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Adaptive disturbance rejection control for nonlinear stochastic systems: an application to bioreactor system

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***Abstract*— In this paper, an adaptive scheduling H_∞ disturbance rejection controller is designed for a class of nonlinear Markov jump systems with nonhomogeneous Markov jump process, in which the transition probabilities are time-varying. To estimate and reject the disturbance, a disturbance observer is considered, such that a disturbance rejection state-feedback control law is designed. Under the designed controller, a sufficient condition is presented to ensure that the resulting closed-loop system is stochastically stable and a prescribed H_∞ performance index is satisfied. Finally, a simulation example is given to illustrate the effectiveness of the developed techniques proposed.**

I. INTRODUCTION

Markov jump systems (MJSs), as a special kind of hybrid system, has attracted much attention in recent years, due to its application in modeling much practical systems, such as in manufacturing systems, economic systems, and electrical systems [1]. Much work has been obtained for this type of stochastic systems, such as stochastic stability and stabilization [2], control [3], [4], fault detection and filtering [5]–[7], etc. However, all the works mentioned above are all under the assumption that transition probabilities for the MJSs are fully available although this information may not be fully known in practice.

On the other hand, disturbance in system may lead to very serious result, even make stable system unstable. Therefore, disturbance rejection is crucial to controller design in realistic control systems. To reduce or rejection disturbance, some attempt work has been made, one typical method is based on disturbance observer, which was originally presented in [8]. It is an effective way to handle this problem. The basic idea is to construct an observer to estimate the dynamic of the disturbance, and then a feed-forward compensator is applied to compensate the disturbance based on the output information of the observer. This has attracted much attention in the control community [9]–[12]. Based on our team's previous work [13], [14], in this paper, we will design adaptive continuous gain-scheduling H_∞ disturbance rejection controller for a class of nonlinear MJSs with nonhomogeneous jump processes. The rest of the paper is organized as follows: Problem statement and preliminaries are given in Section 2. In Section 3, stochastic stability analysis of linear stochastic systems is addressed. In Section 4, a set of mode-dependent H_∞ disturbance rejection controllers for linear Markov jump systems is designed, and then, continuous gain-scheduling disturbance rejection controller for the entire nonlinear stochastic system is designed in Section 5. A numerical example is shown to illustrate the effectiveness of our approach in Section 6. Finally, some concluding remarks are given in Section 7.

The notation $\mathbb{R}^n$ stands for an n-dimensional Euclidean space, the transpose of a matrix is denoted by A^{T}, $E\{\cdot\}$ denotes the mathematical statistical expectation of the s-tochastic process or vector, $L_2^n[0,\infty)$ stands for the space of n-dimensional square integrable vector valued functions over $[0,\infty)$, a positive-definite matrix is denoted by $P>0$, I is the unit matrix with appropriate dimension, and $*$ means the symmetric term in a symmetric matrix.

II. PROBLEM STATEMENT AND PRELIMINARIES

Let (M,Q,P) be a probability space, where M, Q and P represent, respectively, sample space, algebra of events and probability measure. Consider the following nonlinear Markov jump system (NMJS) with nonhomogeneous process

$$x_{k+1}=f_1(x_k,u_k,d_{1k},d_{2k},r_k) \tag{1}$$

where $f_1(\cdot)$ is a nonlinear function, $x_k\in\mathbb{R}^n$ represents the state of the system, $u_k\in\mathbb{R}^l$ represents the control input, d_{1k} is an external disturbance to the input of such system and it satisfies Assumption 2.1 to be defined later, $d_{2k}\in L_2^q[0,\infty)$ is another type of external disturbance to the system. $\{r_k,k\geq 0\}$ is a discrete time Markov stochastic process which takes values from a finite set of state $\Lambda=\{1,2,3,\ldots,N\}$, and r_0 represents the initial mode, the transition probability matrix is defined as $\Pi(k)=\{\pi_{ij}(k)\}$, $i,\ j\in\Lambda$, $\pi_{ij}(k)=P(r_{k+1}=j|r_k=i)$ is the transition probability from mode i at time k to mode j at time $k+1$, which satisfies $\pi_{ij}(k)\geq 0$ and $\sum\limits_{j=1}^{N}\pi_{ij}(k)=1$.

