# Iterative Learning Control for Batch-varying References

Se-Kyu Oh, Jung Hun Kim, En Sup Yoon, Chang Jun Lee and Jong Min Lee*

***Abstract*— This paper presents iterative learning control (ILC) schemes for batch-varying references. Generally, reference or target trajectory must be identical for all iterations to implement the ILC. However, references can be changed in dynamic systems such as robotics and chemical processes according to cycle or batch. ILC schemes for batch-varying references are proposed in three forms which are inverse of model-based ILC (I-ILC), quadratic-criterion-based ILC (Q-ILC), and general norm optimal ILC form. These control schemes are studied for discrete linear time invariant (LTI) system. A numerical example is provided to demonstrate the performance of the proposed algorithms.**

## I. INTRODUCTION

Iterative learning control (ILC) is an effective control technique for improving tracking performance of batch process under model uncertainty. ILC was originally introduced in 1984 by Arimoto et al. for robot manipulators [1], then ILC has been implemented in many industrial processes such as semiconductor manufacturing [2] and chemical processes [3]. All the previous studies on ILC have only considered the batch-process with identical reference trajectories for all iterations.

In this paper, we propose ILC schemes for batch-varying references and present three modified ILC forms for batch-varying references case: inverse of model-based ILC, quadratic-criterion-based ILC, and general norm optimal ILC. The most important issue in the proposed ILC schemes is to estimate the precise model. We cannot guarantee a perfect convergence without a perfect model. The difference between the current desired input and next desired input with respect to each different reference trajectory should be required. For the issue, numerical algorithms for state-space subspace system identification (N4SID) is used at the end of each iteration to estimate a precise model.

The remainder of the paper is organized as follows. In Section II, lifted system representation and a brief description of the general ILC schemes are introduced. In Section III, ILC schemes for batch-varying references are presented and the reason why a perfect model is needed for convergence is also discussed. Finally, numerical illustration is provided in Section IV.

Se-Kyu Oh is with School of Chemical Engineering, Institute of Chemical Processes, Seoul National University, 1 Gwanak-ro, Gwanak-gu, Seoul, 151-744, Korea sekyu@snu.ac.kr

Jung Hun Kim is with School of Chemical Engineering, Institute of Chemical Processes, Seoul National University, 1 Gwanak-ro, Gwanak-gu, Seoul, 151-744, Korea zeus4997@snu.ac.kr

En Sup Yoon is with School of Chemical Engineering, Institute of Chemical Processes, Seoul National University, 1 Gwanak-ro, Gwanak-gu, Seoul, 151-744, Korea esyoon@pslab.snu.ac.kr

Chang Jun Lee is with Department of Safety Engineering, Pukyong National University, 100 Yongdang-dong, Nam-gu, Busan 608-739, Korea changjunlee@pknu.ac.kr

*Jong Min Lee is with School of Chemical Engineering, Institute of Chemical Processes, Seoul National University, 1 Gwanak-ro, Gwanak-gu, Seoul, 151-744, Korea jongmin@snu.ac.kr

## II. PRELIMINARY

Consider the following discrete linear time-invariant system which operates on an interval $t \in [0,N]$:

$$\begin{aligned} x_k(t+1) &= Ax_k(t)+Bu_k(t) \\ y_k(t) &= Cx_k(t) \end{aligned} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is a state vector; $u_k \in \mathbb{R}^m$ is an input vector; $y_k \in \mathbb{R}^p$ is an output vector; $t$ is a time index; $k$ is a batch index; and the matrices $A, B$, and $C$ are real matrices of appropriate dimensions and assumed to be time-invariant. Since finite time intervals $[0,N]$ are considered in ILC, this system can be rewritten as a lifted system:

$$y_k = G_p u_k \tag{2}$$

with $x_k(0) = 0$ and the plant matrix $G_p = \mathbb{R}^{(pN)\times(mN)}$ defined as

$$G_p = \begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{N-1}B & CA^{N-2}B & \cdots & CB \end{bmatrix} \tag{3}$$

and the vectors $y_k, y_0 \in \mathbb{R}^{pN}$, and $u_k \in \mathbb{R}^{mN}$ are defined as

$$y_k = \begin{bmatrix} y_k^T(1) & y_k^T(2) & \cdots & y_k^T(N) \end{bmatrix}^T \tag{4}$$

$$u_k = \begin{bmatrix} u_k^T(0) & u_k^T(1) & \cdots & u_k^T(N-1) \end{bmatrix}^T \tag{5}$$

