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# Design of a Multi-Loop Self-Tuning PID Controller for Multivariable Coupled Processes

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***Abstract*— In many industrial systems, the multivariable system which has mutual interaction is often treated. Therefore, decoupling the multivariable system is an important issue to construct a control system. Although various decoupling methods are proposed from the former, the present condition is that industrial application is not carried out positively. In such a background, the decoupling method in which an inverted decoupler was used, attracts attention due to the handling being easy as compared to the conventional method in recent years. That design approach has a desirable design for the continuous time system. However, from a practical viewpoint, the design approach in a discrete time system is desirable. In this paper, an inverted decoupler in a discrete-time system is constructed, and the design method of the multiple loop PID controller to the decoupling system is considered. Specifically, self-tuning control system designing the inverted decoupler and PID controller at the same time is constructed by identification one by one of the system. The effectiveness of this proposed method is shown by a numerical simulation.**

## I. INTRODUCTION

In the most industrial system, the multivariable system with mutual interaction has to be dealt with. Therefore, decoupling the multivariable system is one of the important issue to design a control system. However, conventionally, the present situation is not industrial application positively despite the various decoupling methods have been proposed. In recently years, decoupling methods using a inverted decoupler[1], [2] has attracted attention since the handling is simple as compared with the conventional method. However, the design method is intended for continuous-time system, design approach in discrete-time system is desirable from a practical point of view. Consideration has not been such inverted decoupler design method in discrete-time system.

On the other hand, there is not many at all that a property for the controlled system is grasped enough beforehand when a controlled system for a practical system is designed. As the control method for such case, self-tuning control[3], [4] was suggested, and various research papers have been accomplished. However, self-tuning control tends to come to have many control parameters by the order and time delay of a controlled system, and the increase in number of the control parameters becomes the factor to cause the deterioration of transient properties of the controlled system. Therefore, in reference[6], by designing the control parameters based on Generalized Minimum Variance Control(GMVC), and replacing the control parameter with PID gain[5] approximately, the number of control parameters are decreased in three parameters.

In this paper, the design method of the multiple loop PID controller to the decoupling system is considered. Specifically, the inverted decoupler for a discrete-time system and self-tuning PID control system is simultaneously designed by identifying for a controlled system sequentially online. This design method of PID controller is designed based on GMVC. Furthermore, the effectiveness of this method is verified by a numerical simulation.

## II. DESIGN OF CONTROL SYSTEM

### A. System Description

A control object is assumed to be described as followings.

$$\boldsymbol{A}(z^{-1})\boldsymbol{y}(t) = \tilde{\boldsymbol{B}}(z^{-1})\boldsymbol{u}(t) + \frac{\boldsymbol{\xi}(t)}{\Delta} \tag{1}$$

In equation (1), $\boldsymbol{A}(z^{-1})$ and $\tilde{\boldsymbol{B}}(z^{-1})$ are matrix polynomials expressed with equation (2) and (3).

$$\left.\begin{array}{l} \boldsymbol{A}(z^{-1}) := \mathrm{diag}\left[A_1(z^{-1}),\ A_2(z^{-1}),\ \cdots,\ A_p(z^{-1})\right] \\ A_i(z^{-1}) := 1 + a_{i1}z^{-1} + a_{i2}z^{-2} \\ \qquad (i = 1,\ 2,\ \cdots, p) \end{array}\right\} \tag{2}$$

$$\left.\begin{array}{l} \tilde{\boldsymbol{B}}(z^{-1}) := \begin{bmatrix} \tilde{B}_{11}(z^{-1}) & \cdots & \tilde{B}_{1p}(z^{-1}) \\ \vdots & \ddots & \vdots \\ \tilde{B}_{p1}(z^{-1}) & \cdots & \tilde{B}_{pp}(z^{-1}) \end{bmatrix} \\ \tilde{B}_{jk}(z^{-1}) := z^{-(d_{jk}+1)}B_{jk}(z^{-1}) \\ B_{jk}(z^{-1}) := b_{jk,0} + b_{jk,1}z^{-1} + \cdots + b_{jk,m}z^{-m} \\ \qquad (j = 1,\ 2,\ \cdots, p,\ k = 1,\ 2,\ \cdots, p) \end{array}\right\} \tag{3}$$