In order to linearize system (1) in the vicinity of these selected operating states, gradient linearization procedure [15] is applied here, and let $x_k^{(m)}$, $m\in\Psi$, $\Psi=\{1,2,3\ldots h\}$ be the selected working points of system (1), where h represents the number of selected operating states, $(x_k^{(m)})^\top=\begin{bmatrix}(x_1^{(m)})_k & (x_2^{(m)})_k & \ldots & (x_n^{(m)})_k\end{bmatrix}$. Then, system (1) can be linearized at these selected working points in respective time interval.

Thus, a series of Markov jump linear systems (MJLSs) are obtained as follows:

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$$x_{k+1} = A_m(r_k)x_k + B_m(r_k)[u_k + d_{1k}] + H_m(r_k)d_{2k} \quad (2)$$

where $A_m(r_k)$, $B_m(r_k)$ and $H_m(r_k)$ are mode-dependent constant matrices for the m-th linear stochastic system.

In this paper, nonhomogeneous jump process, in which transition probability of system (2) is time-varying, is described by a polytope with several vertices, which is given below:

$$\Pi(k) = \sum_{s=1}^{w} \alpha_s(k)\Pi^s$$

where Π^s are given matrices and $s = 1, \ldots, w$, w represents the number of the vertices, and

$$0 \le \alpha_s(k) \le 1, \quad \sum_{s=1}^{w} \alpha_s(k) = 1$$

Assumption 2.1: The disturbance d_{1k} under consideration in (2) is generated by the system given below:

$$\begin{cases} w_{k+1} = W(r_k)w_k + M(r_k)d_{3k} \\ d_{1k} = V(r_k)w_k \end{cases} \quad (3)$$

where $W(r_k)$, $M(r_k)$ and $V(r_k)$ are mode-dependent constant matrices with appropriate dimensions at the working instant k, $d_{3k} \in L_2^q[0,\infty)$ is an external disturbance to the system arise due to uncertainties or system noises.

Remark 2.1: Comparing with those considered in the existing literature, disturbance d_{1k} in this paper is a periodic noise, which is more realistic in practical systems.

Assumption 2.2: For systems (1) and (2), it holds that 1) (A_m, B_m) is controllable; and 2) $(W, B_m V)$ is observable.

For brevity, when $r_k = i$, $i \in \Lambda$, the matrices $A_m(r_k)$, $B_m(r_k)$ and $H_m(r_k)$ are denoted as $A_m(i)$, $B_m(i)$ and $H_m(i)$, respectively. The same principle is applied to all other parameter matrices.

Under the assumption that all the system states are available, we need only to estimate d_{1k}. For this, we consider the following reduced-order observer.

$$\begin{cases} \hat{d}_{1(k+1)} = V(i)\hat{w}_k \\ \hat{w}_{k+1} = v_k - L(i)x_k \\ v_{k+1} = (W(i) + L(i)B_m(i)V(i))[v_k - L(i)x_k] \\ \qquad + L(i)[A_m(i)x_k + B_m(i)u_k] \end{cases} \quad (4)$$

The controller in this system is designed as:

$$u_k = -\hat{d}_{1k} + K_m(i)x_k$$

Denote

$$e_k = w_k - \hat{w}_k$$

Then, we have

$$\begin{aligned} e_{k+1} &= (W(i) + L(i)B_m(i)V(i))e_k \\ &\quad + M(i)d_{3k} + L(i)H_m(i)d_{2k} \end{aligned} \quad (5)$$

Let

$$\xi_k^{\mathrm{T}} = \begin{bmatrix} x_k^{\mathrm{T}} & e_k^{\mathrm{T}} \end{bmatrix}$$

