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# Design of Kreisselmeier Adaptive Observer using Projection Algorithm for Continuous-time Linear Fractional Order Systems

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*Abstract*— **Many complex systems containing fractal structure can be described by fractional order derivative. By expressing the complex system as fractional order system (system containing fractional order derivative), system's complex behavior can be simply described with small number of parameters. However, in order to control or identify such system containing unknown variable parameters, online parameter estimation mechanism targeting at the fractional order system is needed. In this paper, we propose kreisselmeier adaptive observer applicable to fractional order system, and we introduce the projection algorithm method to proposed observer.**

## I. INTRODUCTION

Fractional calculus is the operation expanding the order of differential and integral operation from integer to non-integer order. By using fractional calculus, many complicated dynamics like visco-erastic body's responce or amorphous semiconductor's electric behavior can be described as the simple system containing fractional order derivative (fractional order system)[1],[2]. The system described as fractional order system can express the plant's behavior more accurately by fewer parameters than the system treated as integer order system whose order is simply increased. In addition, the coefficient of fractional order derivative in the fractional order system has many important information. In the case of visco-erastic body, the coefficient of fractional order derivative ($s^{1/2}$) has the information about temperature and state of molecules, and the plant's behavior is influenced by them. So, in order to obtain these information and control the system dicribed by fractional calculus, adaptive observer for such system is needed. In this study, we designed the Kreisselmeier adaptive observer for the fractional calculus system. Then, using proposed adaptive observer, we constructed state-feedback control system.

## II. FRACTIONAL CALCULUS

### A. Definition of Fractional order Caputo derivative

Fractional order Caputo derivative is given by

$$ {}_a^C D_t^q [f(t)] = \int_a^t \frac{(t-\tau)^{n-q-1}}{\Gamma(n-q)} \frac{d^n f(\tau)}{d\tau^n} d\tau \tag{1}$$

where $q$ is the order of the fractional derivative such that $n-1 < q < n$, $n$ is the integer, and $\Gamma(\cdot)$ is the gamma function, which is the function expanding the factorial to arbitraly order.we defined fractional derivative $D^q$ as fractional order Caputo derivative $\left(D^q \equiv {}_0^C D_t^q\right)$.

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### B. Approximation Method of Fractional Calculus

In the case of constructing the control system using the fractional order transfer function, it requires a lot of time to calculate the convolution from the initial time. For reducing the time on calculation, Manabe proposed the approach to approximate the fractional order transfer function to the superposition of the integer order transfer functions on the bode diagram around a specified frequency domain[3].We used this approximation method to calculate the fractional order derivative for the numerical simulation.The transfer function of $1/s^q$ at $1 < q < 2$ can be approximated to

$$\frac{1}{s^q} = \frac{1}{s} \cdot \Pi_{i=1}^{j} \frac{s+a_i}{s+b_i} \cdot \Pi_{i=1}^{j} \frac{1+b_i s}{1+a_i s} \tag{2}$$

$$\Omega_{low} < \omega < \Omega_{high} \tag{3}$$

where

$$\delta = 20 \log_{10} \alpha \tag{4}$$

$$\beta = \alpha^{-\frac{2}{(2-q)(q-1)}} \tag{5}$$

$$a_1 = \alpha^{-\frac{1}{q-1}} \tag{6}$$

$$a_{i+1} = a_i \beta \tag{7}$$

$$b_i = a_i \alpha^{-\frac{2}{2-q}} \tag{8}$$

$$\Omega_{low} = \alpha_{j+1} \tag{9}$$

$$\Omega_{high} = \frac{1}{\alpha_{k+1}} \tag{10}$$

$\Omega_{low} < \omega < \Omega_{high}$ is the approximated frequency domain. In the case of $1/s^r$ at $0 < r < 1$, the approximated transfer function can be obtained by multiplying (2) by $s$. It becomes

$$\frac{1}{s^r} = \frac{1}{s^q} \cdot s = \Pi_{i=1}^{j} \frac{s+a_i}{s+b_i} \cdot \Pi_{i=1}^{j} \frac{1+b_i s}{1+a_i s} \tag{11}$$

where $r = 1 - q$.

