

Self-repairing nonlinear control and its application to chemical reactor

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Abstract—Against sensor failures, this paper presents a new design method for a self-repairing nonlinear control system (SRNCS). The proposed system can automatically replace the failed sensor with healthy backup if the sensor failure is found. The nonlinear detection filter guarantees exact and early fault detection, and the failure can be detected within a finite time prescribed by a design parameter of the filter. Also, in this paper, the proposed SRNCS is applied to a well-known continuous stirred-tank reactor, and the effectiveness is confirmed through several numerical simulations.

I. INTRODUCTION

Sensor failure often causes serious damage to system stability and control performance. In order to recover from failure fundamentally, control systems ought to find failures and replace the failed sensors with the healthy ones. Hence, such a self-repairing function should be provided against failures in advance.

To realize self-repairing, fault detectors are needed. A large number of design methods for fault detectors have been developed. From the viewpoint of reliability, the deterministic approaches to fault detection might be preferred to ensure exact fault detection [1], [2], [3]. However, in many of them, mathematical models of plants are often exploited to design the detection filters. This unfortunately, raises the problem of the complexities of the constructed detection filters. In addition, if there is a slight mismatch between actual and estimated models, then exact fault detection cannot be guaranteed theoretically.

In the framework of the self-repairing control, our previous works have developed a new design concept of the detection filter against a stuck-type sensor failures [4]. The detection filter has a finite escape time in only faulty situation. Hence, just monitoring the filtered signal makes it possible to find the failures exactly. Furthermore, by choosing the finite escape time, the detection time can be shortened arbitrarily. Also, because the detection filter is of the first order, the structure of the overall fault-tolerant control system does not depend on the plant order but also becomes extremely simple.

However, the preceding detection filter has parameters on plants explicitly, the precise information about plants are necessary to design the filter. To solve this problem, this paper presents a new design method for a self-repairing nonlinear control system (SRNCS). The nonlinear detection filter presented here, contains plant parameters implicitly, and so rough and few information is required to construct the filter and the control system. Of course, it can guarantee

exact and early fault detection, and the failure can be detected within a finite time prescribed by a design parameter of the filter.

Furthermore, in this paper, the proposed SRNCS is applied to a well-known continuous stirred-tank reactor (CSTR) [7], [8], and the effectiveness is confirmed through several numerical simulations.

In this paper, the sign function is defined as follows:

$$\text{sgn}[e] = \begin{cases} 1 & (e \geq 0) \\ -1 & (e < 0) \end{cases}$$

This definition is slightly different from the ordinary one.

II. PROBLEM STATEMENT

Consider the following minimum-phase system of $n \in \mathbb{I}^+$ -th order and the relative degree one.

$$\begin{aligned} \Sigma_P \quad & \dot{y} = ay + bu + \mathbf{h}^T \mathbf{z} \\ & \dot{\mathbf{z}} = \mathbf{F}\mathbf{z} + \mathbf{g}y \end{aligned} \quad (1)$$

where $y \in \mathbb{R}$ is the actual output, and $u : \mathbb{R}^+ \rightarrow \mathbb{R}$ is the control input. The above representation can be obtained by transforming the state-space equation of the plant. From the minimum-phase property, the system matrix $\mathbf{F} \in \mathbb{R}^{(n-1) \times (n-1)}$ is a stable matrix (all eigenvalues lie in the left half complex plane) [5], [6].

Here, we define the error $e : \mathbb{R}^+ \rightarrow \mathbb{R}$ by

$$e \triangleq r - y \quad (2)$$

where $r : \mathbb{R}^+ \rightarrow \mathbb{R}$ is the reference input whose time derivative \dot{r} is bounded. In this paper, the tracking problem is considered, that is, the control objective is to make the preceding error e small in the practical sense.