Combining systems (1), (2) and (3), we obtain an error estimation system:

$$\xi_{k+1} = \mathscr{A}_m(i)\xi_k + \mathscr{H}_m(i)d_k \quad (6)$$

where

$$d_k^{\mathrm{T}} = \begin{bmatrix} d_{2k}^{\mathrm{T}} & d_{3k}^{\mathrm{T}} \end{bmatrix}$$

$$\begin{aligned} &\mathscr{A}_m(i) \\ &= \begin{bmatrix} A_m(i) + B_m(i)K_m(i) & B_m(i)V(i) \\ 0 & W(i) + L(i)B_m(i)V(i) \end{bmatrix} \end{aligned}$$

$$\mathscr{H}_m(i) = \begin{bmatrix} H_m(i) & 0 \\ L(i)H_m(i) & M_m(i) \end{bmatrix} \quad \forall i \in \Lambda$$

The reference output of system (5) is set as:

$$z_k = \mathscr{C}_m(i)\xi_k \quad (7)$$

where

$$\mathscr{C}_m(i) = \begin{bmatrix} C_{1m}(i) & C_{2m}(i) \end{bmatrix}$$

The aim of our work is: design an H_∞ controller for system (2), ensure that the error system (6) is stochastically stable and satisfies a prescribed H_∞ performance index. To proceed further, we recall some definitions and present some preliminary work which will be needed to develop our main results in the paper.

Definition 2.1: For any initial mode r_0, and a given initial state ξ_0, system (6) (with $d_k = 0$) is stochastically stable if

$$\lim_{m\to\infty} E\{\sum_{k=0}^{m} \xi_k^{\mathrm{T}}\xi_k | \xi_0, r_0\} < \infty \quad (8)$$

Definition 2.2: For any initial mode r_0, a given initial state ξ_0 and a constant $\gamma > 0$, if there exists a feasible controller u_k and a positive number $\tilde{N}(\xi_0, r_0)$, such that system (6) (with $d_k \neq 0$) satisfies (9) and (10), then, error estimation linear system (6) is said to be stochastically stabilizable with an H_∞ performance index γ, i.e.,

$$\lim_{m\to\infty} E\{\sum_{k=0}^{m} \xi_k^{\mathrm{T}}\xi_k | \xi_0, r_0\} < \tilde{N}(\xi_0, r_0) \quad (9)$$

$$E\left\{\sum_{k=0}^{\infty} z_k^{\mathrm{T}} z_k\right\} \le \gamma^2 E\left\{\sum_{k=0}^{\infty} d_k^{\mathrm{T}} d_k\right\} \quad (10)$$

Sufficient conditions to ensure that system (6) with $d_k = 0$ is stochastically stable are given in the following lemma, where the transition probabilities are time variant matrices and the Lyapunov function is selected as a polytope function.

III. Stochastic Stability

Lemma 3.1: For a given initial condition ξ_0, system (6) (with $d_k = 0$) is stochastically stable, if there exists a set of positive definite symmetric matrices $P_s(i)$ and $P_q(j)$ such that

$$\Xi(i) = \begin{bmatrix} a_{11} & a_{12} \\ * & a_{22} \end{bmatrix} < 0 \qquad \forall i \in \Lambda \tag{11}$$

where

$$a_{11} = -\sum_{s=1}^{w} \alpha_s(k) P_s(i), a_{12} = \mathscr{A}_m^{\mathrm{T}}(i)$$

$$a_{22} = -(\sum_{j=1}^{N}\sum_{s=1}^{w}\sum_{q=1}^{w} \alpha_s(k)\beta_q(k)\pi_{ij}^{s} P_q(j))^{-1}$$

$$0 \le \alpha_s(k) \le 1, \quad \sum_{s=1}^{w} \alpha_s(k) = 1$$

$$0 \le \beta_q(k) \le 1, \quad \sum_{q=1}^{w} \beta_q(k) = 1$$

Proof: State equation of system (6) (with $d_k = 0$) is written as:

$$\xi_{k+1} = \mathscr{A}_m(i)\xi_k \tag{12}$$

Lyapunov function for system (12) is constructed as follows:

$$V(\xi_k, i) = \sum_{s=1}^{w} \alpha_s(k) \xi_k^{\mathrm{T}} P_s(i) \xi_k \quad (i \in \Lambda)$$