### C. Solution, Stability, Controllability and Observability of Fractional Calculus System

Considering the fractonal calculus system shown in following equations.

$$D^q x(t) = Ax(t) + Bu(t), \quad x(0) = x_0 \tag{12}$$

$$y(t) = C^\top x(t) \tag{13}$$

where $q$ is the order of derivative.

By using Mittag-Leffler function, solution of Eq. (12) can be written as

$$\begin{aligned} x(t) &= E_{q,1}(At^q)x(0) \\ &+ \int_0^t (t-\tau)^{q-1}E_{q,q}(A(t-\tau)^q)Bu(\tau)d\tau \end{aligned} \tag{14}$$

$$E_{\alpha,\beta}(Az) = \sum_{k=0}^{\infty} \frac{(Az)^k}{\Gamma(k\alpha+\beta)} \tag{15}$$

$E_{\alpha,\beta}(Az)$ is Mittag-Leffler function. If $\alpha = 1$, $\beta = 1$, $E_{1,1}(Az)$ becomes

$$E_{1,1}(Az) = e^{Az} \tag{16}$$

So, in the case $q = 1$, the solution of $q$ order fractional calculus system Eq. (14) becomes

$$\begin{aligned} x(t) &= e^{At}x_0 \\ &+ \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau \end{aligned} \tag{17}$$

Eq. (17) is the same solution as the solution of integer order system.

As the stability, controllability and observability condition of this system, following conditions are known[4],[5].

*Lemma 1:* Fractional order system described by Eqs. (12) - (13) is stable if and only if

$$|\arg(\lambda_i)| > \frac{q\pi}{2}, \quad (i = 1, 2, \cdots, n) \tag{18}$$

where $_i$ $(i = 1, 2, \cdots, n)$ is eigenvalues of $A$.

*Lemma 2:* Fractional order system described by Eqs. (12) - (13) is controllable if and only if

$$\operatorname{rank} W = n \tag{19}$$

$$W = \begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix} \tag{20}$$

*Lemma 3:* Fractional order system described by Eqs. (12) - (13) is observable if and only if

$$\operatorname{rank} N = n \tag{21}$$

$$N = \begin{bmatrix} C^\top & (CA)^\top & \cdots & (CA^{n-1})^\top \end{bmatrix}^\top \tag{22}$$

## III. ADAPTIVE OBSERVER

### A. Design of Kreisselmeier Adaptive Observer

Single input single output (SISO) plant containing fractional order derivative is given by

$$y^{\left(\frac{n}{m}\right)} + \sum_{j=1}^{n} \alpha_j y^{\left(\frac{n-j}{m}\right)} = \sum_{j=1}^{n} \beta_j u^{\left(\frac{n-j}{m}\right)} \tag{23}$$

where $\alpha_j$ and $\beta_j$ are unkown time-invariant parameters, $u$ is the input, $y$ is the plant's output. State space representation of this plant can be described by following equations.

$$\begin{aligned} D^{\frac{1}{m}}x(t) &= Ax(t) + Bu(t) && (24) \\ y(t) &= C^\top x(t) && (25) \end{aligned}$$

where $1/m$ is the order of the fractional derivative. For the system expressed as Eq. (24) and Eq. (25), we make following assumptions.

(A1) Plant is asymptotically stable.

(A2) Plant is a controllable and observable system.

(A3) The highest degree of the plant $n/m$ is a known quantity.

