ADCONIP 2014 Hiroshima

PID Tuning Based on Partial Model Matching Using Closed-Loop Plant Response Data

Yoshihiro Matsui[1], Hideki Ayano and Kazushi Nakano[2]

***Abstract*— This paper proposes a method to estimate plant models and to give the reference model for the partial model matching method using only one-shot closed-loop transient data. The validity of the method is shown by numerical examples in which plant models are estimated, the estimated plant models are used to tune PID controllers and they are evaluated. Since the method does not require additional information other than the same data used in VRFT or FRIT, it is practical and useful.**

I. INTRODUCTION

Many PID controllers are used in various industrial control applications and some of them are poorly tuned. Poorly tuned PID controllers affect the quality of the products or cause needless increase in the production cost. However trial-and-error tuning of them, which is often used in practice, is difficult and time consuming. Therefore practical and useful method to tune PID controller have been required. One of practical PID tuning method is the partial model matching method [1]. However the method requires both the plant model and the reference model to be matched to the closed-loop system with the tuned PID controller.

In recent years, in order to tune PID controllers mainly, data-driven controller design methods [2], [3], [4] have been extensively developed with the improvements of the ability of computers and non-linear optimization methods, and could save the cost and the time to model plants to tune PID controllers because they do not require plant models. The Virtual Reference Feedback Tuning (VRFT) [2] and the Fictitious Reference Iterative Tuning (FRIT) [4] are such data-driven methods. Since they use the closed-loop transient data directly to design controllers instead of the plant models, they do not require the plant models. However they require the reference models yet, and the way to give appropriate and realizable reference models has not been shown.

It has been shown that the frequency characteristics of plants could be estimated with one-shot closed-loop transient data [5]. The fact suggests that the data has useful information in frequency domain and it also must be available to estimate the plant model in time domain. If the plant model is obtained, appropriate and realizable reference models for it can be chosen easily.

[1]Yoshihiro Matsui and Hideki Ayano are with the Department of Electrical Engineering, Tokyo National College of Technology, 1220-2 Kunugida-machi, Hachioji, Tokyo 193-0997, JAPAN {matsui, ayano}@tokyo-ct.ac.jp

[2]Kazushi Nakano is with the Department of Mechanical Engineering and Intelligent Systems, Graduate School of Informatics and Engineering, The University of Electro-Communications, 1-5-1 Chofu-ga-oka, Chofu, Tokyo 182-8585, JAPAN nakano@ee.uec.ac.jp

This paper proposes a method to estimate plant models and to give the reference model for the partial model matching method using only one-shot closed-loop transient data. The validity of the method is shown by numerical examples in which plant models are estimated, the estimated plant models are used to tune PID controllers and they are evaluated. Since the method does not require additional information other than the same data used in VRFT or FRIT, it is practical and useful.

II. PID TUNING BASED ON PARTIAL MODEL MATCHING

This paper deals with the PID controller $C(\boldsymbol{\rho}, s)$ and its parameter $\boldsymbol{\rho}$ given by (1) and (2), respectively.

$$C(\boldsymbol{\rho}, s) = \frac{c_2 s^2 + c_1 s + c_0}{s(s+d_1)}. \tag{1}$$

$$\boldsymbol{\rho} = \begin{bmatrix} c_2 & c_1 & c_0 & d_1 \end{bmatrix}^{\mathrm{T}}. \tag{2}$$

The controller $C(\boldsymbol{\rho}, s)$ can be rewritten by

$$C(\boldsymbol{\rho}, s) = K_p \left(1 + \frac{1}{T_i s} + \frac{T_d s}{T_f s + 1}\right), \tag{3}$$

where K_p, T_i, T_d and T_f are the proportional gain, the integral time, the derivative time and the time constant of the low pass filter, respectively, and they are given by following equations:

$$K_p = \frac{c_1 d_1 - c_0}{d_1^2} \tag{4}$$

$$T_i = \frac{c_1 d_1 - c_0}{c_0 d_1} \tag{5}$$

$$T_d = \frac{c_2 d_1^2 - c_1 d_1 + c_0}{c_1 d_1^2 - c_0 d_1} \tag{6}$$

$$T_f = \frac{1}{d_1} \tag{7}$$

Let us consider the model matching problem using a reference model in the denominator expansion form given by

$$T_r(s) = \frac{1}{1 + \alpha_1 \tau s + \alpha_2 \tau^2 s^2 + \alpha_3 \tau^3 s^3 + \cdots}, \tag{8}$$