Further, $\boldsymbol{y}(t)$, $\boldsymbol{u}(t)$ and $\boldsymbol{\xi}(t)$ denote the system output vector, the control input vector and gaussian white noise vector, and are defined by

$$\boldsymbol{y}(t) := [y_1(t), y_2(t), \cdots, y_p(t)]^T \tag{4}$$

$$\boldsymbol{\xi}(t) := [\xi_1(t), \xi_2(t), \cdots, \xi_p(t)]^T \tag{5}$$

$$\boldsymbol{u}(t) := [u_1(t), u_2(t), \cdots, u_p(t)]^T \tag{6}$$

In addition, $z^{-1}$ is defined as a backward shift operator, $d_{jk}$ and $m$ express time delay and a order of $B_{jk}(z^{-1})$ in equation (3). At this time, the control object is the stable and smallest phase system, values of $m$ and $d_{jk}$ is known.

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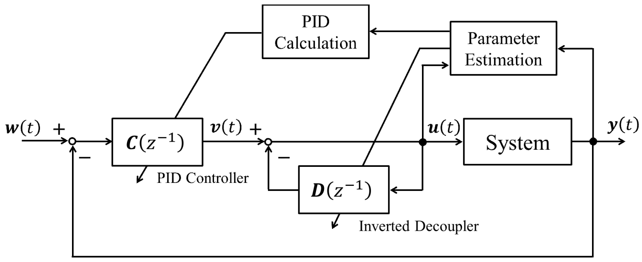


Fig. 1. Block diagram of control system

*B. Design of Inverted Decoupler in Discrete-Time*

The controlled system including the inverted decoupler is shown in Fig.1, where, $\boldsymbol{w}(t)$, $\boldsymbol{v}(t)$, $\boldsymbol{u}(t)$ and $\boldsymbol{y}(t)$ are the set value, the signal to the inverted decoupler, the control input, the system output respectively. Furthermore, $\boldsymbol{C}(z^{-1})$ and $\boldsymbol{D}(z^{-1})$ are respectively the controller and the inverted decoupler.

First, in Fig.1, the control input $\boldsymbol{u}(t)$ can be express as follows:

$$\boldsymbol{u}(t) = \left(I + \boldsymbol{D}(z^{-1})\right)^{-1} \boldsymbol{v}(t), \tag{7}$$

where, $I$ is an unit matrix of $p \times p$. Next, the following equation is provided by substituting equation (7) for equation (1).

$$\boldsymbol{A}(z^{-1})\boldsymbol{y}(t) = \tilde{\boldsymbol{B}}(z^{-1}) \left(I + \boldsymbol{D}(z^{-1})\right)^{-1} \boldsymbol{v}(t) + \frac{\boldsymbol{\xi}(t)}{\Delta} \tag{8}$$

Here, decoupling for the controlled system has to satisfy the following conditions.

$$\tilde{\boldsymbol{B}}_{\text{diag}} = \tilde{\boldsymbol{B}}(z^{-1}) \left(I + \boldsymbol{D}(z^{-1})\right)^{-1} \tag{9}$$

In addition $\tilde{\boldsymbol{B}}_{\text{diag}}$ is expressed as follows:

$$\tilde{\boldsymbol{B}}_{\text{diag}} := \text{diag}\left[\tilde{B}_{11}(z^{-1}),\ \tilde{B}_{22}(z^{-1}),\ \cdots,\ \tilde{B}_{pp}(z^{-1})\right] \tag{10}$$

Therefore, the inverted decoupler $\boldsymbol{D}(z^{-1})$ is provided by the following equation when equation (9) is solved.