where

$$0 \le \alpha_s(k) \le 1, \quad \sum_{s=1}^{w} \alpha_s(k) = 1$$

Then, $\Delta V(\xi_k, i)$ for system (12) is obtained as:

$$\begin{aligned}
&\Delta V(\xi_k, i) = E\{V(\xi_{k+1}, i)\} - V(\xi_k, i) \\
&= \xi_k^{\mathrm{T}}[\mathscr{A}_m^{\mathrm{T}}(i) \textstyle\sum_{j=1}^{N}\sum_{s=1}^{w}\sum_{s=1}^{w} \alpha_s(k)\alpha_s(k+1)\pi_{ij}^{s} P_s(j)\mathscr{A}_m(i)]\xi_k \\
&- \textstyle\sum_{s=1}^{w} \alpha_s(k)\xi_k^{\mathrm{T}} P_s(i)\xi_k
\end{aligned}$$

Denote

$$\sum_{s=1}^{w} \alpha_s(k+1) P_s(j) = \sum_{q=1}^{w} \beta_q(k) P_q(j)$$

Then, we have

$$\begin{aligned}
&\Delta V(\xi_k, i) = \\
&\xi_k^{\mathrm{T}}[\mathscr{A}_m^{\mathrm{T}}(i)(\textstyle\sum_{j=1}^{N}\sum_{s=1}^{w}\sum_{q=1}^{w} \alpha_s(k)\beta_q(k)\pi_{ij}^{s} P_q(j))\mathscr{A}_m(i)]\xi_k \\
&-\xi_k^{\mathrm{T}} \textstyle\sum_{s=1}^{w} \alpha_s(k) P_s(i)\xi_k \\
&= \xi_k^{\mathrm{T}} \Xi(i)\xi_k
\end{aligned}$$

For system (12), condition (11) implies

$$\Delta V(\xi_k, i) < 0 \quad \forall i \in \Lambda$$

Let

$$\eta = \min_k\{\lambda_{min}(-\Xi(i))\} \quad \forall i \in \Lambda$$

where $\lambda_{min}(-\Xi(i))$ is the minimal eigenvalue of $-\Xi(i)$. Then,

$$\Delta V(\xi_k, i) \le -\eta \xi_k^{\mathrm{T}} \xi_k$$

Thus,

$$\begin{aligned}
E\{\textstyle\sum_{k=0}^{T} \Delta V(\xi_k, i)\} &= E\{V(\xi_{T+1}, i)\} - V(\xi_0, i) \\
&\le -\eta E\{\textstyle\sum_{k=0}^{T} \|\xi_k\|^2\}
\end{aligned}$$

and the following inequality holds

$$\begin{aligned}
E\{\textstyle\sum_{k=0}^{T} \|\xi_k\|^2\} &\le \tfrac{1}{\eta}\{V(\xi_0, i) - E\{V(\xi_{T+1}, i)\}\} \\
&\le \tfrac{1}{\eta} V(\xi_0, i)
\end{aligned}$$

which, in turn, implies that

$$\lim_{T\to\infty} E\{\sum_{k=0}^{T} \|\xi_k\|^2\} \le \frac{1}{\eta} V(\xi_0, i)$$

From Definition 2.1, system (6) (with $d_k = 0$) is stochastically stable, and this concludes the proof.

Theorem 3.1: For a given initial condition ξ_0, $\forall i \in \Lambda$, suppose that there exists a set of positive definite symmetric matrices $P_s(i)$ and $Q_q(j)$ such that

$$\Phi_1(i) = \begin{bmatrix} b_{11} & b_{12} & \dots & b_{13} \\ * & -Q_q(1) & 0 & 0 \\ * & * & \ddots & 0 \\ * & * & * & -Q_q(N) \end{bmatrix} < 0 \tag{13}$$

where

$$Q_s(i) = P_s^{-1}(i), \quad Q_q(j) = P_q^{-1}(j)$$

$$b_{11} = -G^{\mathrm{T}}(i) + Q_s(i) - G(i)$$

$$b_{12}=\sqrt{\pi_{i1}^{s}}G^{\mathrm{T}}(i)\mathscr{A}_m^{\mathrm{T}}(i), b_{13}=\sqrt{\pi_{iN}^{s}}G^{\mathrm{T}}(i)\mathscr{A}_m^{\mathrm{T}}(i)$$

Then, system (6) is stochastically stable with $d_k=0$.

