# Design of Adaptive I-PD Control System for 2-DOF Flexible Link Robot

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*Abstract*— **Proportional-Integral-Derivative (PID) controllers are the most employed controllers in industry. As one of the PID parameter tuning methods, partial model matching method is proposed by Kitamori. In this method, we can get the PID parameters calculated systematically. However, if we cannot get the enough information about controlled object in advance, the performance of sysytem may become bad. So, in this paper, we propose an adaptive I-PD control system and show the effectiveness by the experiment using 2-DOF Serial Flexible Link Robot.**

## I. INTRODUCTION

Although various control schemes based on modern control theory are studied, in the industrial world, the PID control based on classical control theory has still accounted for the 80 percent or more. Its reasons are, the structure of PID control is easy, it is connecting each operation called proportionality, integration, and differentiation to the concept of future, in the past, now, and it is mentioned that it is easy to understand intuitively.

During the past five decades, a comprehensive PID tuning literature has been developed. Roughly speaking, there are two different approaches to obtain PID and PID-like controller parameters.

First, tune the parameters of the PID structure by one of the following available tuning techniques: Ziegler-Nichol method[3], the CHR method[4], internal-model-control-based methodbib1, optimization method[6], and gain-phase margin method[7]. For single-input/single-output (SISO) plants, satisfactory control can be achieved by using established tuning rules.

Second, assume that the controller has a PID structure, and find the PID parameters by using some well-known optimization methods, e.g., $H_\infty$[8], mixed $H_2/H_\infty$[9], and semidefinite programming approaches[10]. These methods can be used to obtain the PID controller parameters such that the controllers have good time-domain performance and frequency-domain robustness. The main problem with this approach is that the resulting controllers are statespace controllers of high-order rather than low-order controllers with a fixed structure. Although one can reduce or approximate it with a PID-like structure, it is not so far the reduced-order controller.

Although there are many methods of a preparation of a typical PID parameter, at the actual operation spot, it is performed by an engineer's trial and error in many cases.

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There is a partial model matching method[11] proposed by Kitamori by making this PID parameter adjustment into the way of performing systematically. It is the characteristic of this technique to match sequentially from a low order term in the feedback system and reference model of a controlled object, and to interrupt matching for the suitable degree according to the complexity of the control device. Since matching is not necessarily completely performed to the highest term, it is called a partial model matching method. The purpose of the control design in this way is as follows. (i) Making steady-state error to zero. (ii) Having a suitable damping characteristic. (iii) After fulfilling the above-mentioned characteristic, rise time becomes the minimum. However, since this partial model matching method uses a fixed PID parameter, when the information on the controlled object obtained by beforehand has uncertainty, the above-mentioned control performance may be unable to demonstrate it.

So, in this paper, we propose an adaptive I-PD control system to overcome the model uncertainty. And we show the effectiveness of proposed adaptide tuning method of PID parameters by the experiment using 2-DOF Serial Flexible Link Robot.

## II. PROBLEM STATEMENT

Consider the SISO, linear time-invariant systems in Fig. 1 described by

$$y(t) = G(s)u(t) \tag{1}$$

where $u(t), y(t) \in \Re$ are the control input and the plant output respectively. Next, we choose the following reference model.

$$y_M(t) = G_M(s)r(t) \tag{2}$$

where $r(t), y_M(t) \in \Re$ are the reference input and the reference model output respectively. The control objective is to

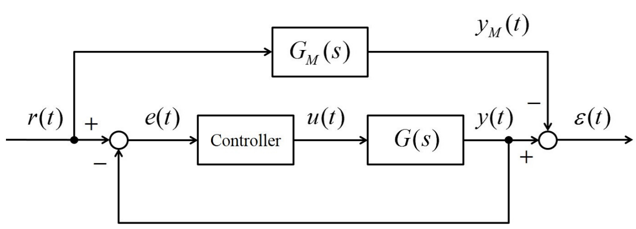


Fig. 1. Control system

design $u(t)$ such that asymptotically $y(t)$ tracks $y_M(t)$ with all generated signals remaining bounded. That means when we define the tracking error as

$$e(t) = r(t) - y(t) \tag{3}$$
$$\varepsilon(t) = y(t) - y_M(t) \tag{4}$$

the control objective is to make the tracking error to go to zero, In other words,

$$\lim_{t\to\infty} \varepsilon(t) = 0 \tag{5}$$

## III. Partial model matching method

### A. Denominator Series Expression

Although there are various methods in how to choose a reference model, the Kitamori's method[12] is adopted here. First, the mathematical model of a controlled object is denoted by denominator series expression.

