that it isn't able to make the parameter converge to the perfect optimal value. It only keeps the parameter around the optimal value. So, we will also consider eliminating this drawback.

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