Proof: From Lemma 3.1, $\Xi(i)<0$ implies

$$\Phi_2(i)=\begin{bmatrix} c_{11} & c_{12} \\ * & c_{22}\end{bmatrix}<0 \qquad \forall i\in\Lambda \tag{14}$$

where

$$c_{11}=-P_s(i)$$
$$c_{12}=\left[\begin{array}{ccc} \mathscr{A}_m^{\mathrm{T}}(i) & \dots & \mathscr{A}_m^{\mathrm{T}}(i)\end{array}\right]_N$$
$$c_{22}=-diag\left[\begin{array}{ccc}(\pi_{i1}^{s})^{-1}Q_q(1) & \dots & (\pi_{ij}^{s})^{-1}Q_q(j)\end{array}\right]$$

which, in turn, implies that

$$\Phi_3(i)=\begin{bmatrix} -P_s(i) & \sqrt{\pi_{i1}^{s}}\mathscr{A}_m^{\mathrm{T}}(i) & \dots & \sqrt{\pi_{iN}^{s}}\mathscr{A}_m^{\mathrm{T}}(i) \\ * & -Q_q(1) & 0 & 0 \\ * & * & \ddots & 0 \\ * & * & * & -Q_q(N)\end{bmatrix}$$

Multiplying $\Phi_3(i)$ by $\mathscr{G}^{\mathrm{T}}(i)$ and $\mathscr{G}(i)$ on the left hand side and the right hand side, respectively,

where

$$\mathscr{G}(i)=diag\left\{\begin{array}{cccc} G(i) & I & \cdots & I\end{array}\right\},$$

we have

$$\Phi_4(i)=\begin{bmatrix} d_{11} & d_{12} & \dots & d_{13} \\ * & -Q_q(1) & 0 & 0 \\ * & * & \ddots & 0 \\ * & * & * & -Q_q(N)\end{bmatrix}<0 \tag{15}$$

where

$$d_{11}=-G^{\mathrm{T}}(i)P_s(i)G(i)$$
$$d_{12}=\sqrt{\pi_{i1}^{s}}G^{\mathrm{T}}(i)A_m^{\mathrm{T}}(i)$$
$$d_{13}=\sqrt{\pi_{iN}^{s}}G^{\mathrm{T}}(i)A_m^{\mathrm{T}}(i)$$

It follows that

$$G^{\mathrm{T}}(i)P_s(i)G(i)\geq G^{\mathrm{T}}(i)-Q_s(i)+G(i)$$

Therefore, $\Phi_1(i)<0$ guarantees $\Phi_4(i)<0$. This implies that system (6) (with $d_k=0$) is stochastically stable, and hence completing the proof.

Next, in order to minimize the influences of the disturbances, we will design a robust H_∞ controller under which system (6) subject to all admissible disturbances is stochastically stable and has a prescribed H_∞ performance index.

IV. DISTURBANCE REJECTION H_∞ CONTROLLER DESIGN

Theorem 4.1: For a given constant $\gamma>0$, suppose that there exists a set of positive definite symmetric matrices $P_s(i)$ and $Q_q(j)$ such that

$$\Omega(i)=\begin{bmatrix} -P_s(i) & 0 & f_1 & \dots & f_2 & f_5 \\ * & -\gamma^2 I & f_3 & \dots & f_4 & 0 \\ * & * & -Q_q(1) & 0 & 0 & 0 \\ * & * & * & \ddots & 0 & 0 \\ * & * & * & * & -Q_q(N) & 0 \\ * & * & * & * & * & -I\end{bmatrix}<0$$

where

$$Q_q(j)=P_q^{-1}(j), \quad f_1=\sqrt{\pi_{i1}^{s}}\mathscr{A}_m^{\mathrm{T}}(i)$$
$$f_5=\mathscr{C}_m^{\mathrm{T}}(i), \quad f_2=\sqrt{\pi_{iN}^{s}}\mathscr{A}_m^{\mathrm{T}}(i)$$
$$f_3=\sqrt{\pi_{i1}^{s}}\mathscr{H}_m^{\mathrm{T}}(i), \quad f_4=\sqrt{\pi_{iN}^{s}}\mathscr{H}_m^{\mathrm{T}}(i).$$