where $\alpha_1, \alpha_2, \alpha_3, \cdots$ and τ are the constants which determine the damping characteristics of the model and its response speed, respectively. $T_r(s)$ is also a complementary sensitivity function in the denominator expansion form. Since the complimentary sensitivity function consists of $P(s)$ and $C(s)$ is given by

$$T(s) = \frac{P(s)C(\boldsymbol{\rho}, s)}{1 + P(s)C(\boldsymbol{\rho}, s)} = \frac{1}{1 + P^{-1}(s)C^{-1}(\boldsymbol{\rho}, s)}, \tag{9}$$

the model matching method is to find the controller parameter vector $\boldsymbol{\rho}$ so that $T(j\omega) \simeq T_r(j\omega)$. However, it should be noted that the band-width where the model matching is achievable is restricted due to the order and the form of the plant model.

If $T(j\omega) = T_r(j\omega)$, by comparing the right-hand side of (8) with the most right-hand side of (9), we have

$$P^{-1}(s)C^{-1}(\boldsymbol{\rho}, s) = \alpha_1 \tau s + \alpha_2 \tau^2 s^2 + \alpha_3 \tau^3 s^3 + \cdots . \quad (10)$$

If the plant model in the denominator expansion form given by

$$P(s) = \frac{1}{p_0 + p_1 s + p_2 s^2 + p_3 s^3 + \cdots} \quad (11)$$

is able to be obtained, by substituting (1) and (11) into (10), we have

$$\begin{aligned} & s(d_1 + s)(p_0 + p_1 s + p_2 s^2 + p_3 s^3 + \cdots) \\ &= (c0 + c_1 s + c_2 s^2)(\alpha_1 \tau s + \alpha_2 \tau^2 s^2 + \alpha_3 \tau^3 s^3 + \cdots). \quad (12) \end{aligned}$$

And by comparing the coefficients of s on both sides of (12) from lower order ones, we have

$$\boldsymbol{\theta} = \boldsymbol{A}\boldsymbol{\rho} \quad (13)$$

where

$$\boldsymbol{A} = \begin{bmatrix} \alpha_2 \tau^2 & \alpha_3 \tau^3 & \alpha_4 \tau^4 & -p_3 \\ \alpha_1 \tau & \alpha_2 \tau^2 & \alpha_3 \tau^3 & -p_2 \\ 0 & \alpha_1 \tau & \alpha_2 \tau^2 & -p_1 \\ 0 & 0 & \alpha_1 \tau & -p_0 \end{bmatrix} \text{and} \quad (14)$$

$$\boldsymbol{\theta} = \begin{bmatrix} p_3 & p_2 & p_1 & p_0 \end{bmatrix}^{\mathrm{T}}. \quad (15)$$

Thus we are able to obtain the solution of the partial model matching problem which is defined by $\boldsymbol{\rho}_d$ and it is given by

$$\boldsymbol{\rho}_d = \boldsymbol{A}^{-1}\boldsymbol{\theta}. \quad (16)$$

Since the method uses the restricted order reference model and plant model so that the frequency response of the closed-loop system with $\boldsymbol{\rho}_d$ is close to that of the reference model only at low frequencies, the method is called the partial model matching method. In order to determine PID controller given by (1) by the method, the plant parameters $p_0 \sim p_3$ in (11) and the 4th order reference model are necessary as shown in (14).

III. Plant model estimation

As shown in the previous section, in order to solve the partial model matching problem, the plant model $P_M(\boldsymbol{\theta}, s)$ given by (17) is necessary.

$$P_M(\boldsymbol{\theta}, s) = \frac{1}{p_0 + p_1 s + p_2 s^2 + p_3 s^3} \quad (17)$$

In this section the method to obtain $\boldsymbol{\theta}$ given by (15) is shown.

A. Properties of closed-loop transient data

Before discussing the plant parameter estimation, the properties of the closed-loop transient data used for the estimation is investigated. Let us consider the closed-loop system shown in Fig.1. The signals r, e, n, u and y in the system are the reference, the error, the measurement noise, the input and the output of the plant,respectively, and r and n are assumed as a step function and a white noise, respectively.