To measure the actual output y , we prepare the two sensors; one is the primary sensor #1, and the other is the backup #2 for sensor-repairing. Then the feedback signal $y_S : \mathbb{R}^+ \rightarrow \mathbb{R}$ is given by

$$y_S = \begin{cases} y_1 & (t \leq t_D) \\ y_2 & (t > t_D) \end{cases} \quad (3)$$

where $y_i \in \mathbb{R}$ ($i = 1, 2$) is the output signal measured by the sensor # i , and $t_D \in \mathbb{R}^+$ is the detection time which will be defined later. The primary sensor #1 is usually utilized, and replaced with the backup #2 if the failure is detected. Because of “cold standby”, the backup (spare) sensor is maintained to be healthy out of the control loop until it is activated and inserted.

From (3), the measured error $e_S : \mathbb{R}^+ \rightarrow \mathbb{R}$ is given by

$$e_S \triangleq r - y_S \quad (4)$$

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Further calculation yields

$$\begin{aligned} \dot{S} \leq & -\frac{1}{2} \underbrace{\left(p - a - \frac{2}{\lambda} - \lambda \|\mathbf{P}\|^2 \right)}_{\alpha_1} e^2 \\ & - \frac{\lambda}{2} \underbrace{\left\{ 2\lambda_{\min}[\mathbf{Q}] - 2\|\mathbf{h}\|^2 - 2\|\mathbf{g}\|^2 \right\}}_{\alpha_2} \|z\|^2 \\ & - \frac{1}{2} \underbrace{\left(p + a - 2 - \frac{2}{\lambda} \right)}_{\alpha_3} v^2 \\ & + \frac{\lambda}{2} (|\dot{r} - ar|^2 + \|\mathbf{P}\|^2 r^2 + \gamma^2) \end{aligned} \quad (15)$$

Regarding the last term in the R.H.S. of (14), because of boundedness of r and \dot{r} , there exists a finite constant $\beta \in \mathbb{R}^+$ such that

$$\beta > |\dot{r} - ar|^2 + \|\mathbf{P}\|^2 r^2 + \gamma^2 \quad (16)$$

Therefore, we have

$$\dot{S} \leq -\alpha S + \frac{\lambda\beta}{2} \quad (17)$$

where

$$\alpha = \min \left\{ \alpha_1, \frac{\alpha_2}{\lambda_{\max}[\mathbf{P}]}, \alpha_3 \right\} \quad (18)$$

Thus, the solution S obeys

$$S \leq S(0) \exp(-\alpha t) + \frac{\lambda\beta}{2\alpha} \quad (19)$$

which means that all the signals, e , v and z are bounded on $[0, t_F]$. The proof is completed. ■

From Lemma 1, in the healthy situation, there exists a finite constant $\Gamma \in \mathbb{R}^+$ such that

$$|v(t)| < \Gamma, \quad t \in [0, t_F] \quad (20)$$

On the other hand, however, in the faulty situation ($t \leq t_F$), the behavior of the filtered signal v obeys

$$\dot{v} = \text{sgn}[e_S] \{ |v| + \gamma + \text{sgn}[e_S] (\dot{r} - \dot{\varphi}) + p|e_S| \} \quad (21)$$

This implies

$$\dot{v} \geq |v| + \varepsilon \geq \varepsilon \quad (\text{for } \text{sgn}[e_S] \geq 0) \quad (22)$$

and

$$\dot{v} \leq -|v| - \varepsilon \leq -\varepsilon \quad (\text{for } \text{sgn}[e_S] < 0) \quad (23)$$

where $\varepsilon = \gamma - \eta$. From these results, for both cases, the filtered signal v tends to diverge. Hence, if the failure occurs and the failed sensor is not replaced, then the inequality (18) holds no longer. Therefore, the detection time t_D is defined by

$$t_D \triangleq \min \{ t \mid |v(t)| \geq \Gamma \} \quad (24)$$

By this detection rule, the failure can be exactly detected. At the same time, the failed sensor is replaced with the healthy backup and the system stability and performance recovery.

Thus, we can summarize the results in the following theorem as a main result.

Theorem 1: Consider the SRNCR constructed by (3), (4), (8), (9) and (23). Then, the SRNCR has the following properties;

(P1) After the failure, there is a finite detection time $t_D (> t_F)$ such that

$$t_D \leq t_F + \frac{2\Gamma}{\varepsilon} \quad (25)$$

(P2) All the signals, e , v and z are bounded on $[0, \infty)$.