The system matrix $G_p$ is a markov matrix which is a lower triangular Toeplitz matrix [4].

### A. *Inverse of Model-based ILC (I-ILC)*

The most general input update law of the ILC is represented by

$$u_{k+1} = u_k + He_k \tag{6}$$

where $H$ is a learning gain matrix, $e_k = r - y_k$, and $r$ is a reference trajectory [5].

**Theorem 1** *[6] Consider the linear system (2) and the ILC controller (6). Then, the ILC system is monotonic convergent if $H$ is chosen such that $\|I - G_pH\| < 1$.*

*Proof:* The error at the $(k+1)$-th batch is derived as

$$\begin{aligned} r - y_{k+1} &= r - G_p u_{k+1} \\ &= r - G_p(u_k + H(r-y_k)) \\ &= (I - G_pH)(r-y_k) \end{aligned} \tag{7}$$

Then, it leads to the inequality

$$\|e_{k+1}\| \leq \|I - G_p H\| \|e_k\| \tag{8}$$

Consequently, the monotonic convergence of error is guaranteed if $\|I - G_p H\| < 1$. ■

In this case, inverse of model matrix $G$ is generally chosen as $H$, that is, $H = G^{-1}$. However, the learning gain matrix $H$ based on the model inverse $G^{-1}$ becomes high-sensitive to high-frequency components in $e_k$. Therefore, excessive input change can occur.

### B. Quadratic-criterion-based ILC (Q-ILC)

Q-ILC [7], [8] was proposed to use the following objective which has a penalty term on the input change between two adjacent batches:

$$\min_{\Delta u_{k+1}} J = \frac{1}{2}\{e_{k+1}^T Q e_{k+1} + \Delta u_{k+1}^T R \Delta u_{k+1}\} \tag{9}$$

where $Q$ and $R$ are positive-definite matrices and $\Delta u_{k+1} = u_{k+1} - u_k$. By calculating the derivative of $J$ with respect to $\Delta u_{k+1}$, we obtain the following input update law of the Q-ILC:

$$u_{k+1} = u_k + H_Q e_k \tag{10}$$

where

$$H_Q = (G^T Q G + R)^{-1} G^T Q \tag{11}$$

Although the control law (10) is derived using the plant matrix $G_p$, it is applied using the model matrix $G$ because we do not know the precise plant matrix.

**Theorem 2** *[7] Consider the linear system (2) and the Q-ILC controller (10). Then, the system is monotonic convergent if G is chosen such that* $\|I - G_p H_Q\| < 1$.

*Proof:* The input-output relationship between $k$-th and $k+1$-th batch is written as

$$y_{k+1} = y_k + G_p(u_{k+1} - u_k) \tag{12}$$

Then, the following expression can be derived:

$$e_{k+1} = e_k - G_p \Delta u_{k+1} \tag{13}$$

By substituting eq. (10) for $\Delta u_{k+1}$ in eq. (13), we can obtain the following expression:

$$e_{k+1} = (I - G_p(G^T Q G + R)^{-1} G^T Q) e_k \tag{14}$$

Then, it leads to the inequality

$$\|e_{k+1}\| \leq \|(I - G_p(G^T Q G + R)^{-1} G^T Q)\| \|e_k\| \tag{15}$$

Therefore, the monotonic convergence of error is guaranteed if $\|I - G_p H_Q\| < 1$. ■