$$\begin{aligned} G(s) &= \frac{b(s)}{a(s)} \\ &= \frac{b_0 + b_1 s + b_2 s^2 + b_3 s^3 + \cdots}{a_0 + a_1 s + a_2 s^2 + a_3 s^3 + \cdots} \\ &= \frac{1}{\beta_0 + \beta_1 s + \beta_2 s^2 + \beta_3 s^3 + \cdots} \\ &= \frac{1}{\beta(s)} \end{aligned} \tag{6}$$

where we can calculate each coefficient as:

$$\beta_0 = \frac{a_0}{b_0} \tag{7}$$
$$\beta_1 = \frac{a_1 - \beta_1}{b_0} \tag{8}$$
$$\beta_2 = \frac{a_2 - b_1\beta_1 - b_2\beta_0}{b_0} \tag{9}$$
$$\beta_3 = \frac{a_3 - b_1\beta_2 - b_2\beta_1 - b_3\beta_0}{b_0} \tag{10}$$
$$\vdots$$
$$\beta_i = \frac{a_i - b_1\beta_{i-1} - b_2\beta_{i-2} - \cdots - b_i\beta_0}{b_0} \tag{11}$$

Next, a reference model is similarly given by the model of denominator series expression.

$$\begin{aligned} G_M(s) &= \frac{1}{1 + \sigma s + \alpha_2(\sigma s)^2 + \alpha_3(\sigma s)^3 + \cdots} \\ &= \frac{1}{\alpha(s)} \end{aligned} \tag{12}$$

The $\sigma$ in the above model is a time-scaling parameter, at the same time, and a measure of response time because it is the first-order moment of the impulse response, that is, an average delay of the impulse response. The smaller the value of $\sigma$ is, the higher the response speed is. The value of $\sigma$ is left indeterminate in the model because the speed of designed system depends upon the speed of controlled object and ability of the compensator/controller used. The value is determined in the course of matching.

The $\alpha_i$'s are parameters to adjust the damping characteristics of designed system. Some recommendable sets of values for $\alpha_i$'s are known. A set is given as

$$\{\alpha_2, \alpha_3, \alpha_4, \cdots\} = \{0.5, 0.15, 0.03, \cdots\} \tag{13}$$

which gives rise to step responses of about 10 percent overshoot with good damping[12]. Some others are

$$\{\alpha_2, \alpha_3, \alpha_4, \cdots\} = \{0.425, 0.0975, 0.014344, \cdots\} \tag{14}$$
$$\{\alpha_2, \alpha_3, \alpha_4, \cdots\} = \{0.375, 0.0625, 0.003906, \cdots\} \tag{15}$$

The former, proposed by Shigemasa[13], gives rise to quicker step responses with negligi ble overshoot.The latter is the fourth order critical damping.

### B. Control Law

Let's consider the system expressed with the denominator expansion form in Fig. 2. The control objective is to design

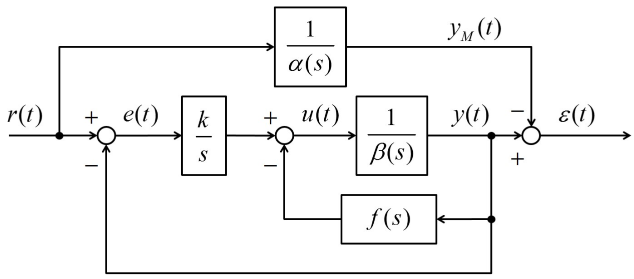


Fig. 2. I-PD type control system

$u(t)$ such that asymptotically $y(t)$ tracks $y_M(t)$ with all generated signals remaining bounded. We want to calculate the transfer function $W(s)$ from $r(t)$ to $y(t)$. From block diagram in Fig. 2,

$$y(t) = G(s)\left[\frac{k}{s}\{r(t) - y(t)\} - f(s)y(t)\right] \tag{16}$$

where

$$f(s) = f_0 + f_1 s + f_2 s^2 + f_3 s^3 + \cdots \tag{17}$$

So the closed-loop transfer function $W(s)$ is

$$W(s) = \frac{1}{1 + \frac{s}{k}\{\beta(s) + f(s)\}} \tag{18}$$

From (12) and (18), if the following equation is satisfied, then $W(s) = G_M(s)$.