(P3) Regarding the tracking performance, for arbitrarily small $\delta \in \mathbb{R}^+$, there exists a $p_0 \in \mathbb{R}^+$ such that for $p > p_0$ the following inequality holds,

$$\limsup_{t \rightarrow \infty} |e(t)| \leq \delta \quad (26)$$

(Proof) Re-consider the behavior of v in the faulty situation. From (21), it follows that

$$v \geq \varepsilon(t - t_F) + v(t_F) \triangleq \tilde{v} \quad (27)$$

Clearly, there is a finite time $\tilde{t}_D > t_F$ such that $\tilde{v}(\tilde{t}_D) = \Gamma$, and it satisfies

$$\tilde{t}_D - t_F \leq \frac{\Gamma + |v(t_F)|}{\varepsilon} \leq \frac{2\Gamma}{\varepsilon} \quad (28)$$

Because $t_D \leq \tilde{t}_D$, the inequality (24) holds. Also, from (22), in the case of $\text{sgn}[e_S] < 0$, by the same manner as above, the inequality (24) holds. Thus, (P1) is true.

Furthermore, from Lemma 1, on the period $[0, t_F]$, all the signals, e , v and z are bounded. On the period $[t_F, t_D]$ where the sensor fails, from (23), v is bounded. Hence, u is bounded. For bounded u , the plant Σ_P does not have a finite escape time, and so the signals e and z are bounded. After repairing sensor, the control system recovers its stability and the tracking performance. Finally, we can conclude that all the signals, e , v and z are bounded on $[0, \infty)$. Hence, (P2) is proven.

At last, regarding (P3), before the failure, or after repairing sensor, the behavior of e , v and z obey the differential equations shown in (10). Therefore, from (18), it follows that

$$\limsup_{t \rightarrow \infty} |e| \leq \sqrt{\frac{\lambda\beta}{\alpha}} \quad (29)$$

Here, take sufficiently large p such that

$$\min\{\alpha_1, \alpha_3\} > 1 \quad (30)$$

Then, we have

$$\alpha = \min \left\{ 1, \frac{\alpha_2}{\lambda_{\max}[\mathbf{P}]} \right\} \quad (31)$$

The constants α and β do not depend on λ . Therefore, we can choose sufficiently small λ so that

$$\sqrt{\frac{\lambda\beta}{\alpha}} \leq \delta \quad (32)$$

This means that (P3) is true.

Thus, the theorem 1 can be proven. ■

Remark 1: The candidate of Γ is given by

$$\Gamma = \left(2S(0) + \frac{\lambda\beta}{\alpha} \right)^{\frac{1}{2}} \quad (33)$$

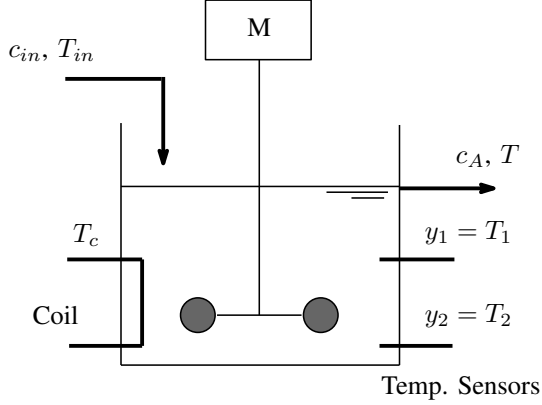


Fig. 2. Illustration of the CSTR with the two temperature sensors.

which is independent of ε . Hence, from (24), the detection time, $t_D - t_F$ can be arbitrarily shortened by choosing ε . This is one of the advantages of the proposed method.

Remark 2: The systems stability and the control performance are guaranteed based on high-gain feedback [5], [6]. Regarding to the tracking performance (25), from (29) and (31), it is shown that for smaller δ (small λ), the feedback-gain p should be higher.