## III. ITERATIVE LEARNING CONTROL FOR BATCH-VARYING REFERENCES

In the conventional ILC formulation, the output $y_k$ converges to the reference $r$ for all batches. Hence, it is possible to make the output converge as long as we know the values of the error and the model which satisfies the convergence condition. If the reference trajectories are varied in batches, we should know not only the values of the error but also the input variation necessary to move the output from the current reference $r_k$ to the next reference $r_{k+1}$. The desired input of $(k+1)$-th batch can be expressed as the following form:

$$u_{k+1} = u_k + (u_{r(k)} - u_k) + (u_{r(k+1)} - u_{r(k)}) \tag{16}$$

where $u_{r(k)}$ is the desired input for current reference $r_k$ and $u_{r(k+1)}$ is the desired input for next reference $r_{k+1}$. With the plant description of $y_k = G_P u_k$, eq. (16) can be rewritten as:

$$u_{k+1} = u_k + \underbrace{G_p^{-1}(r_k - y_k)}_{convergence\ term} + \underbrace{G_p^{-1}(r_{k+1} - r_k)}_{reference\ tracking\ term} \tag{17}$$

In the ILC problem, it is assumed that the plant matrix $G_p$ is not known exactly or $G_p$ is not invertible. Hence, we introduce learning gain matrices to obtain input update law of the ILC for batch-varying references:

$$u_{k+1} = u_k + H_c(r_k - y_k) + H_r(r_{k+1} - r_k) \tag{18}$$

where $H_c$ is the learning gain matrix for convergence and $H_r$ is the learning gain matrix for reference tracking.

### A. I-ILC for Batch-varying References

For using the ILC algorithm for batch-varying references, we should choose two learning gain matrices, $H_c$ and $H_r$.

**Theorem 3** *Consider the linear system (2) and the ILC controller in (18). Then, in the ILC system,* $e_k \to 0$ *as* $k \to \infty$ *if* $H_c$ *is chosen such that* $\|I - H_r^{-1} H_c\| < 1$ *and* $H_r$ *is chosen such that* $\|H_r^{-1} G_p^{-1} - I\| = 0$.

*Proof:* Pre-multiplying the input update law (18) by the plant matrix $G_p$ yields

$$G_p u_{k+1} = G_p u_k + G_p H_c(r_k - y_k) + G_p H_r(r_{k+1} - r_k) \tag{19}$$

then, the following can be derived by using $y_k = G_p u_k$.

$$y_{k+1} = y_k + G_p H_c(r_k - y_k) + G_p H_r(r_{k+1} - r_k) \tag{20}$$

Adding $G_p H_r y_{k+1} + G_p H_r y_k$ to eq. (20) yields

$$\begin{aligned} y_{k+1} + G_p H_r y_{k+1} + G_p H_r y_k = y_k + G_p H_c(r_k - y_k) \\ + G_p H_r(r_{k+1} - r_k) + G_p H_r y_{k+1} + G_p H_r y_k \end{aligned} \tag{21}$$

then, this equation can be rearranged to:

$$G_p H_r e_{k+1} = (G_p H_r - G_p H_c) e_k + (I - G_p H_r)(y_{k+1} - y_k) \tag{22}$$

From this, it follows that

$$e_{k+1} = (I - H_r^{-1} H_c) e_k + (H_r^{-1} G_p^{-1} - I) \Delta y_{k+1} \tag{23}$$

where $\Delta y_{k+1} = y_{k+1} - y_k$. Then, it leads to the inequality

$$\begin{aligned}\|e_{k+1}\| \leq &\|I - H_r^{-1}H_c\|\|e_k\| \\ &+ \|H_r^{-1}G_p^{-1} - I\|\|\Delta y_{k+1}\|\end{aligned} \tag{24}$$