$$\alpha(s) = 1 + \frac{s}{k}\{\beta(s) + f(s)\} \tag{19}$$

This model matching condition is an identical equation for $s$, and can be rewritten as:

$$\frac{\beta_0 + f_0}{k} = \sigma \tag{20}$$
$$\frac{\beta_1 + f_1}{k} = \alpha_2\sigma^2 \tag{21}$$
$$\frac{\beta_2 + f_2}{k} = \alpha_3\sigma^3 \tag{22}$$
$$\frac{\beta_3 + f_3}{k} = \alpha_4\sigma^4 \tag{23}$$
$$\vdots$$

And now we can obtain the error equation.

$$\varepsilon(t) = \frac{s}{k\alpha(s)}\left[u(t) - \theta^{\mathrm{T}}\zeta(t)\right] \tag{24}$$

where

$$\theta = [k, f_0, f_1, f_2, \cdots]^{\mathrm{T}} \tag{25}$$

$$\zeta(t) = \left[\frac{1}{s}e(t), -y(t), -\dot{y}(t), \cdots\right]^{\mathrm{T}} \tag{26}$$

Hence, the control law is the following $u(t)$ when the parameters of plant are known.

$$u(t) = \theta^{\mathrm{T}}(t)\zeta(t) \tag{27}$$

*1) I-P Control:* When we design I-P control system, let $f_1, f_2 = 0$ in eq(20)-(22),

$$\beta_0 + f_0 = k\sigma \tag{28}$$
$$\beta_1 = k\alpha_2\sigma^2 \tag{29}$$
$$\beta_2 = k\alpha_3\sigma^3 \tag{30}$$

From these equations, we can obtain the following control parameters.

$$\sigma = \frac{\alpha_2\beta_2}{\alpha_3\beta_1} \tag{31}$$
$$k = \frac{\beta_1}{\alpha_2\sigma^2} \tag{32}$$
$$f_0 = k\sigma - \beta_0 \tag{33}$$

*2) I-PD Control:* Next, when we design I-PD control system, let $f_2, f_3 = 0$ in eq(20)-(23),

$$\beta_0 + f_0 = k\sigma \tag{34}$$
$$\beta_1 + f_1 = k\alpha_2\sigma^2 \tag{35}$$
$$\beta_2 = k\alpha_3\sigma^3 \tag{36}$$
$$\beta_3 = k\alpha_4\sigma^4 \tag{37}$$

So, the control parameters are caluculated as follows.

$$\sigma = \frac{\alpha_3\beta_3}{\alpha_4\beta_2} \tag{38}$$
$$k = \frac{\beta_2}{\alpha_3\sigma^3} \tag{39}$$
$$f_0 = k\sigma - \beta_0 \tag{40}$$
$$f_1 = k\alpha_2\sigma^2 - \beta_1 \tag{41}$$

## IV. Adaptive I-PD Control Design

Figure 3 shows the adaptive control system. When the plant parameters are unknown, true parameters $\theta$ are replaced by the adjustable parameters $\hat{\theta}(t)$. Controller parameters in $\hat{\theta}(t)$ provided by the adaptation law.

$$u(t) = \hat{\theta}^{\mathrm{T}}(t)\zeta(t) \tag{42}$$
$$\varepsilon(t) = \frac{s}{k\alpha(s)}\left[\tilde{\theta}^{\mathrm{T}}(t)\zeta(t)\right] \tag{43}$$
$$\tilde{\theta}(t) = \hat{\theta}(t) - \theta \tag{44}$$

However, the choice of adaptation law given by

$$\dot{\hat{\theta}}(t) = -\Gamma\zeta(t)\varepsilon(t) \tag{45}$$
$$\tag{46}$$

cannot be used, because now the error transfer function $\frac{s}{\alpha(s)}$ is no longer strictly positive real. A famous technique called error augmentation can be used to avoid this difficulty in finding an adaptation law for this error model. The basic idea of the technique is to consider a so-called augmented error $\varepsilon'(t)$ which correlates to the parameter error in a more desirable way than the tracking error $\varepsilon(t)$.