From (A2), fractional order system can be transformed to the observable canonical form.

$$\begin{aligned} D^{\frac{1}{m}}x_O(t) &= A_O x_O(t) + B_O u(t) && (26) \\ y(t) &= C_O{}^\top x_O(t) && (27) \end{aligned}$$

$$A_O = \left[ \begin{array}{c|c} & I_{n-1} \\ \alpha & \\ & 0 \end{array} \right] \tag{28}$$

$$\alpha^\top = \begin{bmatrix} -\alpha_1 & -\alpha_2 & \cdots & -\alpha_n \end{bmatrix} \tag{29}$$

$$B_O{}^\top = \beta^\top = \begin{bmatrix} \beta_1 & \beta_2 & \cdots & \beta_n \end{bmatrix} \tag{30}$$

$$C_O{}^\top = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \tag{31}$$

By introducing adequate vector $f$, the observable canonical form of fractional order plant Eq. (26) - Eq. (31) can be transformed to following equations.

$$D^{\frac{1}{m}}x_O(t) = Fx_O(t) + (\alpha - f)y(t) + \beta u(t) \tag{32}$$

$$F = \left[ \begin{array}{c|c} & g^\top \\ f & \\ & K \end{array} \right] \tag{33}$$

$$f^\top = \begin{bmatrix} f_1 & f_2 & \cdots & f_n \end{bmatrix} \tag{34}$$

$$g^\top = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \tag{35}$$

$$K = \begin{bmatrix} 0 & & & \\ \vdots & & I_{n-2} & \\ \vdots & & & \\ 0 & \cdots & \cdots & 0 \end{bmatrix} \tag{36}$$

where $f$ is given to make $F$ stable. In order to design the Kreisselmeier adaptive observer, we can rewrite Eq. (32) - Eq. (36) by using following theorem.

*Theorem 1:* Fractional order system described by Eq. (32) - Eq. (36) can be transformed to the following equations.

$$\begin{aligned} x_O(t) &= \sum_{j=0}^{\infty} \frac{F^j t^{\frac{1}{m}j}}{\Gamma\left(\frac{1}{m}j+1\right)} x_O(0) \\ &\quad + R_y(t)(\alpha - f) + R_u(t)\beta \end{aligned} \tag{37}$$

$$y(t) = z(t) + \theta^\top \xi(t) \tag{38}$$

where

$$D^{1/m}R_y(t) = FR_y(t) + I_n y(t), \; R_y(0) = 0 \tag{39}$$

$$D^{1/m}R_u(t) = FR_u(t) + I_n u(t), \; R_u(0) = 0 \tag{40}$$

$$\xi_y(t) = R_y{}^\top(t)C_O \tag{41}$$

$$\xi_u(t) = R_u{}^\top(t)C_O \tag{42}$$

$$D^{1/m}\xi_y(t) = F^\top \xi_y(t) + C_O y(t) \tag{43}$$

$$D^{1/m}\xi_u(t) = F^\top \xi_u(t) + C_O u(t) \tag{44}$$

$$z(t) = C^\top \sum_{j=0}^{\infty} \frac{F^j t^{\frac{1}{m}j}}{\Gamma\left(\frac{1}{m}j+1\right)} x(0) \tag{45}$$

$$\theta^\top = \left[\begin{array}{cc} (\alpha - f)^\top & \beta^\top \end{array}\right] \tag{46}$$

$$\xi^\top = \left[\begin{array}{cc} \xi_y{}^\top & \xi_u{}^\top \end{array}\right] \tag{47}$$

Proof: From the solution of fractional calculus system Eqs. (14) - (15), the solution of Eq. (32) is expressed as follow.

$$\begin{aligned} x(t) &= E_{\frac{1}{m},1}(Ft^{\frac{1}{m}})x(0) \\ &+ \int_0^t (t-\tau)^{\frac{1}{m}-1} E_{\frac{1}{m},\frac{1}{m}}(F(t-\tau)^{\frac{1}{m}})(\alpha - f)y(\tau)d\tau \\ &+ \int_0^t (t-\tau)^{\frac{1}{m}-1} E_{\frac{1}{m},\frac{1}{m}}(F(t-\tau)^{\frac{1}{m}})\beta u(\tau)d\tau \end{aligned} \tag{48}$$