In this case, $\Delta y_{k+1}$ cannot be 0 since the references can vary for all batches. Hence, $H_r^{-1}$ should equal to $G_p$ so that $\|H_r^{-1}G_p^{-1} - I\|\|\Delta y_{k+1}\| = 0$. Then, the convergence condition follows that

$$\|e_{k+1}\| \leq \|I - G_p H_c\|\|e_k\| \tag{25}$$

Consequently, the error $e_k \to 0$ as $k \to \infty$ if $\|I - G_p H_c\| \leq 0$ and $H_r^{-1} = G_p$. ■

### *B. Q-ILC for Batch-varying References*

In many control applications, smooth control are preferable for stable operation. Therefore, we need to obtain control law of the Q-ILC form, which has the quadratic objective function involving both regulation error and input change. The Q-ILC is the special case of norm optimal ILC. First, we need to derive $e_{k+1}$ to use the following objective:

$$\min_{\Delta u_{k+1}} J = \frac{1}{2}\{e_{k+1}^T Q e_{k+1} + \Delta u_{k+1}^T R \Delta u_{k+1}\} \tag{26}$$

The input-output relationship between two adjacent batches is

$$y_{k+1} = y_k + G_p \Delta u_{k+1} \tag{27}$$

then, adding $(r_{k+1} + r_k)$ to eq. (27), the following error dynamics can be obtained.

$$e_{k+1} = e_k - G_p \Delta u_{k+1} + r_{k+1} - r_k \tag{28}$$

substituting eq. (28) for $e_{k+1}$ in eq. (26) and applying $\partial J/\partial \Delta u_{k+1} = 0$ yield

$$\begin{aligned}u_{k+1} =& u_k + (G_p^T Q G_p + R)^{-1} G_p^T Q e_k \\ &+ (G_p^T Q G_p + R)^{-1} G_p^T Q \Delta r_{k+1}\end{aligned} \tag{29}$$

where $\Delta r_{k+1} = r_{k+1} - r_k$. Since the precise plant model is hardly known, we use $G$ instead of $G_p$. Then, we have the following input update law of the Q-ILC for batch-varying references.

$$\begin{aligned}u_{k+1} =& u_k + \underbrace{(G_c^T Q G_c + R)^{-1} G_c^T Q e_k}_{convergence\ term} \\ &+ \underbrace{(G_r^T Q G_r + R)^{-1} G_r^T Q \Delta r_{k+1}}_{reference\ tracking\ term}\end{aligned} \tag{30}$$

Even if we formulate the input update law (30) without any assumption about a basic form of control law, eq. (30) is the same form as eq. (18) which was the input update law first introduced. The input update law can be simply expressed in the following form:

$$u_{k+1} = u_k + H_{Q_c} e_k + H_{Q_r} \Delta r_{k+1} \tag{31}$$

**Theorem 4** *Consider the linear system (2) and the Q-ILC controller (31). Then, in the system, $e_k \to 0$ as $k \to \infty$ if $H_{Q_c}$ is chosen such that $\|I - G_p H_{Q_c}\| \leq 1$ and $H_{Q_r}$ is chosen such that $\|I - G_p H_{Q_r}\| = 0$.*