When the closed-loop system in Fig.1 is stabilized by the $C(\boldsymbol{\rho}, s)$ with $\boldsymbol{\rho} = \boldsymbol{\rho}_0$, the step reference response data sets $r(t)$, $u(t)$ and $y(t)$ from the closed-loop system are acquired and saved as $r_0(t)$, $u_0(t)$ and $y_0(t)$, respectively. The frequency characteristics of $y_0(t)$ and $u_0(t)$ are given by (18) and (19), respectively

$$y_0(j\omega) = \frac{1}{1 + L_0(j\omega)} \{L_0(j\omega) r_0(j\omega) + n(j\omega)\} \quad (18)$$

$$u_0(j\omega) = \frac{C_0(j\omega)}{1 + L_0(j\omega)} \{r_0(j\omega) - n(j\omega)\} \quad (19)$$

In (19) and (18), $L_0(s)$ and $C_0(s)$ are given by

$$L_0(s) = C(\boldsymbol{\rho}_0, s)P(s) \quad (20)$$

and

$$C_0(s) = C(\boldsymbol{\rho}_0, s), \quad (21)$$

respectively.

If the system is controlled stably with $C(\boldsymbol{\rho}_0, s)$, we can assume that the first term of the right-hand side of (19) is dominant, that is, the relation given by

$$|r(t)| \gg |n(t)| \quad (22)$$

holds at least at frequencies lower than the gain crossover frequency of the closed-loop system with the $C(\boldsymbol{\rho}_0.s)$. Since $|P(j\omega)C(\boldsymbol{\rho}_0, j\omega)| > 1$ at the frequencies lower than the gain crossover frequency, the first term of the right-hand side of (18) is dominant at the frequencies, too. Therefore the data sets of $u_0(t)$ and $y_0(t)$ could have the information to estimate the plant parameters at least at the frequencies lower than the gain crossover frequency.

The information of the data in frequency domain is useful to check the validity of the plant parameters estimated. The frequency spectra of $y_0(t)$ and $u_0(t)$ are able to be estimated by

$$\hat{y}_0(j\omega) = \frac{\mathscr{F}[F(s) y_0(t)]}{F(j\omega)} \quad (23)$$

and

$$\hat{u}_0(j\omega) = \frac{\mathscr{F}[F(s) u_0(t)]}{F(j\omega)}, \quad (24)$$

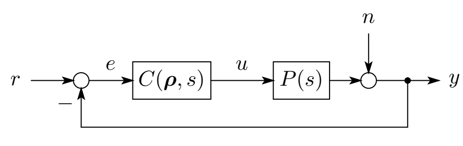

Fig. 1. Closed-loop system

respectively, where the symbol $\mathscr{F}$ denotes Fourier transform and $F(s)$ is the bandpass filter which is employed so that the Fourier Transform is able to apply to $y_0(t)$ and $u_0(t)$. The filter is given by

$$F(s) = \frac{100Ts}{(100Ts+1)(10Ts+1)}, \tag{25}$$

where T is the sampling period of the data.

Here the frequency ω_{nf} is defined so that at lower frequencies than $\omega < \omega_{nf}$ it can be assumed that neither $u_0(t)$ nor $y_0(t)$ are contaminated by the measurement noise. As shown in numerical examples later, $\omega < \omega_{nf}$ can be found easily from the gain plots of $\hat{u}_0(j\omega)$ and $\hat{y}_0(j\omega)$. Moreover at $\omega < \omega_{nf}$ the frequency characteristics of the plant can be estimated by

$$\hat{P}(j\omega) = \frac{\hat{y}_0(j\omega)}{\hat{u}_0(j\omega)}, \tag{26}$$

using $\hat{y}_0(j\omega)$ and $\hat{u}_0(j\omega)$, and it is used to evaluate the plant model estimated by the method explained next.

B. Plant parameter estimation

Considering the relationships in time domain between y_0 and r_0, and u_0 and r_0 given by (18) and (19), respectively, we have two errors for the parameter estimation.

$$e_y(\boldsymbol{\theta},t) = C_0(s)\left\{y_0(t) - \frac{C_0(s)P_M(\boldsymbol{\theta},s)}{1+C_0(s)P_M(\boldsymbol{\theta},s)}r_0(t)\right\} \tag{27}$$

$$e_u(\boldsymbol{\theta},t) = u_0(t) - \frac{C_0(s)}{1+C_0(s)P_M(\boldsymbol{\theta},s)}r_0(t) \tag{28}$$

The reason why $C_0(s)$ is multiplied to the right hand side of (27) is that the amplitudes of the terms of the measurement noise n in (18) and (19) are to be same in (27) and (28).