$$\boldsymbol{D}(z^{-1}) = \begin{bmatrix} 0 & \frac{\tilde{B}_{12}(z^{-1})}{\tilde{B}_{11}(z^{-1})} & \cdots & \frac{\tilde{B}_{1p}(z^{-1})}{\tilde{B}_{11}(z^{-1})} \\ \frac{\tilde{B}_{21}(z^{-1})}{\tilde{B}_{22}(z^{-1})} & 0 & \cdots & \frac{\tilde{B}_{2p}(z^{-1})}{\tilde{B}_{22}(z^{-1})} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\tilde{B}_{p1}(z^{-1})}{\tilde{B}_{pp}(z^{-1})} & \frac{\tilde{B}_{p2}(z^{-1})}{\tilde{B}_{pp}(z^{-1})} & \cdots & 0 \end{bmatrix} \tag{11}$$

*C. Derivation of GMVC law*

Control parameters are calculated based on GMVC. In this subsection, GMVC law is shown as follows: First, by the inverted decoupler provided by equation (11), the controlled system of equation (1) is indicated by the following non-interactive system.

$$A_i(z^{-1})y_i(t) = z^{-(d_{ii}+1)}B_{ii}(z^{-1})u_i(t) + \frac{\xi_i(t)}{\Delta}, \tag{12}$$

where, $i = 1, 2, \cdots, p$. Therefore, GMVC is applied for each loop because system of equation (12) is a non-interactive system. Next, for the system of equation (12), the control law is derived to minimize the evaluation criterion shown as follows:

$$J_i = E[\phi_i^2(t + d_{ii} + 1)] \tag{13}$$

However $\phi_i$ is the generalized output, and it is defined by

$$\phi_i(t + d_{ii} + 1) = P_i(1)w_i(t) - P_i(z^{-1})y(t) + \lambda u_i(t) \tag{14}$$

Here, $\lambda_i$ indicates the weight for each input, $w_i(t)$ shows the reference with piece constant for each output. In addition, $\boldsymbol{P}(z^{-1})$ is the design polynomial[7] expressed by

$$\left.\begin{aligned} P_i(z^{-1}) &= 1 + p_{i,1}z^{-1} + p_{i,2}z^{-2} \\ p_{i,1} &:= -2e^{-\frac{\rho_i}{2\mu_i}} \cos\left(\frac{\sqrt{4\mu_i - 1}}{2\mu_i}\rho_i\right) \\ p_{i,2} &:= e^{-\frac{\rho_i}{\mu_i}} \\ \rho_i &:= T_s/\sigma_i \\ \mu_i &:= 0.25(1 - \delta_i) + 0.51\delta_i \end{aligned}\right\} \tag{15}$$

In equation (15), $T_s$, $\delta_i$ and $\sigma_i$ respective indicate sampling time, related to rise-time and dumping property.

However, when present time is assumed $t$, generalized output, $\phi_i(t + d_{ii} + 1)$ is future value of $d_{ii} + 1$ step ahead. Therefore, $\phi_i(t + d_{ii} + 1)$ cannot be obtained the value in present time $t$. Accordingly, Diophantine equation is introduced to be given in following equation to get $d_{ii} + 1$ ahead point predict in generalized output $\phi_i(t)$.

$$P_i(z^{-1}) = \Delta A_i(z^{-1})E_i(z^{-1}) + z^{-(d_{ii}+1)}F_i(z^{-1}), \tag{16}$$

where, $E_i(z^{-1})$ and $F_i(z^{-1})$ are polynomials shown as following equations.

$$\left.\begin{aligned} E_i(z^{-1}) &:= 1 + e_{i,1}z^{-1} + e_{i,2}z^{-2} + \cdots + e_{i,d_{ii}}z^{-d_{ii}} \\ F_i(z^{-1}) &:= f_{i,0} + f_{i,1}z^{-1} + f_{i,2}z^{-2} \end{aligned}\right\} \tag{17}$$