First, let us define an auxiliary error $\eta(t)$ by

$$\eta(t) = \hat{\theta}^{\mathrm{T}}(t)\left[\frac{s}{\alpha(s)}\zeta(t)\right] - \frac{s}{\alpha(s)}\left(\hat{\theta}^{\mathrm{T}}(t)\zeta(t)\right) \tag{47}$$

It is useful to note two features about the auxiliary error. First, this error $\eta(t)$ can be computed on-line, since the estimated parameter vector $\hat{\theta}(t)$ and the signal vector $\zeta(t)$ are both available on-line manner. Secondly, this error $\eta(t)$ is caused by time-varying nature of the estimated parameters $\hat{\theta}(t)$, in the sense that when the estimated parameters $\hat{\theta}(t)$ is replaced by the constant parameter vector, then we have $\eta(t) = 0$.

This also implies that the auxiliary error can be written

$$\eta(t) = \tilde{\theta}^{\mathrm{T}}(t)\left[\frac{s}{\alpha(s)}\zeta(t)\right] - \frac{s}{\alpha(s)}\left(\tilde{\theta}^{\mathrm{T}}(t)\zeta(t)\right) \tag{48}$$

Now let us define an augmented error $\varepsilon'(t)$, by combining the tracking error $\varepsilon(t)$ with the auxiliary error $\eta(t)$ as

$$\varepsilon'(t) = \varepsilon(t) + \hat{h}(t)\eta(t) \tag{49}$$
$$h = \frac{1}{k} \tag{50}$$

where $\hat{h}(t)$ is a time-varying parameter to be determined by adaptation. Note that $\hat{h}(t)$ is not a controller parameter, but only a parameter used in forming the new error $\varepsilon'(t)$.

$$\begin{aligned}
\varepsilon'(t) &= \varepsilon(t) + \hat{h}(t)\eta(t) \\
&= \frac{sh}{\alpha(s)}\left[\tilde{\theta}^{\mathrm{T}}(t)\zeta(t)\right] + \hat{h}(t)\eta(t) + h\eta(t) - h\eta(t) \\
&= \frac{sh}{\alpha(s)}\left[\tilde{\theta}^{\mathrm{T}}(t)\zeta(t)\right] + \hat{h}(t)\eta(t) \\
&\quad + h\left(\tilde{\theta}^{\mathrm{T}}(t)\left[\frac{s}{\alpha(s)}\zeta(t)\right] - \frac{s}{\alpha(s)}\left(\tilde{\theta}^{\mathrm{T}}(t)\zeta(t)\right)\right) \\
&\quad - h\eta(t) \\
&= h\tilde{\theta}^{\mathrm{T}}(t)\left[\frac{s}{\alpha(s)}\zeta(t)\right] + \tilde{h}(t)\eta(t)
\end{aligned} \tag{51}$$

We obtain

$$\varepsilon'(t) = h\tilde{\theta}^{\mathrm{T}}(t)\xi(t) + \tilde{h}(t)\eta(t) \tag{52}$$
$$\xi(t) = \frac{s}{\alpha(s)}\zeta(t) \tag{53}$$

This implies that the augmented error can be linearly parameterized by the parameter errors $\tilde{\theta}(t)$ and $\tilde{h}(t)$. Then a number of standard techniques such that the gradient method or the least-squares method can be used to update the parameters.

Using the gradient method with normalization, the controller parameters $\hat{\theta}(t)$ and the parameter $\hat{h}(t)$ for forming the augmented error are updated by

$$\dot{\hat{\theta}}(t) = -\frac{\Gamma\xi(t)\varepsilon'(t)}{\rho+\xi^{\mathrm{T}}(t)\xi(t)} \tag{54}$$

$$\dot{\hat{h}}(t) = -\frac{\gamma\eta(t)\varepsilon'(t)}{\rho+\xi^{\mathrm{T}}(t)\xi(t)} \tag{55}$$

where $\Gamma=\Gamma^{\mathrm{T}}>0$, $\gamma>0$ are adaptive gains, and $\rho$ is positive number.