By a least squares approach using $e_y(t)$ and $e_u(t)$, the plant parameters can be estimated by

$$\boldsymbol{\theta}_d = \arg\min_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) \tag{29}$$

and

$$J(\boldsymbol{\theta}) = \sum_{k=0}^{N-1}\left\{e_y^2(\boldsymbol{\theta},kT) + e_u^2(\boldsymbol{\theta},kT)\right\}, \tag{30}$$

where T and N are the sampling period and the data length, respectively.

As (27), (28) and (30) show, any nonlinear optimization method is necessary to obtain (29). In this paper, MATLAB's function "fminsearch" is employed. The search of $\boldsymbol{\theta}_d$ can be carried out in comparatively short period of time by the function, and the validity of the solutions by the method can be easily evaluated in the time and frequency domains as mentioned below.

IV. Validation of tuned controller

The plant model with $\boldsymbol{\theta}_d$ and the controller with $\boldsymbol{\rho}_d$ must be confirmed to be valid before the controller is implemented into the actual control system. The confirmation can be done using (24), (23) and (26) in frequency domain.

The comparison of $\hat{P}(j\omega)$ and $P_M(\boldsymbol{\theta}_d, j\omega)$ give us the information about the validity of $\boldsymbol{\theta}_d$, and the comparisons of

$$y_M(t) = \frac{P_M(\boldsymbol{\theta}_d,s)C_0(s)}{1+P_M(\boldsymbol{\theta}_d,s)C_0(s)}r_0(t) \tag{31}$$

and $y_0(t)$, and

$$u_M(t) = \frac{C_0(s)}{1+P_M(\boldsymbol{\theta}_d,s)C_0(s)}r_0(t) \tag{32}$$

and $u_0(t)$ in time domain are also useful. The estimated frequency characteristics $\hat{y}_M(j\omega)$ and $\hat{u}_M(j\omega)$ of $y_M(t)$ and $u_M(t)$ are obtained in similar way for $\hat{y}(j\omega)$ and $\hat{u}(j\omega)$ calculated by (23) and (24), respectively.

And we are able to estimate the frequency characteristics of the sensitivity function with $C(\boldsymbol{\rho}_d,s)$ given by

$$\hat{S}(j\omega) = \frac{1}{1+\hat{P}(j\omega)C(\boldsymbol{\rho}_d,j\omega)}. \tag{33}$$

Since $|S(j\omega)| = |1+P(j\omega)C(\boldsymbol{\rho}_d,j\omega)|^{-1}$ shows the inverse of the distance in the complex plain between $P(j\omega)C(\boldsymbol{\rho}_d,j\omega)$ and the critical point $-1+j0$ for Nyquist stability criterion, the maximum value of $|\hat{S}(j\omega)|$ shows the estimated stability margin and is usually recommended to be less than $2(\simeq 6\,\mathrm{dB})$.

V. Numerical examples

The coefficients of the reference model given by (8) are determined as the 4th order binomial standard form, that is, they are given by

$$\{\alpha_1,\alpha_2,\alpha_3,\alpha_4\} = \{4,\ 6,\ 4,\ 1\}. \tag{34}$$

The relation between τ of (8) and the gain crossover frequency ω_{gc} of the loop transfer function corresponding to the complimentary sensitivity function given by (8) is given by

$$\tau = \frac{0.24798}{\omega_{gc}}. \tag{35}$$

A. Example I

Let us consider the plant given by

$$P(s) = \frac{2e^{-s}}{s+1} \tag{36}$$

and the controller given by

$$C_0(s) = \frac{0.1(s+5)}{s}. \tag{37}$$

The step reference responses $y_0(t)$ and $u_0(t)$ of the closed-loop system consists of (36) and (37) shown in Fig.2 were saved and used to estimate the plant by the method. The estimated plant model using $y_0(t)$ and $u_0(t)$ in Fig.2 is given by

$$P_M(\boldsymbol{\theta}_d,s) = \frac{1}{0.5080+0.9632s+0.6830s^2+0.2643s^3}. \tag{38}$$

The $y_M(t)$ and $u_M(t)$ calculated from (31) and (32) with $P_M(\boldsymbol{\theta}_d,s)$ given by (38), respectively, are also shown in Fig.2. The figure shows that $y_M(t)$ and $u_M(t)$ are very similar