Then, by equation (1), (14) and (16), the following relation is obtained.

$$\begin{aligned} \phi_i(t + d_{ii} + 1|t) =& G_i(z^{-1})\Delta u_i(t) + F_i(z^{-1})y_i(t) \\ & - P_i(1)w_i(t) \\ & + E_i(z^{-1})\xi_i(t + d_{ii} + 1) \end{aligned} \tag{18}$$

Here, $\phi_i(t + d_{ii} + 1|t)$ indicates value of $d_{ii} + 1$ ahead point predict in generalized output $\phi_i(t)$, and $G_i(z^{-1})$ is expressed as follows:

$$G_i(z^{-1}) = E_i(z^{-1})B_{ii}(z^{-1}). \tag{19}$$

From equation (13) and (18), the following equation is provided.

$$\begin{aligned} J_i =& E\left[\left\{G_i(z^{-1})\Delta u_i(t) + F_i(z^{-1})y_i(t)\right.\right. \\ & \left.\left. - P_i(1)w_i(t) + E_i(z^{-1})\xi_i(t + d_{ii} + 1)\right\}^2\right] \end{aligned} \tag{20}$$

As the result, the following control law is provided by taking partial differentiation for $J_i$ in $\Delta u_i(t)$ so that equation (20) is minimized.

$$\Delta u_i(t) = \frac{1}{G_i(z^{-1}) + \lambda_i}\left\{P_i(1)w_i(t) - F_i(z^{-1})y_i(t)\right\} \tag{21}$$

*D. Replacement to PID gains*

By the static gain $G_i(1)$ given by replacing $G_i(z^{-1})$ in equation (21), the following equation is obtained.

$$\Delta u_i(t) = \frac{1}{G_i(1)+\lambda_i}\left\{P_i(1)w_i(t) - F_i(z^{-1})y_i(t)\right\} \quad (22)$$

Then, proportion and differential precedence type PID control law is shown in following equation.

$$\Delta u_i(t) = K_{Ii}e_i(t) - \Delta K_{Pi}y_i(t) - \Delta^2 K_{Dii}y_i(t) \quad (23)$$

Where, $e_i(t)$ is error of each loop and expressed as follows:

$$e_i(t) = w_i(t) - y_i(t) \quad (24)$$

Therefore, to compare the coefficients in (22) and (23), the following relationship between the control parameters and PID gains can be obtained.

$$K_{Pi} = -\nu_i\left(f_{i,1} + 2f_{i,2}\right) \quad (25)$$

$$K_{Ii} = \nu_i\left(f_{i,0} + f_{i,1} + f_{i,2}\right) \quad (26)$$

$$K_{Pi} = \nu_i f_{i,2} \quad (27)$$

$$\nu_i := \frac{1}{G_i(1)+\lambda_i} \quad (28)$$

*E. Combination with the Self-Tuning Control*

Based on inverted decoupler design method in II-B and the control law in II-C, II-D, self-tuning control system is designed such as figure 1. Specifically, the control system which realizes desired control performance by using the system parameters to be provided in self-tuning control for not only the adjustment of the PID gains but also the design of the inverted decoupler is designed.

## III. SIMULATION

It is evaluated the effectiveness of the proposed method using a numerical simulation. First, the controlled system assume a 2-input/2-output system and describe the system parameters as follows:

$$\left.\begin{aligned} A_1(z^{-1}) &= 1 - 0.95z^{-1} + 0.10z^{-2} \\ A_2(z^{-1}) &= 1 - 0.85z^{-1} + 0.09z^{-2} \\ \tilde{B}_{11}(z^{-1}) &= 0.63z^{-4} + 0.31z^{-5} \\ \tilde{B}_{21}(z^{-1}) &= 0.36z^{-5} + 0.28z^{-6} \\ \tilde{B}_{12}(z^{-1}) &= 0.48z^{-5} + 0.40z^{-6} \\ \tilde{B}_{22}(z^{-1}) &= 0.51z^{-4} + 0.31z^{-5} \end{aligned}\right\} \quad (29)$$