IV. APPLICATION OF A CSTR

In this section, an application of the proposed method to the well-known continuous stirred-tank reactor (CSTR) is shown to confirm the effectiveness of the SRNCS.

Consider a first order, irreversible, exothermic, chemical reaction (where $A \rightarrow B$) [7], [8]. The mass and energy balances are expressed as follows:

$$\begin{aligned} \dot{c}_A &= -\kappa(t, T)c_A + \frac{Q}{V}(c_{in} - c_A) \\ \dot{T} &= \frac{(-\Delta H)}{C}\kappa(t, T)c_A \\ &\quad + \frac{Q}{V}(T_{in} - T) + \frac{UA}{VC}(T_c - T) \end{aligned} \quad (34)$$

where $c_A \in \mathbb{R}$ is the concentration of species A, and $T \in \mathbb{R}^+$ is the reactor temperature which is the controlled variable. T_c is the manipulated temperature as a control input. Commonly, $\kappa(t, T)$ is a nonlinear term, and is given by Arrhenius relation, $\kappa(t, T) = \kappa_0 \exp\{-E/(RT)\}$. The parameters are shown in Table 1.

The control objective is to make the temperature T track the desired set-point $T_r \in \mathbb{R}^+$ in the presence of the failure of the temperature sensor.

Now, set $y = T$, $z = c_A$ and $u = T_c$. Then from (33) we can obtain a simplified model as follows.

$$\begin{aligned} \dot{y} &= -\tilde{a}y + \tilde{b}u + \rho_1 z + \rho_2 \\ \dot{z} &= -(\tilde{f} + \rho_3)z + \rho_4 \end{aligned} \quad (35)$$

where \tilde{a} , \tilde{b} and \tilde{f} are some positive constants, and $\rho_i : \mathbb{R}^+ \rightarrow \mathbb{R}$ ($i = 1, 2, 3, 4$) are bounded functions with respect to time t . Although the above mathematical model (34) is slightly

different from (1), the proposed SRNCS design is applied to the CSTR.

The design parameters for the detector Σ_D and the controller Σ_C are given as follows.

$$\gamma = 3, \quad p = 5$$

The feedback-gain p is chosen by trial and error. Throughout several numerical simulation, the threshold Γ for fault detection is set as

$$\Gamma = 13$$

To measure T , two temperature sensors are prepared as shown in Figure 2, where $y_1 = T_1$ [K] and $y_2 = T_2$ [K] are the temperatures measured by the sensors. The failure scenario is supposed to be a stuck-type as

$$y_1(t) = y_1(t_F) = T(t_F), \quad t_F = 10 \text{ [min]}$$

Hence, because of $\eta = 0$, from (24), the maximum detection time is estimated by

$$t_D \leq t_F + \frac{2\Gamma}{\varepsilon} < 10 + 27/3 = 19 \triangleq \tilde{t}_D \text{ [min]}$$

In the simulation, the initial values of the CSTR are given as

$$T(0) = y(0) = 380 \text{ [K]}$$

$$c_A(0) = z(0) = 0.2 \text{ [mol/L]}$$

The set-point T_r is chosen as

$$T_r = 383.7 \text{ [K]}$$

At this temperature, the desired concentration is $c_A^* = 0.1$ [mol/L].

The simulation results are shown in Figures 3 and 4. In Figure 3, the upper figure shows the actual temperature $T = y$, and also the lower figure illustrates the concentration c_A . In Figure 4, the upper shows the manipulated temperature T_c , and the lower shows the absolute value of the filtered signal v (solid line) and the threshold Γ (dashed line).