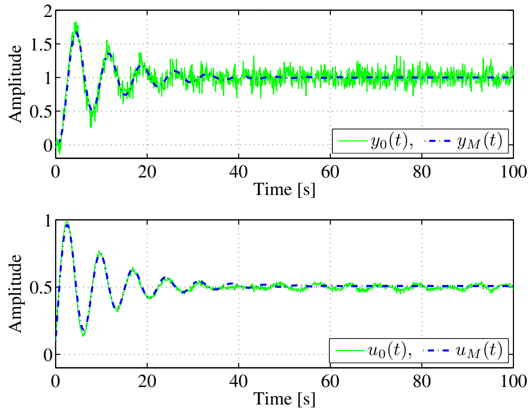

Fig. 2. Step reference responses of $y_0(t)$, $y_M(t)$, $u_0(t)$ and $u_M(t)$

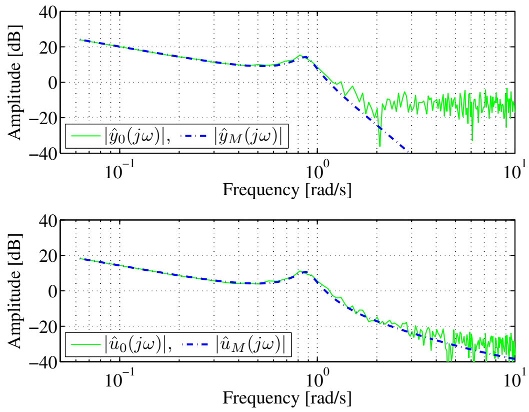

Fig. 3. Estimated frequency spectra of $y_0(t)$, $y_M(t)$, $u_0(t)$ and $u_M(t)$

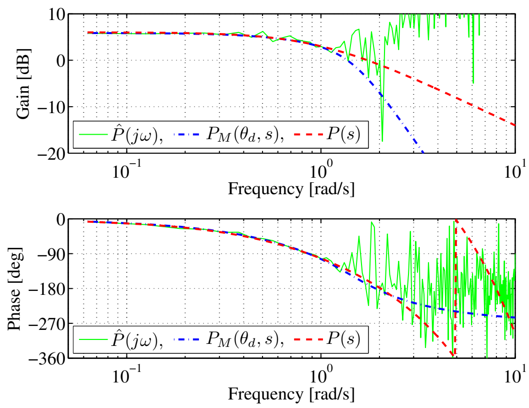

Fig. 4. Bode plots of $\hat{P}(j\omega)$, $P_M(\boldsymbol{\theta}_d,s)$ and $P(s)$

to $y_0(t)$ and $u_0(t)$, respectively, considering that $y_M(t)$ and $u_M(t)$ are calculated without the measurement noise. Fig.3 shows the estimated frequency spectra of them. It is obvious that the frequency spectra of $y_M(t)$ and $u_M(t)$ are very similar to those of $y_0(t)$ and $u_0(t)$, respectively, at lower frequencies than $\omega_{nf} = 1\,\mathrm{rad/s}$.

The Bode plots of $\hat{P}(j\omega)$ estimated by (26), $P_M(\boldsymbol{\theta}_d,s)$ and $P(s)$ are shown in Fig. 4. The figure shows that they are very similar to each other at lower frequencies than ω_{nf}, $\hat{P}(j\omega)$ is

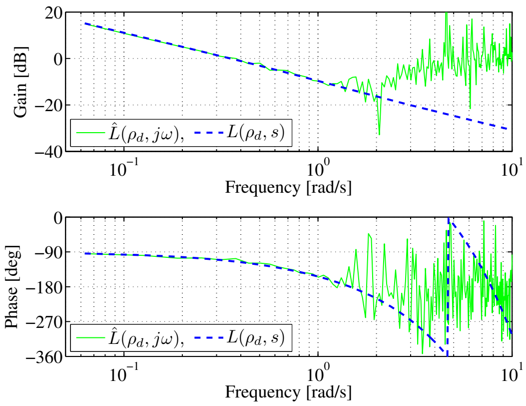

Fig. 5. Bode plots of $\hat{L}(\boldsymbol{\rho}_d, j\omega)$ and $P(\boldsymbol{\rho}_d,s)$