The solution of Eq. (39) and Eq. (40) becomes

$$R_y(t) = \int_0^t (t-\tau)^{\frac{1}{m}-1} E_{\frac{1}{m},\frac{1}{m}}(F(t-\tau)^{\frac{1}{m}})y(\tau)d\tau \tag{49}$$

$$R_u(t) = \int_0^t (t-\tau)^{\frac{1}{m}-1} E_{\frac{1}{m},\frac{1}{m}}(F(t-\tau)^{\frac{1}{m}})u(\tau)d\tau \tag{50}$$

From Eqs. (49) - (50), Eq. (48) can be transformed to the following equation.

$$\begin{aligned} x(t) &= E_{\frac{1}{m},1}(Ft^{\frac{1}{m}})x(0) \\ &+ R_y(t)(\alpha - f) + R_u(t)\beta \end{aligned} \tag{51}$$

Following equation can be obtained by multiplying Eq. (51) by $C_O{}^\top$.

$$\begin{aligned} y(t) = C^\top \sum_{j=0}^{\infty} \frac{F^j t^{\frac{1}{m}j}}{\Gamma\left(\frac{1}{m}j+1\right)} x(0) \\ +\xi_y{}^\top(t)(\alpha -) + \xi_u{}^\top(t)\beta \end{aligned} \tag{52}$$

From Eqs. (49) - (50), following equations can be obtained.

$$FR_y(t) = R_y(t)F \tag{53}$$
$$FR_u(t) = R_u(t)F \tag{54}$$

From Eqs. (53) - (54), $D^{\frac{1}{m}}\xi_y{}^\top(t)$ and $D^{\frac{1}{m}}\xi_u{}^\top(t)$ can be calculated as follows.

$$\begin{aligned} \xi_y{}^{(\frac{1}{m})}(t) &= R_y{}^{\top(\frac{1}{m})}(t)C_O \\ &= (FR_y(t) + I_n y)^\top C_O \\ &= F^\top(t)R_y{}^\top(t)C_O + C_O y(t) \\ &= F^\top(t)\xi_y(t) + C_O y(t) \end{aligned} \tag{55}$$

$$\begin{aligned} \xi_u{}^{(\frac{1}{m})}(t) &= R_u{}^{\top(\frac{1}{m})}(t)C \\ &= (FR_u(t) + I_n u(t))^\top C_O \\ &= F^\top R_u{}^\top(t)C_O + C_O u(t) \\ &= F^\top \xi_u(t) + C_O u(t) \end{aligned} \tag{56}$$

[Q. E. D.]

$\theta$ is the unknown parameter, so, by using estimated parameter $\hat{\theta}(t)$, we construct $\hat{y}(t)$ as follow.

$$\hat{y}(t) = \hat{\theta}^\top(t)\xi(t) \tag{57}$$
$$\hat{\theta}^\top(t) = \left[\begin{array}{cc} (\hat{\alpha}(t) - f)^\top & \hat{\beta}^\top(t) \end{array}\right] \tag{58}$$

where $\hat{\alpha}$ is estimated $\alpha$ parameter, and $\hat{\beta}$ is estimated $\beta$ parameter. Then, $e_1(t) = \hat{y}(t) - y(t)$ can be described by the following equations.

$$e_1(t) = \phi^\top(t)\xi(t) - z(t) \tag{59}$$
$$\begin{aligned} \phi^\top(t) &= \hat{\theta}^\top(t) - \theta^\top \\ &= \left[\begin{array}{cc} (\hat{\alpha}(t) - \alpha)^\top & (\hat{\beta}(t) - \beta)^\top \end{array}\right] \end{aligned} \tag{60}$$

where $\phi$ is estimation error of plant parameters.
$F$ is stable. Thus,

$$z(t) = 0 \quad (t \to \infty) \tag{61}$$

$$e_1(t) = \phi^\top(t)\xi(t) \quad (t \to \infty) \tag{62}$$

Then, it becomes $e_1(t) = 0 (t \to \infty)$ if adaptive control law is given as

$$\begin{aligned} &\dot{\hat{\theta}}_i(t) = -\frac{\gamma_i \xi_i(t) e_1(t)}{\nu + \xi^\top(t)\xi(t)}, \quad (\nu > 0, \gamma_i \geq 0) \\ &(i = 1, 2, \cdots, 2n) \end{aligned} \tag{63}$$

If we give adequate input $u(t)$ satisfying persistently exciting condition, it becomes $\phi = 0$ $(t \to 0)$. Then, proposed adaptive observer can accomplish parameter identification and state observation of fraction order system.