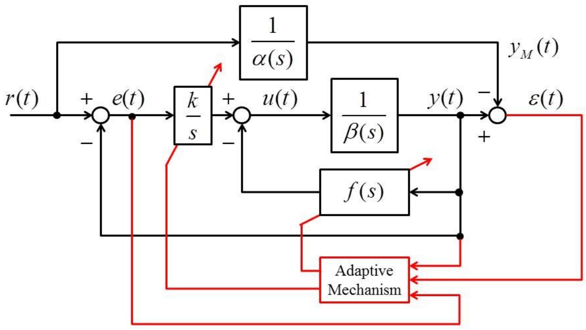


Fig. 3. Model reference adaptive control

## V. Stability Analysis

Consider the function

$$V = \frac{1}{2}h\tilde{\theta}^{\mathrm{T}}(t)\Gamma^{-1}\tilde{\theta}(t)+\frac{1}{2\gamma}\tilde{h}^2(t) \tag{56}$$

as a Lyapunov candidate for the system. The time derivative $\dot{V}$ is calculated as

$$\begin{aligned} \dot{V} &= h\tilde{\theta}^{\mathrm{T}}(t)\Gamma^{-1}\dot{\tilde{\theta}}(t)+\frac{1}{\gamma}\tilde{h}(t)\dot{\tilde{h}}(t) \\ &= -h\tilde{\theta}^{\mathrm{T}}(t)\frac{\xi(t)\varepsilon'(t)}{\rho+\xi^{\mathrm{T}}(t)\xi(t)}-\tilde{h}(t)\frac{\eta(t)\varepsilon'(t)}{\rho+\xi^{\mathrm{T}}(t)\xi(t)} \\ &= -\frac{h\tilde{\theta}^{\mathrm{T}}(t)\xi(t)+\tilde{h}(t)\eta(t)}{\rho+\xi^{\mathrm{T}}(t)\xi(t)}\varepsilon'(t) \\ &= -\frac{\varepsilon'^2(t)}{\rho+\xi^{\mathrm{T}}(t)\xi(t)} \end{aligned} \tag{57}$$

Therefore, estimated error $\tilde{\theta}(t)$ and $\tilde{h}(t)$ are bounded.

## VI. Experiment of 2-DOF Flexible Link Robot

### A. 2-DOF Serial Flexible Link Robot

Controlled object is 2-DOF Serial Flexible Link Robot in fig.4. We define the first (shoulder) driving shaft absolute angular position as $\phi_1$, the second (elbow) driving shaft angular position relative to link 1 as $\phi_2$, the first flexible link relative end-effector angular position as $q_1$, and the second flexible link end-effector angular position relative to link 1 as $q_2$ like in fig.5. At this time, the linear decoupled model for every link is as follows. First, the state vector about the 1st link is defined as

$$x_1^{\mathrm{T}} = \left[\phi_1(t), q_1(t), \frac{d}{dt}\phi_1(t), \frac{d}{dt}q_1(t)\right] \tag{58}$$