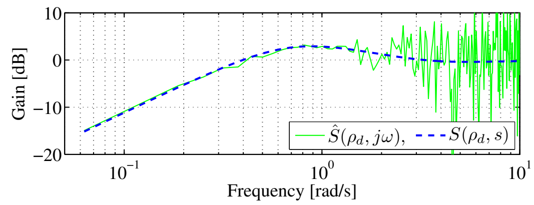

Fig. 6. Gain plots of $\hat{S}(\rho_d, j\omega)$ and $S(\rho_d,s)$

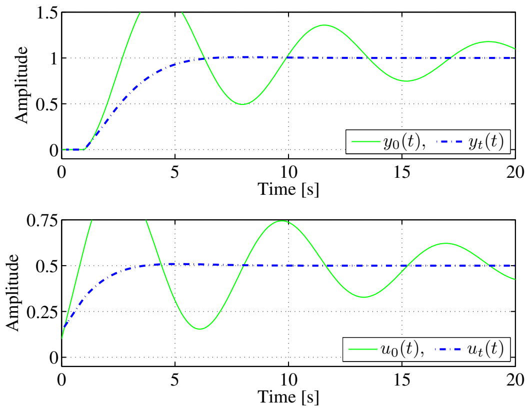

Fig. 7. Step reference responses of $y_0(t)$, $y_t(t)$, $u_0(t)$ and $u_t(t)$

useful to detect ω_{nf} and to confirm the validity of $P_M(\boldsymbol{\theta}_d,s)$, and the deference between $P_M(\boldsymbol{\theta}_d, j\omega)$ and $P(j\omega)$ becomes larger as the frequency is higher due to the restricted order and form of $P_M(\boldsymbol{\theta}_d,s)$.

Now we have the plant model $P_M(\boldsymbol{\theta}_d,s)$ whose validity is confirmed at frequencies lower than $\omega_{nf} = 1\,\mathrm{rad/s}$. Let us design a PID controller with the partial model matching. The gain crossover frequency ω_{gc} of the reference model is set to $0.35\,\mathrm{rad/s}$ which is lower enough than ω_{nf} in order to avoid the affects of the modeling error. The controller obtained from (16) with $\boldsymbol{\theta}_d$, (34) and (35) is given by

$$C(\boldsymbol{\rho}_d,s) = \frac{0.1437(s^2+2.0086s+0.9135)}{s(s+0.7323)}. \tag{39}$$

Fig.5 shows the Bode plots of $\hat{L}(j\omega) = \hat{P}(j\omega)C(\boldsymbol{\rho}_d, j\omega)$

and $L(s) = P(s)C(\boldsymbol{\rho}_d, s)$. It is obvious from the figure that the estimated frequency response $\hat{L}(j\omega) = \hat{P}(j\omega)C(\boldsymbol{\rho}_d, j\omega)$ is useful to confirm the validity of the controller $C(\boldsymbol{\rho}_d, s)$ since it is very similar to $L(s) = P(s)C(\boldsymbol{\rho}_d, s)$ which is true, and the gain crossover frequency of the loop transfer function with $C(\boldsymbol{\rho}_d, j\omega)$ is 0.35 rad/s which is specified and enough phase margin which is larger than 60° is ensured. Fig.6 shows the gain plots of the sensitivity functions $\hat{S}(j\omega)$ and $S(s)$ which are corresponding to the loop transfer functions $\hat{L}(j\omega)$ and $L(s)$ shown in Fig.5. It is inferable that $C(\boldsymbol{\rho}_d, s)$ has enough stability margin since the maximum value of $\hat{S}(j\omega)$ is seemed to be less than 6 dB.

Fig.7 shows the step reference responses. In the figure, $y_t(t)$ and $u_t(t)$ are the responses of the closed loop with $C(\boldsymbol{\rho}_d, s)$, and $y_0(t)$ and $u_0(t)$ are the responses of the closed loop with $C_0(s)$. In the simulations the responses were simulated without the measurement noise in order to make clear the difference of the responses by the controllers. The figure shows that the responses are improved by the controller designed by the method.

B. Example II

Let us consider the non-minimum phase plant given by

$$P(s) = \frac{-s+1}{s^2+s+1} \tag{40}$$

and the controller given by

$$C_0(s) = \frac{0.08(s+5)}{s}. \tag{41}$$

The step reference responses $y_0(t)$ and $u_0(t)$ of the closed-loop system consists of (40) and (41) shown in Fig.8 were saved and used to estimate the plant by the method. The estimated plant model using $y_0(t)$ and $u_0(t)$ in Fig.8 is given by

$$P_M(\boldsymbol{\theta}_d, s) = \frac{1}{0.9996 + 1.771s + 1.957s^2 + 1.468s^3}. \tag{42}$$

The $y_M(t)$ and $u_M(t)$ calculated from (31) and (32) with $P_M(\boldsymbol{\theta}_d, s)$ given by (42), respectively, are also shown in Fig.8. The figure shows that $y_M(t)$ and $u_M(t)$ are similar to $y_0(t)$ and $u_0(t)$, respectively, considering that $y_M(t)$ and $u_M(t)$ are calculated without the measurement noise. Fig.9 shows the estimated frequency spectra of them. It is obvious that the frequency spectra of $y_M(t)$ and $u_M(t)$ are similar to those of $y_0(t)$ and $u_0(t)$, respectively, at lower frequencies than $\omega_{nf} = 0.8$ rad/s.