### B. Projection Algorithm

Plant's parameters $\theta$ is unknown. However, in the case the range of $\theta$ is known a priori, adaptive observer can estimate $\theta$ more efficiently by using "Projection Algorithm."

Projection Algorithm can be given as

$$\dot{\hat{\theta}}_i = -\frac{\gamma_i \xi_i(t) e_1(t)}{\nu + \xi^\top(t)\xi(t)} \tag{64}$$

$$\left(\hat{\theta}_i > \theta_{iu}, \xi_i(t)e_1(t) > 0 \text{ or } \hat{\theta}_i < \hat{\theta}_{ib}, \xi_i(t)e_1(t) < 0\right)$$

$$\dot{\hat{\theta}}_i = 0 \tag{65}$$

$$\left(\hat{\theta}_i > \hat{\theta}_{iu}, \xi_i(t)e_1(t) \leq 0 \text{ or } \hat{\theta}_i < \hat{\theta}_{ib}, \xi_i(t)e_1(t) \geq 0\right)$$

By using Eqs. (64) - (65), Adaptive observer can estimate plant's parameters restricting the range of $\hat{\theta}$ to $\left[\hat{\theta}_b, \hat{\theta}_u\right]$.

## IV. STATE-FEEDBACK CONTROL SYSTEM USING ADAPTIVE OBSERVER

Considering the fractional order system described by Eqs. (23) - (25) as controlled object, Eq. (24) can be transformed to controllable canonical form expressed as following equations.

$$D^{\frac{1}{m}} x_C(t) = A_C x_C(t) + B_C u(t) \tag{66}$$

$$A_C = \left[\begin{array}{cccc} 0 & & & \\ \vdots & & I_{n-1} & \\ 0 & & & \\ -\alpha_n & \cdots & \cdots & -\alpha_1 \end{array}\right] \tag{67}$$

$$B_C{}^\top = \left[\begin{array}{cccc} 0 & \cdots & 0 & 1 \end{array}\right] \tag{68}$$

$$T = \begin{bmatrix} t_1{}^\top & t_2{}^\top & \cdots & t_n{}^\top \end{bmatrix} \tag{69}$$

$$t_i{}^\top = \left( A^{n-i} + \sum_{k=i}^{n-1} \alpha_k A^{n-k-1} \right) B \tag{70}$$

$$A_C = T^{-1}AT \tag{71}$$

$$B_C = T^{-1}B \tag{72}$$

$$x_C = T^{-1}x \tag{73}$$

Then, control input $u(t)$ can be given by

$$u = K_C{}^\top x_C + r \tag{74}$$

$$K_C{}^\top = \begin{bmatrix} k_{C1} & k_{C2} & \cdots & k_{Cn} \end{bmatrix} \tag{75}$$

From Eq. (66) and Eq. (74), state-feedback controlled system can be described by

$$D^{\frac{1}{m}} x_C(t) = \left( A_C(t) + B_C K_C{}^\top \right) x_C(t) + B_C r \tag{76}$$

Characteristic polynomial of this closed-loop system becomes

$$\prod_{k=1}^{n} (s^{\frac{1}{m}} - \lambda_k) = s^{\frac{n}{m}} + \sum_{k=1}^{n} (\alpha_k - k_{Ck}) s^{\frac{n}{m}} \tag{77}$$

where $\lambda_1, \lambda_2, \cdots, \lambda_n$ is the pole of the controlled system. We can design the poles of closed-loop system by calculating controller gain $K_C$. To give the state-feedback controller for the system described by Eq. (24), calculated controller gain $K_C$ can be transformed, and control input $u(t)$ is given by

$$u = K^\top x(t) + r \tag{78}$$

$$K = K_C T^{-1} \tag{79}$$

However, $\alpha_j$ $(j = 1, 2, \cdots, n)$ and $\beta_j$ $(j = 1, 2, \cdots, n)$ are unknown parameters. So, we calculate controller gain $\hat{K}$ by replacing $\alpha_j$, $\beta_j$ $(j = 1, 2, \cdots, n)$ with estimated parameters $\hat{\alpha}_j$, $\hat{\beta}_j$ $(j = 1, 2, \cdots, n)$, and we give the control input $u(t)$ as follow.