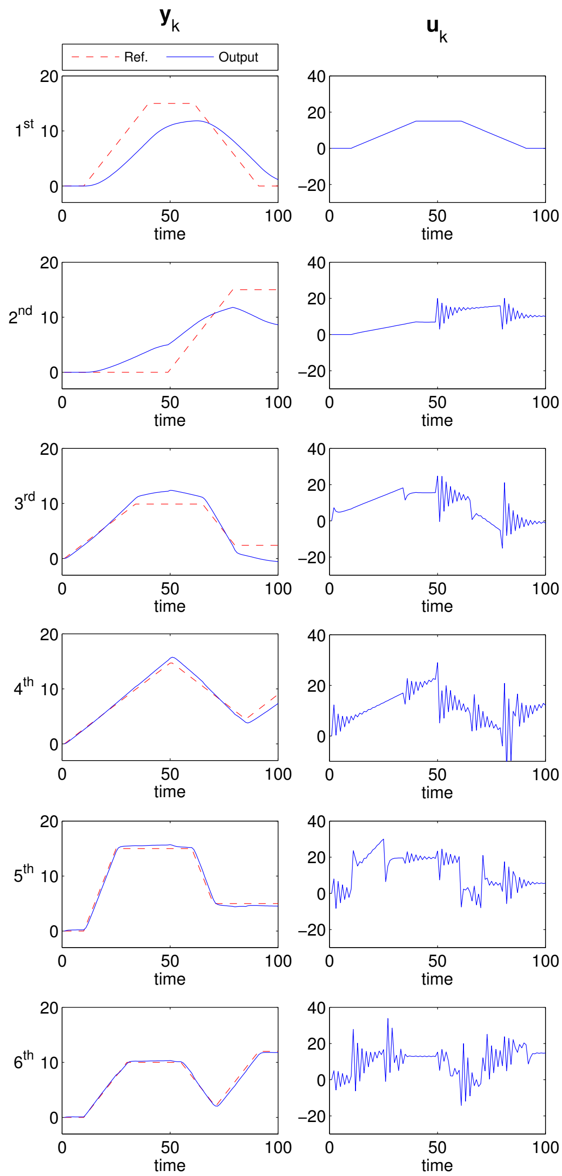



Fig. 1. Simulation results of I-ILC for batch-varying references.

*Proof:* The following expression can be obtained by using eqs. (27), (28) and (29).

$$e_{k+1} = (I - G_p H_{Q_c}) e_k + (I - G_p H_{Q_r}) \Delta r_{k+1} \tag{32}$$

It leads to the inequality

$$\|e_{k+1}\| \leq \|I - G_p H_{Q_c}\|\|e_k\| + \|I - G_p H_{Q_r}\|\|\Delta r_{k+1}\| \tag{33}$$

Hence, the error $e_k \to 0$ as $k \to \infty$ if $\|(I - G_p H_{Q_c})\| \leq 1$ and $H_{Q_r}^{-1} = G_p$ ■

### *C. General norm optimal ILC for batch-varying references*

General norm optimal ILC can also minimize control effort by adding penalty term on the input using the following

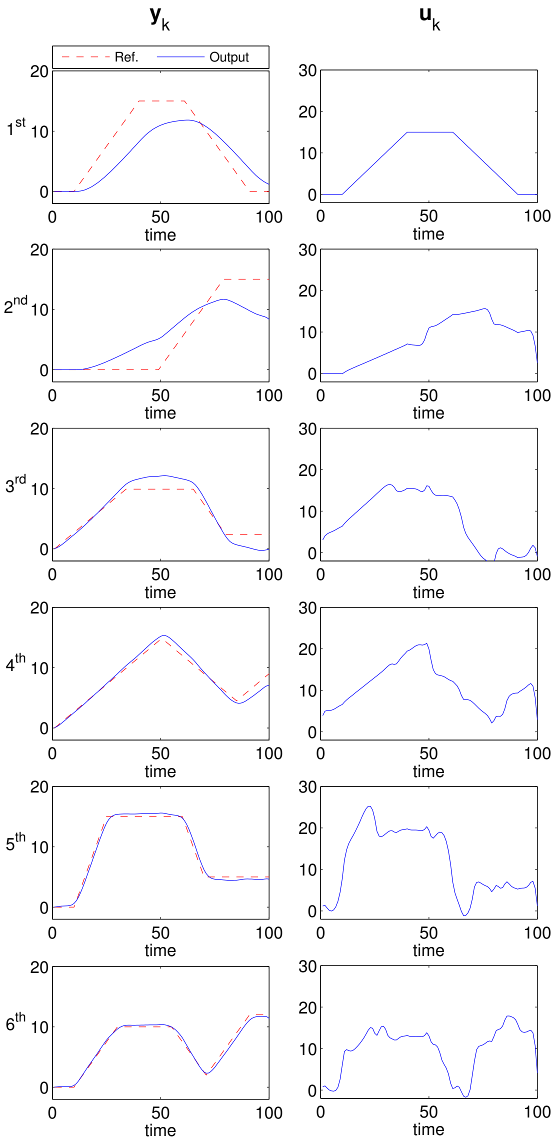



Fig. 2. Simulation results of Q-ILC for batch-varying references.