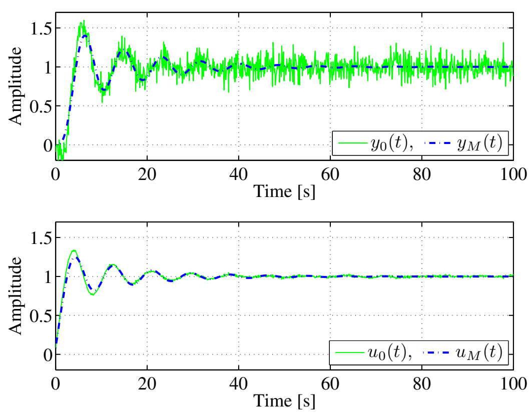

Fig. 8. Step reference responses of $y_0(t)$, $y_M(t)$, $u_0(t)$ and $u_M(t)$

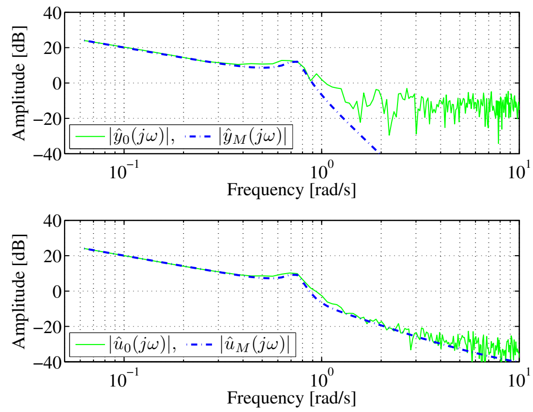

Fig. 9. Estimated frequency spectra of $y_0(t)$, $y_M(t)$, $u_0(t)$ and $u_M(t)$

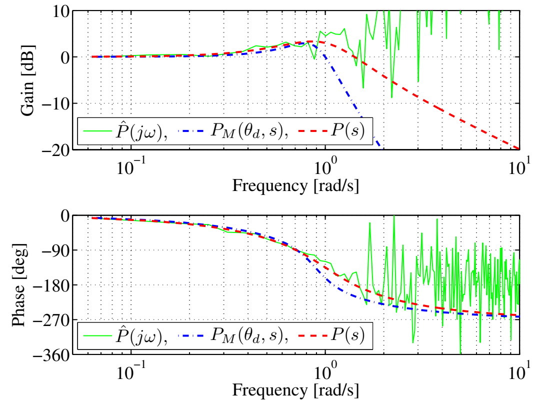

Fig. 10. Bode plots of $\hat{P}(j\omega)$, $P_M(\boldsymbol{\theta}_d, s)$ and $P(s)$

The Bode plots of $\hat{P}(j\omega)$ estimated by (26), $P_M(\boldsymbol{\theta}_d, s)$ and $P(s)$ are shown in Fig. 10. The figure shows that the frequency characteristics of $P_M(\boldsymbol{\theta}_d, s)$ is that of $P(s)$ at lower frequencies than ω_{nf}, $\hat{P}(j\omega)$ is useful to detect ω_{nf} and and to confirm the validity of $P_M(\boldsymbol{\theta}_d, s)$. The reason why the deference between $P_M(\boldsymbol{\theta}_d, j\omega)$ and $P(j\omega)$ becomes larger as the frequency is higher is due to the restricted order and form of $P_M(\boldsymbol{\theta}_d, s)$.