$$u = \hat{K}^\top x(t) + r \tag{80}$$

## V. NUMERICAL SIMULATION

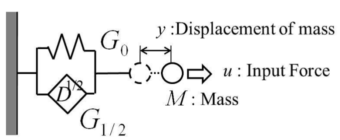


Fig. 1. System connected mass with visco-elastic body.

System connected the mass with visco-elastic body is supposed as shown in Fig. 1. This system can be discribed by

$$MD^2[y(t)] + G_{1/2} D^{1/2}[y(t)] + G_0 y(t) = u(t) \tag{81}$$

If $M = 1$, $G_{1/2} = 1$, $G_0 = 1$, it becomes

$$D^2[y(t)] + D^{1/2}[y(t)] + y(t) = u(t) \tag{82}$$

The state space representation of Eq. (82) can be expressed as follow.

$$D^{\frac{1}{2}} x(t) = Ax(t) + Bu(t) \tag{83}$$

$$y(t) = c^\top x(t) \tag{84}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \end{bmatrix} \tag{85}$$

$$B^\top = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \tag{86}$$

$$c^\top = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \tag{87}$$

To show the effectiveness of proposed adaptive observer, we designed adaptive observer for the system containing three unknown parameters $\alpha_3 = G_{1/2}/M, \alpha_4 = G_0/M, \beta 4 = 1/M$ discribed by Eqs. (83) - (87) and did the numerical simulation in the case 1.

Case 1

$$x(0) = 0 \tag{88}$$

$$\phi^\top(0) = \begin{bmatrix} 0, 0, -10, -10, 0, 0, 0, 10 \end{bmatrix} \tag{89}$$

$$u = \sum_{j=1}^{8} \sin \frac{1+j}{5} t \tag{90}$$

$$f^\top = [-4.9, -21.2, -13.0, -18.8] \tag{91}$$

$$\gamma_i = \begin{cases} 0, & (i = 1, 2, 5, 6, 7) \\ 5.0 \times 10^4, & (i = 3, 4, 8) \end{cases} \tag{92}$$

$$\nu = 1 \tag{93}$$

Fig. 2 show the resalt of parameter identification where $\eta_1 = (\hat{\alpha_3} - \alpha_3)/\alpha_3$, $\eta_2 = (\hat{\alpha_4} - \alpha_4)/\alpha_4$, $\eta_3 = (\hat{\beta_4} - \beta_4)/\beta_4$. Figs. 3 to 6 are resalt of state observation. From Figs. 2 - 6, effectiveness of proposed adaptive observer is shown.

However, by using projection algorithm, response of the proposed adaptive observer can be improved. In this case, the region of the $\theta$ can be estimated as

$$\theta_i \leq -f_i, \quad i = 1, 2, 3, 4 \tag{94}$$

$$0 \leq \theta_i, \quad i = 5, 6, 7, 8 \tag{95}$$

It is because

$$\alpha_i \geq 0, \quad i = 1, 2, 3, 4 \tag{96}$$

$$\beta_i \geq 0, \quad i = 1, 2, 3, 4 \tag{97}$$

To introduce the projection algorithm into the adaptive law, we use Eqs. (64) - (65) and give $\hat{\theta}_u$ and $\hat{\theta}_b$ as follow.

$$\hat{\theta}_u = \begin{bmatrix} -f & \infty & \infty & \infty & \infty \end{bmatrix}^\top \tag{98}$$

$$\hat{\theta}_b = \begin{bmatrix} -\infty & -\infty & -\infty & -\infty & 0 \end{bmatrix}^\top \tag{99}$$

Figs. 7 - 11 show the result of numerical simulation of proposed adaptive observer using projection algorithm. As shown in Figs. 7 - 11, we can confirm the response of adaptive observer can be improved by projection algorithm.