Then, system (6) (with $d_k\neq 0$) is stochastically stable and also satisfies a prescribed H_∞ performance index. The corresponding controller is $u_k=-\hat{d}_{1k}+K_m(i)x_k$.

Proof:

Introduce the following cost function for system (6) (with $d_k\neq 0$).

$$J(T)=E\left\{\sum_{k=0}^{T}z_k^{\mathrm{T}}z_k\right\}-\gamma^2E\left\{\sum_{k=0}^{T}d_k^{\mathrm{T}}d_k\right\} \tag{16}$$

Under zero initial condition, the index $J(T)$ can be rewritten as:

$$J(T)\leq E\left\{\sum_{k=0}^{T}[z_k^{\mathrm{T}}z_k-\gamma^2d_k^{\mathrm{T}}d_k+\Delta V(x_k,i)]\right\} \tag{17}$$

By Lemma 3.1, it follows that

$$\begin{aligned}
J(T)&\leq E\left\{\sum_{k=0}^{T}[z_k^{\mathrm{T}}z_k-\gamma^2d_k^{\mathrm{T}}d_k+\Delta V(x_k,i)]\right\}\\
&=E\left\{\sum_{k=0}^{T}\{(\mathscr{C}_m^{\mathrm{T}}(i)\xi_k)^{\mathrm{T}}\mathscr{C}_m(i)\xi_k-\gamma^2d_k^{\mathrm{T}}d_k+\Delta V(x_k,i)\}\right\}\\
&\leq E\left\{\sum_{k=0}^{T}\{(\mathscr{C}_m^{\mathrm{T}}(i)\xi_k)^{\mathrm{T}}\mathscr{C}_m(i)\xi_k-\gamma^2d_k^{\mathrm{T}}d_k\}\right\}\\
&+E\left\{\sum_{k=0}^{T}[\tilde{A}_m^{\mathrm{T}}(i)(\sum_{j=1}^{N}\sum_{s=1}^{w}\sum_{q=1}^{w}\alpha_s(k)\beta_q(k)\pi_{ij}^{s}P_q(j))\tilde{A}_m(i)]\right\}\\
&-E\left\{\sum_{k=0}^{T}\xi_k^{\mathrm{T}}\sum_{s=1}^{w}\alpha_s(k)P_s(i)\xi_k\right\}
\end{aligned}$$

where

$$\tilde{A}_m(i)=\mathscr{A}_m(i)\xi_k+\mathscr{H}_m(i)d_k$$

By Theorem 3.1 and recalling Schur complement, it holds that

$$J(T)\leq\tilde{x}_k^{\mathrm{T}}\Omega(i)\tilde{x}_k$$

where

$$\tilde{x}_k = \begin{bmatrix} \xi_k^{\mathrm{T}} & d_k^{\mathrm{T}} \end{bmatrix}$$

Clearly, $\Omega(i) < 0$ can be reduced to inequality (13) by denoting $w_k = 0$, so system (6) is stochastically stable. On the other hand, for $T \to \infty$, $\Omega(i) < 0$ results in $J(\infty) < -V(x_\infty, i) < 0$, that is

$$E\left\{\sum_{k=0}^{\infty} z_k^{\mathrm{T}} z_k\right\} \leq \gamma^2 E\left\{\sum_{k=0}^{\infty} d_k^{\mathrm{T}} d_k\right\} \tag{18}$$

Thus, system (6) is stochastically stable and has a prescribed H_∞ performance index where the controller may be chosen as $u_k = -\hat{d}_{1k} + K_m(i)x_k$. This completes the proof.