Now we have the plant model $P_M(\boldsymbol{\theta}_d, s)$ whose validity is confirmed at frequencies lower than $\omega_{nf} = 0.8$ rad/s. Let us design a PID controller with the partial model matching. The gain crossover frequency ω_{gc} of the reference model is set to 0.18 rad/s which is lower enough than ω_{nf} in order to avoid the affects of the modeling error. The controller obtained from (16) with $\boldsymbol{\theta}_d$, (34) and (35) is given by

$$C(\boldsymbol{\rho}_d, s) = \frac{1.4592(s^2 + 1.081s + 1.042)}{s(s+8.618)}. \tag{43}$$

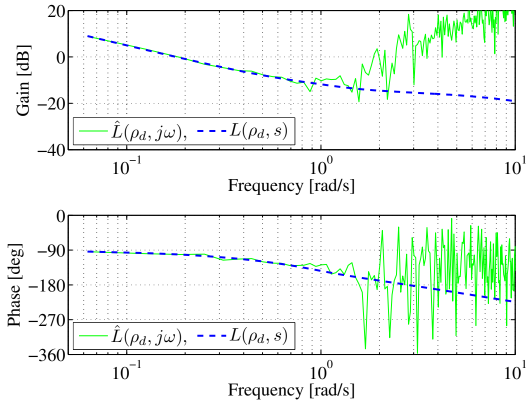

Fig. 11. Bode plots of $\hat{L}(\boldsymbol{\rho}_d, j\omega)$ and $P(\boldsymbol{\rho}_d, s)$

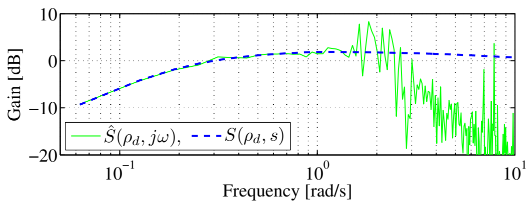

Fig. 12. Gain plots of $\hat{S}(\rho_d, j\omega)$ and $S(\rho_d, s)$

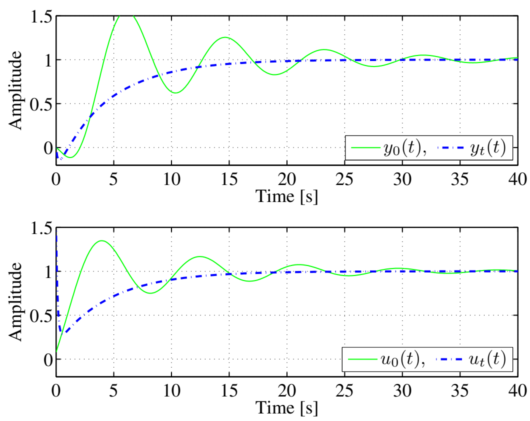

Fig. 13. Step reference responses of $y_0(t)$, $y_t(t)$, $u_0(t)$ and $u_t(t)$

Fig.11 shows the Bode plots of $\hat{L}(j\omega) = \hat{P}(j\omega)C(\boldsymbol{\rho}_d, j\omega)$ and $L(s) = P(s)C(\boldsymbol{\rho}_d, s)$. It is obvious from the figure that the estimated frequency response $\hat{L}(j\omega) = \hat{P}(j\omega)C(\boldsymbol{\rho}_d, j\omega)$ is useful to confirm the validity of the controller $C(\boldsymbol{\rho}_d, s)$ since it is very similar to $L(s) = P(s)C(\boldsymbol{\rho}_d, s)$ which is true, and the gain crossover frequency of the loop transfer function with $C(\boldsymbol{\rho}_d, j\omega)$ is $0.18\,\text{rad/s}$ which is specified and enough phase margin which is larger than 80° is ensured. Fig.12 shows the gain plots of the sensitivity functions $\hat{S}(j\omega)$ and $S(s)$ which are corresponding to the loop transfer functions $\hat{L}(j\omega)$ and $L(s)$ shown in Fig.11. It is inferable that $C(\boldsymbol{\rho}_d, s)$ has enough stability margin since the maximum value of $\hat{S}(j\omega)$ is seemed to be less than $6\,\text{dB}$.

Fig.13 shows the step reference responses. In the figure, $y_t(t)$ and $u_t(t)$ are the responses of the closed loop with $C(\boldsymbol{\rho}_d, s)$, and $y_0(t)$ and $u_0(t)$ are the responses of the closed loop with $C_0(s)$. In the simulations the responses were simulated without the measurement noise in order to make clear the difference of the responses by the controllers. The figure shows that the responses are improved by the controller designed by the method.

VI. Conclusion

A PID tuning method has been proposed. The method requires only one-shot closed-loop data for PID tuning in order to estimate the plant models for the partial model matching. Although the estimated plant models by the method are affected by the measurement noise and its restricted order and form, the frequency ranges where they are valid are also obtained with them. Therefore the plant models by the method are available to tune PID controllers with the partial model matching method.

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