Fig. 4. 2-DOF Serial Flexible Link Robot

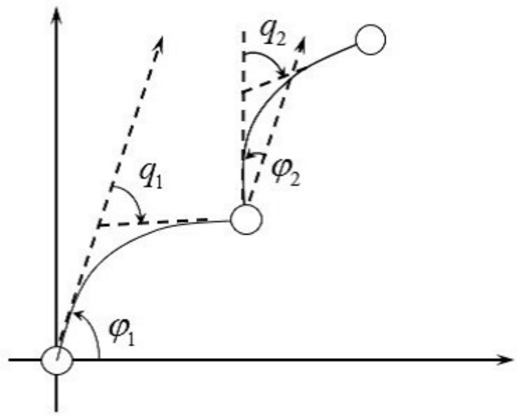


Fig. 5. Definition of $\phi_1, \phi_2, q_1, q_2$

Since an input $u_1$ is current to the motor of 1st link,

$$u_1 = I_1 \tag{59}$$

At this time, the state equation is described as

$$\frac{d}{dt}x_1 = A_1x_1+B_1u_1 \tag{60}$$

$$y_1 = C_1^{\mathrm{T}}x_1 \tag{61}$$

where

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{K_{s1}}{J_{11}} & -\frac{B_{11}}{J_{11}} & \frac{B_{12}}{J_{11}} \\ 0 & -\frac{(J_{11}+J_{12})K_{s1}}{J_{11}J_{12}} & \frac{B_{11}}{J_{11}} & -\frac{B_{12}(J_{11}+J_{12})}{J_{11}J_{12}} \end{bmatrix}, \tag{62}$$

$$B_1^{\mathrm{T}} = \left[0, 0, \frac{K_{t1}}{J_{11}}, -\frac{K_{t1}}{J_{11}}\right], \tag{63}$$

and

$$C_1^{\mathrm{T}} = [1, 0, 0, 0]. \tag{64}$$

Also, the state vector about the 2nd link is defined as

$$x_2^{\mathrm{T}} = \left[\phi_2(t), q_2(t), \frac{d}{dt}\phi_2(t), \frac{d}{dt}q_2(t)\right] \tag{65}$$

Since an input $u_2$ is current to the motor of 2nd link,

$$u_2 = I_2 \tag{66}$$

At this time, the state equation is described as

$$\frac{d}{dt}x_2 = A_2 x_2 + B_2 u_2 \tag{67}$$

$$y_2 = C_2^{\mathrm{T}} x_2 \tag{68}$$

where

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{K_{s2}}{J_{21}} & -\frac{B_{21}}{J_{21}} & \frac{B_{22}}{J_{21}} \\ 0 & -\frac{(J_{21}+J_{22})K_{s2}}{J_{21}J_{22}} & \frac{B_{21}}{J_{21}} & -\frac{B_{22}(J_{21}+J_{22})}{J_{21}J_{22}} \end{bmatrix}, \tag{69}$$

$$B_2^{\mathrm{T}} = \left[0, 0, \frac{K_{t2}}{J_{21}}, -\frac{K_{t2}}{J_{21}}\right], \tag{70}$$

and

$$C_2^{\mathrm{T}} = [1, 0, 0, 0]. \tag{71}$$

### B. Experimental result

Since each link was decoupled respectively, we consider the controlled object as single-input/single-output systems. The PID parameter was set as $\hat{\theta}_1(0) = \hat{\theta}_2(0) = [110.1]^{\mathrm{T}}$ at the beginning noting that the parameter of the controlled object was unknown. From the second time on, the final value of last experiment is used as the initial value of adaptive parameters. The result at the time of giving a step signal as a reference input is shown Fig. 6-9. Here, control parameters are chosen as $\Gamma = I, \gamma = 1, \rho = 0.1$.