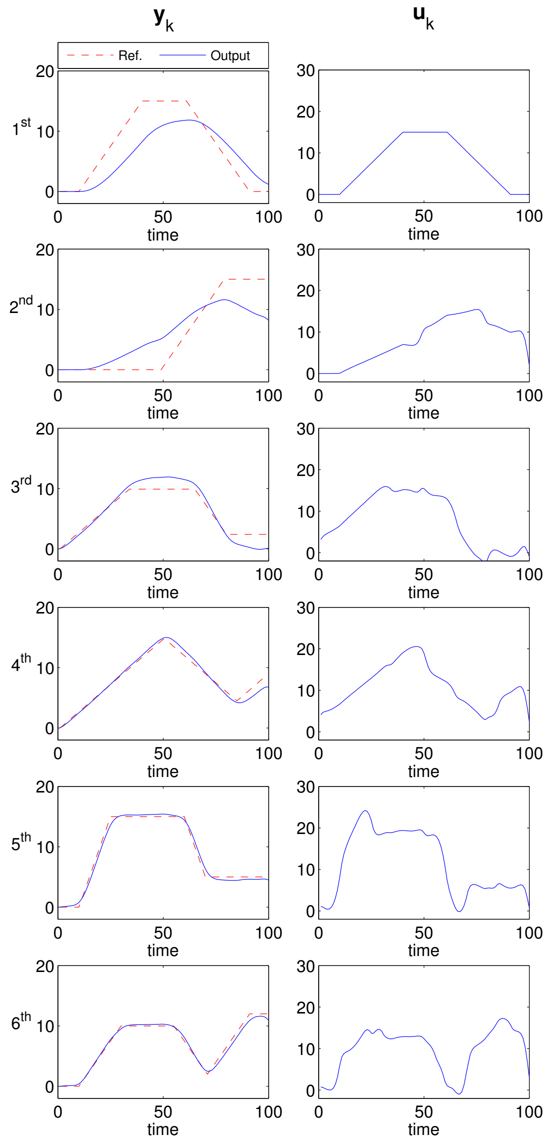



Fig. 3. Simulation results of general norm optimal ILC for batch-varying references.

objective function [9].

$$\min_{u_{k+1}} J = \frac{1}{2}\{e_{k+1}^T Q e_{k+1} + \Delta u_{k+1}^T R \Delta u_{k+1} + u_{k+1}^T S u_{k+1}\} \quad (34)$$

Substituting eq. (28) for $e_{k+1}$ in eq. (34) and applying $\partial J / \partial u_{k+1} = 0$ yield

$$\begin{aligned} u_{k+1} = & (G_c^T Q G_c + R + S)^{-1}(G_c^T Q G_c + R) u_k \\ & + (G_c^T Q G_c + R + S)^{-1} G_c^T Q e_k \\ & + (G_r^T Q G_r + R + S)^{-1} G_r^T Q \Delta r_{k+1} \end{aligned} \quad (35)$$