where, the sampling time Ts is set as 1.0[s], the mean and the variance of the noise $\boldsymbol{\xi}(t)$ are 0 and $10^{-4}$. At this time, the system parameters assume that they are known. Reference values of both systems are switched over to 1.0 or 2.0 each instant of time. In addition, to perform the estimate of the system parameters exactly, the controlled system is inputed irregular signals from control start (1[step]) until 10[step].

First, the result that designed self-tuning PID control system for the controlled system without using a reverse decoupler is shown in Fig.2 and Fig.3. Under the influence of the mutual interference of the controlled system, it is confirmed that the outputs cannot follow the reference values.

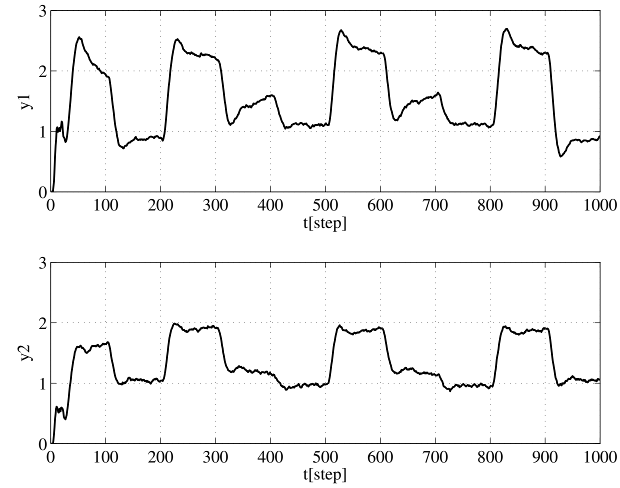


Fig. 2. Output signals corresponding to control inputs shown in Fig.3

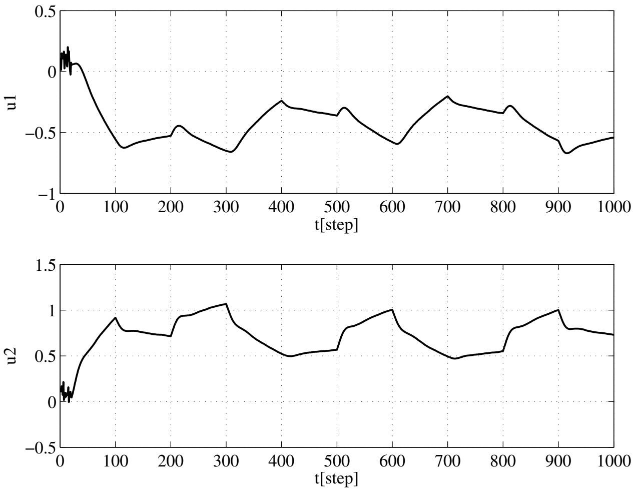


Fig. 3. Control inputs of Fig.2

Each parameters used for the control system design is $\delta_1 = 0$, $\delta_2 = 0$, $\sigma_1 = 20$, $\sigma_2 = 15$, $\lambda_1 = 0.01$, $\lambda_2 = 0.01$.

Next, the result that applied this proposed method is shown in Fig.4, Fig.5 and Fig.6. Each parameters used for the control system design is $\delta_1 = 0$, $\delta_2 = 0$, $\sigma_1 = 20$, $\sigma_2 = 15$, $\lambda_1 = 0.01$, $\lambda_2 = 0.01$ as well as the result in Fig.2 and Fig.3. From these results, a good control result to follow the reference values although control performance is inferior in several decades steps has been provided under the influence of parameters estimate. The control inputs $\boldsymbol{u}(t)$ is coordinated depending on a change of each reference values to suppress the interference, because operation of added the non-interference of the inverted decoupler in $\boldsymbol{v}(t)$ calculated by a multi loop PID controller.