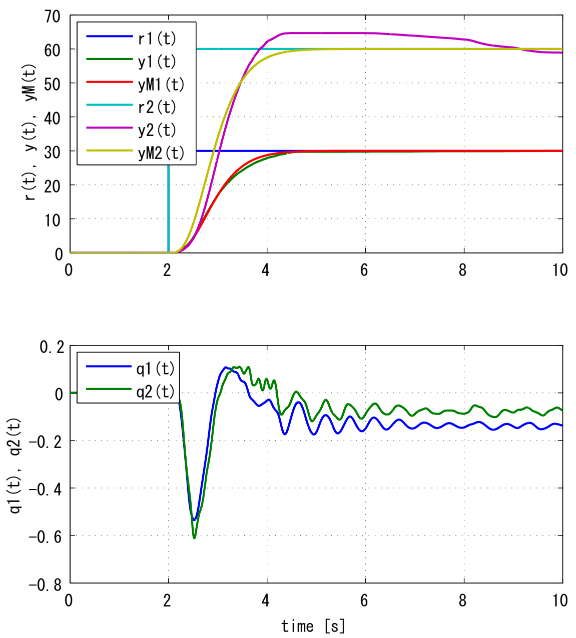


Fig. 6. Control result of 1st trial

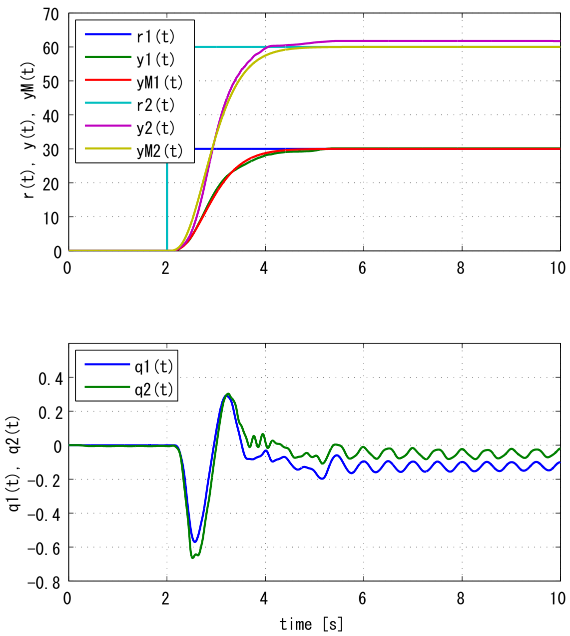


Fig. 7. Control result of 2nd trial

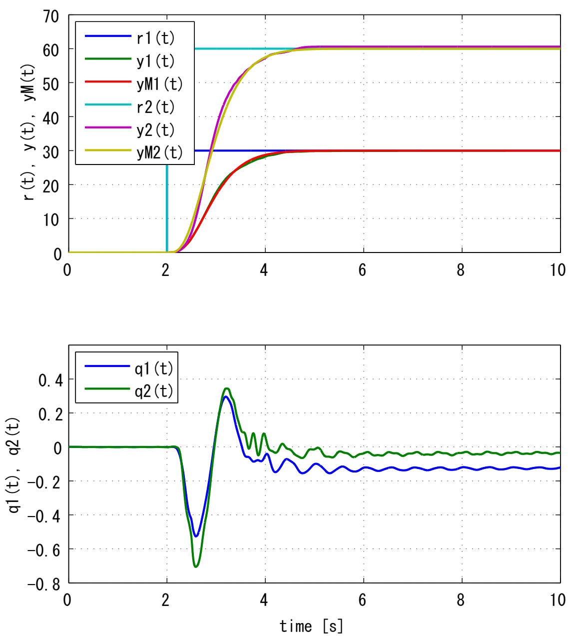


Fig. 8. Control result of 3rd trial

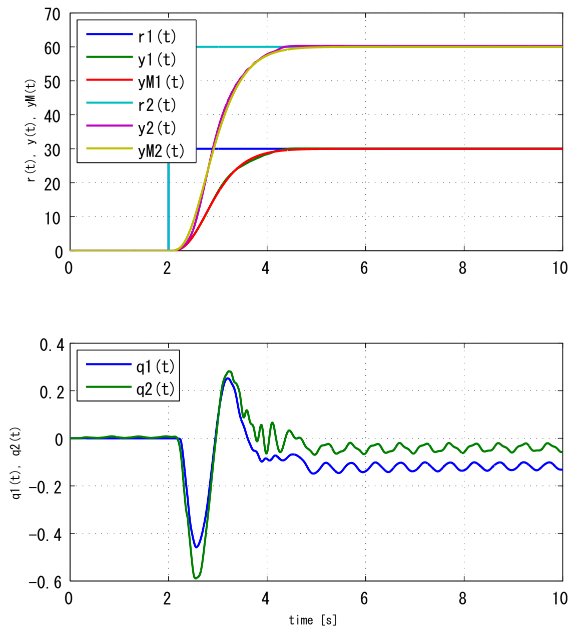


Fig. 9. Control result of 4th trial

From the experiment result, the PID gains $\hat{\theta}_1(t), \hat{\theta}_2(t)$ are gradually tuned, and finally, good tracking performance is achieved.

## VII. Conclusion

In this paper, we propose an adaptive I-PD control system to overcome the model uncertainty. And we show the effectiveness of proposed adaptide tuning method of PID parameters by the experiment using 2-DOF Serial Flexible Link Robot.

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