Next, to show the proposed state-feedback control system using adaptive observer with projection algorithm, we compared the responce of controlled system with system not controlled by the numerical simulation in the case 2.

Case 2

$$
\begin{aligned}
x(0) &= 0 && (100)\\
\phi^{\top}(0) &= \left[\ 0,0,\tfrac{1}{2},-3,0,0,0,1\ \right] && (101)\\
r &= \sum_{j=1}^{8} \sin \frac{1+j}{5} t && (102)\\
f^{\top} &= [-4.9,-21.2,-13.0,-18.8] && (103)\\
\gamma_i &= \begin{cases} 0, & (i=1,2,5,6,7) \\ 8.0\times 10^4, & (i=3,4,8) \end{cases} && (104)\\
\nu &= 1 && (105)
\end{aligned}
$$

In this case, state-feedback gain of the controller was calculated by Eqs. (66) - (80) as the poles of the closed-loop system lie at $\lambda_1 = -1+j$, $\lambda_2 = -1-j$, $\lambda_3 = -0.5+2j$, $\lambda_4 = -0.5-2j$. Fig. 12 show the resalt of parameter identification. Figs. 13 to 16 are state of controlled system and not controlled system. From Figs. 12 to 16, it is shown that proposed control system can change dynamic characteristics of fractional calculus system with identifying plant parameters.

## VI. Conclusion

In this paper, Kreisselmeier adaptive observer using projection algorithm for continuous-time linear fractional calculus system and state-feedback control system using adaptive observer was proposed. By numerical simulation, the effectiveness of proposed adaptive observer and state-feedback control system was shown. Using proposed adaptive control system, fractional order system like visco-elastic body can be control with identifying unknown plant parameters.

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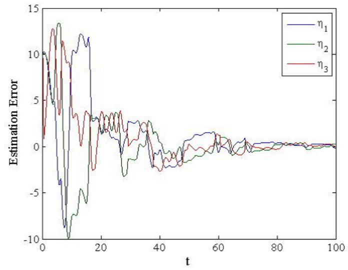


Fig. 2. Estimation Error of Plant Palameters.

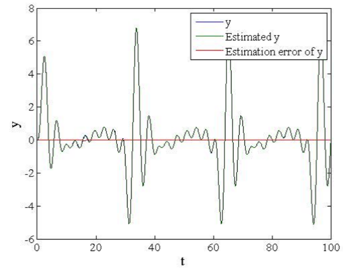


Fig. 3. Estimated $y$ $(y = x_1)$.

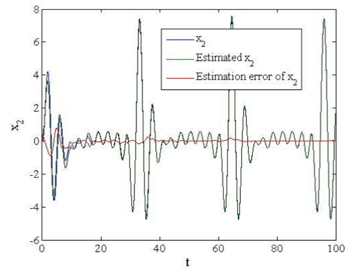


Fig. 4. Estimated $x_2$.

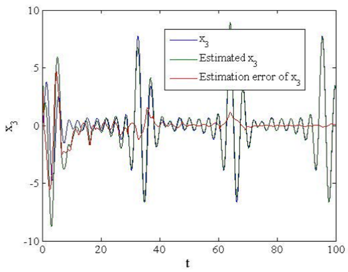


Fig. 5. Estimated $x_3$.

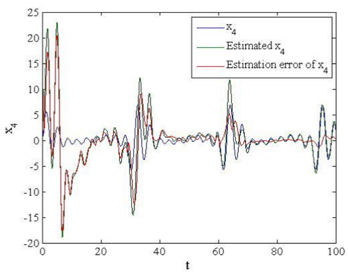


Fig. 6. Estimated $x_4$.