From these results, $|v|$ hits Γ at time $t_D = 10.04$ [min] (see Figure 4), and so sensor failure can be successfully detected earlier than the estimated time \tilde{t}_D . In (24) and also (27), it is supposed that the sign of v at the failure time is unknown. Hence, with the absolute value of v , the estimated time \tilde{t}_D is calculated. This results in the excessively large estimated

TABLE I
CSTR PARAMETERS [6]

Variable	Value	
V	100	L
Q	100	L/min
c_{in}	1	mol/l
κ_0	7.2×10^{10}	min^{-1}
E/R	8750	K
ΔH	-5.0×10^4	J/mol
C	239	J/L K
T_{in}	350	K
UA	5.0×10^4	J/min K

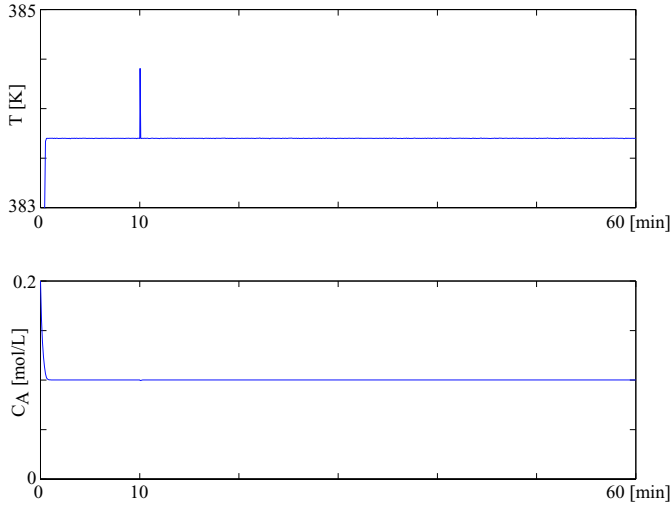


Fig. 3. Simulation results; the temperature T and the concentration c_A .

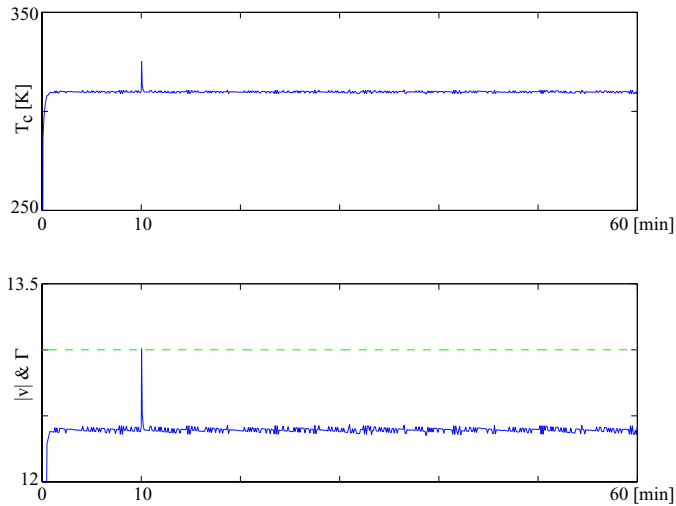


Fig. 4. Simulation results; the manipulated temperature T_c and the absolute value of the filtered signal v with the threshold Γ .

value. In this simulation, because $v(t_F)$ is a positive value, i.e. $v(t_F) \simeq 12.5$, we have

$$t_D \leq t_F + \frac{\Gamma - v(t_F)}{\varepsilon} = 10 + 0.5/3 \simeq 10.17 \text{ [min]}$$

This is the reason why there is a large difference between the above detection time t_D and the estimated maximum time \hat{t}_D . Anyway, the early and exact fault detection can be achieved for the CSTR.

In addition, the controlled temperature T well tracks the set-point T_r so as to maintain the desired concentration in spite of the existence of the nonlinear terms ρ_i in the CSTR model (see Figure 3).

In Figure 4, we can see the small oscillation. This does not come from the effect of noise because any noise is not

inserted in the simulation. This oscillation might be caused by the sign function $\text{sgn}[e_S]$ in the detection filter Σ_D . In spite of oscillatory behaviors of v and T_c , the temperature T and the concentration c_A are well-controlled, and the effect from v and T_c can be suppressed by the high-gain controller.

V. CONCLUSIONS

This paper presents a new design method for an SRNCS for plants with faulty sensors. It is theoretically shown that the SRNCS find the sensor failure exactly within a pre-specified detection time, and repair the sensor (replacing sensors). Furthermore, in this paper, the SRNCS is applied to the CSTR, and the effectiveness is confirmed through several numerical simulations.

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