V. Adaptive Scheduled H_∞ Controller Design

Note that for each linear stochastic system of system (1), we can obtain its state feedback controller, such that a given H_∞ performance index is satisfied. In the following, we will use curve fitting approach to design a continuous gain-scheduled controller $u_k = -\hat{d}_{1k} + K(i)x_k$ for the entire nonlinear stochastic system (1). For this purpose, we describe the procedure for computing the gain scheduling.

First, from Theorem 4.1, the mode-dependent gain matrices $K_m(i) \in R^{l\times n}$ of the controller can be obtained for the $m-th$ linear jump system (1). Denote $K_m(a,b,i)$ as elements of $K_m(i)$, where $a = 1,2,3\ldots,l$; $b = 1,2,3\ldots,n$, and

$$K_m(i) = \{K_m(a,b,i)\}$$

Second, denote the matrix $\tilde{K}(i)$ as:

$$\tilde{K}(i) = \{\tilde{K}(a,b,i)\}$$

$$\hat{x}_e(k) = \begin{bmatrix} x_e^{(1)}(k) & x_e^{(2)}(k) & \ldots & x_e^{(\mathrm{h})}(k) \end{bmatrix}$$

$$e = 1,2,3\ldots n$$

where

$$\tilde{K}(a,b,i) = \begin{bmatrix} K_1(a,b,i) & K_2(a,b,i) & \ldots & K_h(a,b,i) \end{bmatrix}$$

Next, an appropriate fixed value of e is selected and the polynomial fitting approach is applied to matrices $\tilde{K}(a,b,i)$ and $\hat{x}_e(k)$. In this way, each element of the controller is described as a polynomial, and the continuous controller is obtained for the nonlinear jump system (1) as given below:

$$K(i) = \{K(a,b,i)\}$$

$$K(a,b,i) = q_0(a,b) + q_1(a,b)x_e(k) + q_2(a,b)x_e^2(k)$$

$$+q_3(a,b)x_e^3(k) + \ldots + q_g(a,b)x_e^{\mathrm{g}}(k)$$

$q_0(a,b),\ldots,q_g(a,b)$ are fitted coefficients, and g is a selected integer.

VI. Simulation Results

The stochastic bioreactor system [16] with admissible disturbances under consideration is given below:

$$x_1(k+1) = -[u_1(k) + d_1(k)] + x_1(k)(1 - x_2(k))$$

$$\exp(\tfrac{x_2(k)}{\beta}) + 0.2d_2(k)$$

$$x_2(k+1) = -[u_2(k) + d_1(k)]x_2(k) + x_1(k)(1 - x_2(k))$$

$$\tfrac{1+\alpha(i)}{1+\alpha(i)-x_2(k)}\exp(\tfrac{x_2(k)}{\beta})$$

$$z(k) = x_1(k) + u_1(k) + 0.2d_2(k)$$

where $x_1(k)$ and $x_2(k)$ represent the number of cells and the nutrient concentration at time k respectively. $\alpha(i)$ and β are assumed to be known constant parameters which are the growth rates and the nutrient inhibition parameter, the input variable $u(k)$ represents the flow rate through the tank. And the nutrient concentration variable $x_2(k)$ is assumed to evolve on the open intervals $(0, 0.18)$. The Markov jumping process has two jumping modes, and linear parameters are given as $\alpha(1) = 0.02$ and $\alpha(2) = 0.03$,

$$H(1) = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}, \quad H(2) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$$

$$W(1) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad W(2) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$V(1) = \begin{bmatrix} 0.3 & 0.5 \end{bmatrix}, \quad V(2) = \begin{bmatrix} 0.1 & -0.2 \end{bmatrix}$$

$$M(1) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad M(2) = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}$$

$$C_1(1) = \begin{bmatrix} 0.5 & 0.1 \end{bmatrix}, \quad C_1(2) = \begin{bmatrix} 0.1 & 0 \end{bmatrix}$$

$$C_2(1) = \begin{bmatrix} 1.2 & 0.1 \end{bmatrix}, \quad C_2(2) = \begin{bmatrix} 0 & 0.1 \end{bmatrix}$$