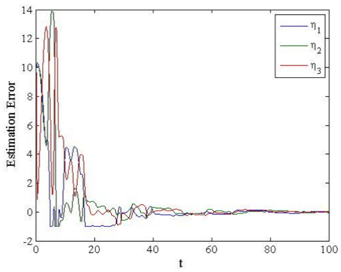


Fig. 7. Estimation Error of Plant Palameters by Projection Algorithm.

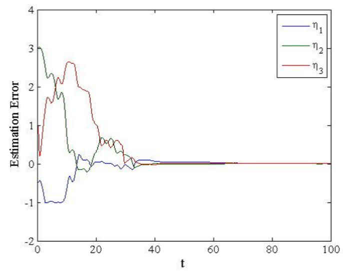


Fig. 12. Estimation Error of Plant Palameters in Adaptive Control system.

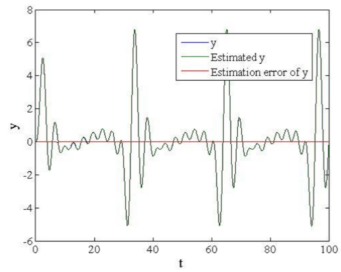


Fig. 8. Estimated $y$ $(y = x_1)$ by Projection Algorithm.

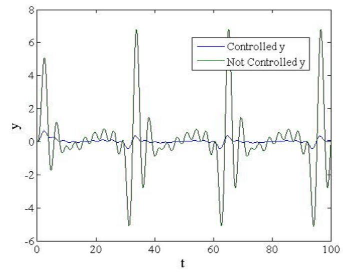


Fig. 13. Controlled $y$ $(y = x_1)$.

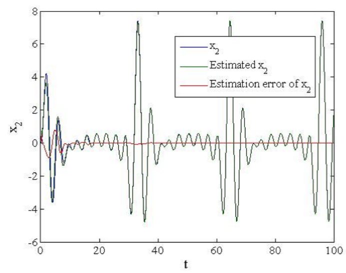


Fig. 9. Estimated $x_2$ by Projection Algorithm.

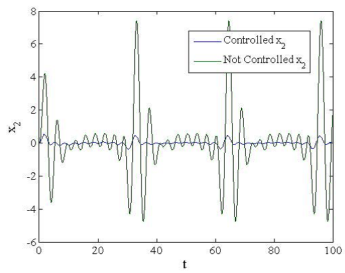


Fig. 14. Controlled $x_2$.

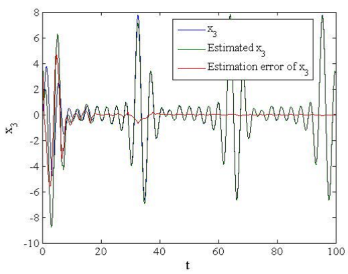


Fig. 10. Estimated $x_3$ by Projection Algorithm.

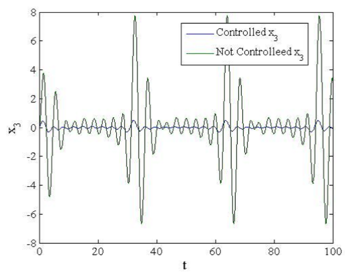


Fig. 15. Controlled $x_3$.

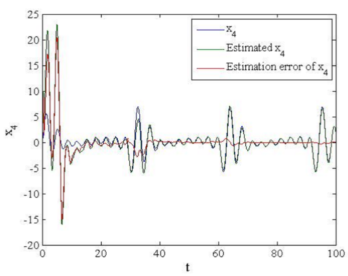


Fig. 11. Estimated $x_4$ by Projection Algorithm.

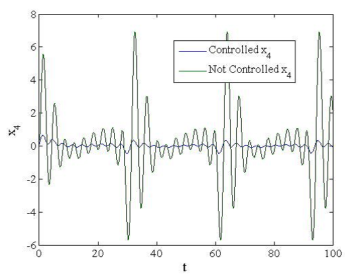


Fig. 16. Controlled $x_4$.