This input update law can be simply expressed in the following form:

$$u_{k+1} = H_{N_u} u_k + H_{N_e} e_k + H_{N_r} \Delta r_{k+1} \quad (36)$$

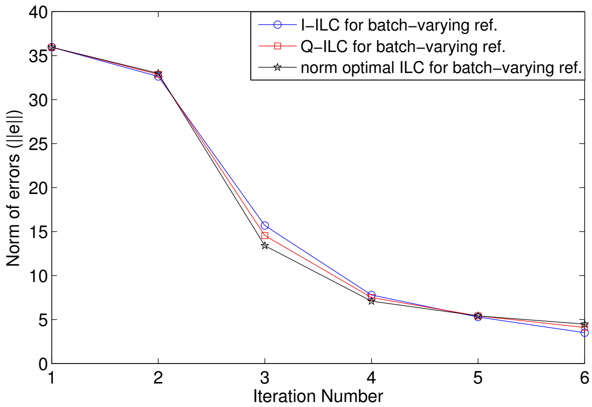
## IV. NUMERICAL ILLUSTRATION

For illustrating performance of the proposed algorithms, the true process $G_p$ and the nominal model $G$ are employed as follows.

$$G_p(s) = \frac{0.8}{(6s+1)(4s+1)} \quad (37)$$

$$G(s) = \frac{1.5}{(8s+1)(2s+1)} \quad (38)$$

Note that there are considerable model errors in the steady state gain as well as in the dynamic gain. We use $r_1$ for the first input signal $u_1$, that is, $u_1 = r_1$. For applying the proposed ILC, we should estimate the true process $G_p$ for reference tracking term update in eqs. (18) and (30). In this study, we use the numerical algorithms for state



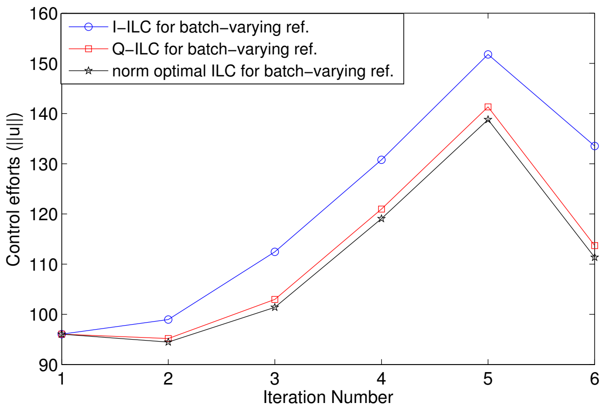




Fig. 4. Errors and control efforts of each ILC input update law.

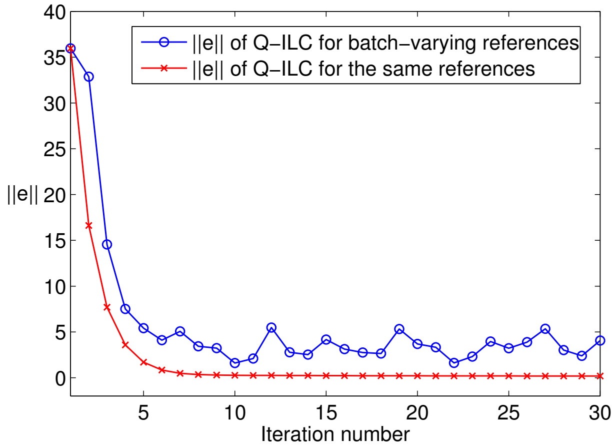



Fig. 5. Errors of Q-ILC for the same references and Q-ILC for batch-varying references.

space subspace system identification (N4SID) [10]. At the end of each iteration, state-space model is estimated by the using input-output data of previous iteration. Then, this model is used for reference tracking term. Fig. 1 shows the performance of I-ILC for batch-varying references. While the outputs converge to each reference, the input signals show severe oscillations and spikes. In Fig. 2, the performance of Q-ILC for batch-varying references is presented. In this case, we used $Q = I$ and $R = 0.01I$. The input signals are smooth even if the result shows the similar performance. Fig. 3 shows the results of norm optimal ILC. In this case, we used $Q = I$ and $R = S = 0.01I$. Owing to the penalty terms, the control efforts decreased as shown in Fig. 4. The proposed ILC method has a limitation of convergence as shown in Fig. 5 because identification method cannot estimate the plant precisely. Therefore, the performance of the proposed ILC depends on the accuracy of the estimated model.

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