The vertices of the time-varying transition probability matrix are given as follows:

$$\Pi^1(k) = \begin{bmatrix} 0.2 & 0.7 & 0.1 \\ 0.35 & 0.2 & 0.45 \\ 0.1 & 0.4 & 0.5 \end{bmatrix}$$

,

$$\Pi^2(k) = \begin{bmatrix} 0.55 & 0.3 & 0.15 \\ 0.48 & 0.22 & 0.3 \\ 0.3 & 0.2 & 0.5 \end{bmatrix}$$

$$\Pi^3(k) = \begin{bmatrix} 0.67 & 0.17 & 0.16 \\ 0.3 & 0.47 & 0.23 \\ 0.26 & 0.1 & 0.64 \end{bmatrix}$$

,

$$\Pi^4(k) = \begin{bmatrix} 0.4 & 0.2 & 0.4 \\ 0.8 & 0.1 & 0.1 \\ 0.25 & 0.25 & 0.5 \end{bmatrix}$$

By solving Theorem 4.1, mode-dependent controller gain matrices $\tilde{K}(a,b,i)$ of system (6) are obtained

The continuous gain-scheduling controller for the uncertain nonlinear stochastic bioreactor system, is designed, and given initial state and parameter γ as $x_0 = \begin{bmatrix} 0.5 & 1 \end{bmatrix}^{\mathrm{T}}$ and $\gamma = 0.42$. The disturbance d_{1k}, estimation disturbance $\hat{d}_{1k}$, and error disturbance $d_{1k} - \hat{d}_{1k}$ are shown in Figure 1. State trajectories of the stochastic system and the controlled output of the system are given in Figures 2-3. Clearly, the stochastic system (1) is stochastically stable under such a controller.

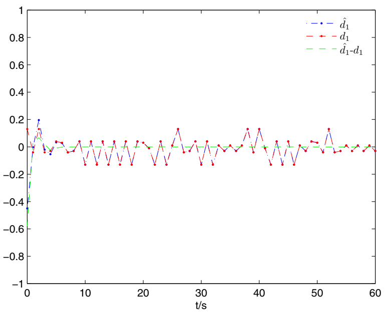

Fig. 1. Disturbance d_1

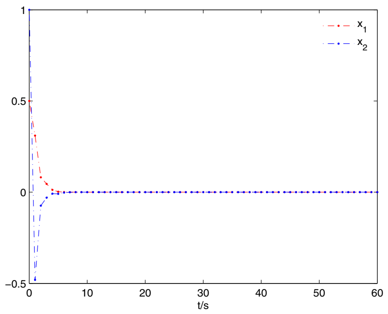

Fig. 2. Trajectory of system states

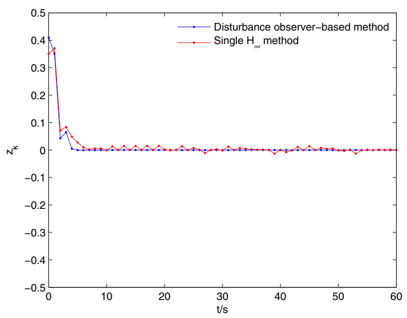

Fig. 3. Trajectory of controlled output z

Remark 6.1: To demonstrate the effectiveness of our results, we carry out comparison in Figure 3 by using a single H_∞ controller, meaning that $u_k = K(i)x_k$. Obviously, in the presence of harmonic disturbance d_{1k}, the single H_∞ controller fails to work.

VII. Conclusions

In this paper, the issue on adaptive disturbance rejection controller design for a class of discrete-time nonlinear Markov jump systems with nonhomogeneous processes is addressed. Transition probability is a time-varying matrix and expressed as being enclosed by a polytope, in which vertices are given. A parameter-dependent Lyapunov function is introduced to investigate the stochastic stability of the systems. Furthermore, it is also shown that a prescribed H_∞ performance index is satisfied. Finally, simulation results obtained show the potential of the approach proposed.

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