## IV. CONCLUSIONS

In this paper, it is considered that the design method multi loop self-tuning PID control system using the inverted decoupler. In this method, The inverted decoupler and self-

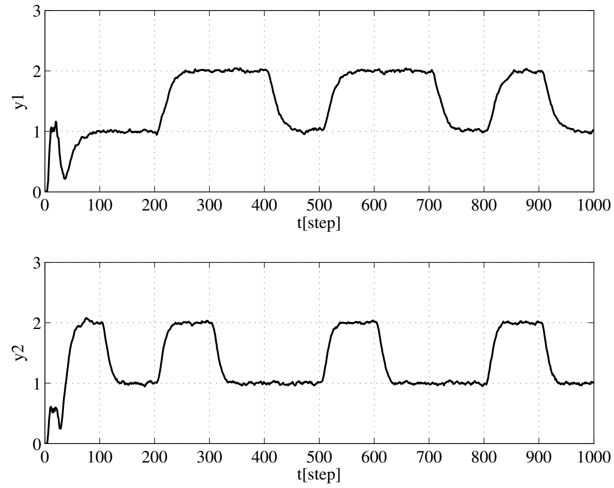


Fig. 4. Output signals corresponding to control inputs shown in Fig.5

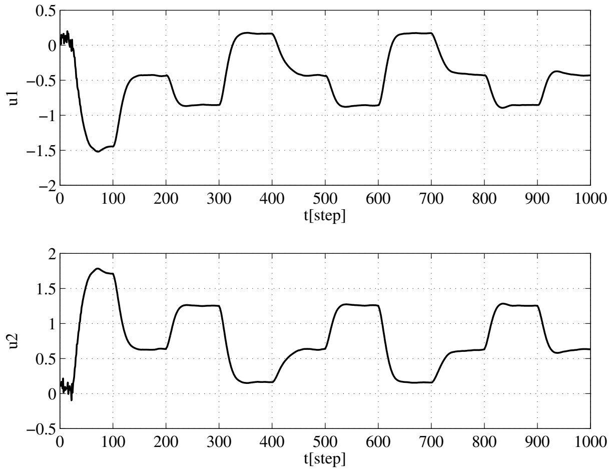


Fig. 5. Control inputs of Fig.4

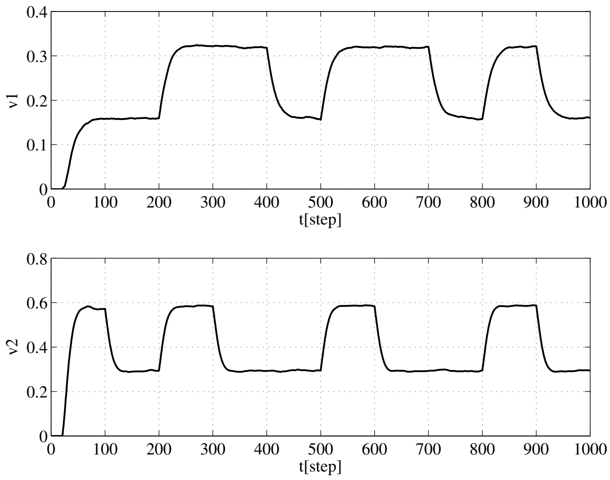


Fig. 6. Signal to inverted decoupler

tuning PID control system are designed at the same time by identifying the system parameters one by one online, and can be obtained desired control performance. Inverted decoupler devised in continuous-time since conventional is converted in discrete-time, PID controller is designed based on GMVC. The effectiveness of this method is shown by a numerical simulation. As for future work, a method determining suitable design parameters $sigma_i$, $\delta_i$ and $\lambda_i$ will be considered, or applied for an unstable system, nonminimum phase system, or experimental system.

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