Crack size estimation using model reduction and genetic algorithm

Brahim Benaissa, Idir Belaidi, Samir Khatir and Abderrachid Hamrani

***Abstract*— In this study we use the material elastic properties as a base. A tow dimensional cracked plate under traction is modelled by finite element method (FEM) then a reduced model is built using the proper orthogonal decomposition method (POD) with help of radial basis functions (RBF) method of interpolation. The crack length is estimated as an inverse identification problem, basing on the deformation obtained from the boundary nodes considered as sensor points. A genetic algorithm (GA) is used for the minimization of the error function which is expressed as the difference between displacement field of the boundaries caused by the crack size value proposed randomly by GA and the field measured at the actual identity. The approach presented accurate results and could guess the real crack size in a precession of 10^{-6} of the fitness function, proving its effectiveness even with a very low number of 4 sensors. To test the stability of the method against uncertainty a white noise was introduced later. The use of the reduced model provides tangible benefits mainly the very low computational cost.**

I. INTRODUCTION

Crack initiation and propagation is an omnipresent fact in all structures undergoing cyclic loads due to the fatigue phenomenon. In most cases, cracks are engaged in a predictable location. Thus maintenance measures give big importance to the crack size, trying to follow its state to prevent reaching the dangerous level.

There are several numerical methods for crack detection [1-3], which use different theoretical bases, thus many of these methods are dedicated to the invention using completely theoretical parameters, where the data are not accessible experimentally.

The existence of a crack changes the behavior of the plate when put under traction, therefore the deformation of the structure, which is also affected by the changes of the crack length. Benefiting from this effect for inverse crack size estimation, the deformation of the structure's border is measured using deformation sensors, positioned at indicated locations as shown in figure 1.

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Inverse problems are defined as the problems where the output is known and the input or source of output remains to be determined. They are contrary to the direct problems, in which output or response are determined using information from input [4]. In the case of the Inverse Elastostatics Problem (IESP) of internal flaw detection, the location, the orientation and the size of the flaw are unknown but the displacements along the boundaries are known. In order to analyze this kind of problems, the boundary displacements, usually called "experimental data", are obtained under known boundary conditions and compared with the calculated ones.

Inverse crack or void identification problems, can be stated as an optimization task. There are several optimization techniques summoned in [5]; the GA is the most popular evolutionary algorithms with a diverse range of applications. It has been employed in inverse crack identification method in [6-8].

The boundary element (BEM) and the finite element (FEM) methods are the two computational methods mainly used to obtain the displacement field. The FEM is a well-established procedure for structural analysis and has formed the basis of most early inverse methods [9]. On the other hand, the BEM has become a popular alternative [10], possessing many advantages, like meshing only on the boundaries and low computational effort. As the weak point of FEM based inverse methods in general is high computational cost, model reduction can be used to solve the FEM difficulties.

POD is a model reduction techniques proceed by the approximation of the problem solution using the appropriate set of approximation functions [11], which contributes to the huge acceleration of the procedure since, once a trained model is built, it computes the system response in a time shorter by about five orders of magnitude compared to FEM [12].

We introduce an inverse problem approach based on boundary measurement [13,14]. The proposed identification procedure is essentially the same as the one used in a traditional approach, except that the simulations required by optimization algorithm, are done with reduced model instead of FEM one. It is capable of estimating the crack's size using GA that compares at every iteration the calculated deformation data and one obtained from deformation sensor posed on the plate's borders.

II. POD-RBF AS A MODEL REDUCTION METHOD

The POD is a powerful statistical method for data analysis considered for model order reduction in many fields. A brief review is accessible in [15,16].In our study, the POD is used to determine the boundary deformation field of a two dimensional elastic structure in variable crack size scenarios, by extracting correlation from results of finite element (FE) simulations of the system with different crack parameter sets, this process is called the method of snapshots. The snapshot consist of the resulting displacement of the boundary's nodes, considered as sensor points data, these data, which are expected to be correlated are stored in matrix U.

$$U = \begin{bmatrix} u_1^1 & u_1^2 & \cdots & u_1^S \\ u_2^1 & u_2^2 & & u_2^S \\ \vdots & \vdots & \ddots & \vdots \\ u_N^1 & u_N^2 & \cdots & u_N^S \end{bmatrix} \quad (1)$$

Where $\boldsymbol{N}$ is the total number of used sensor points and $\boldsymbol{S}$ represent the number of snapshot vectors $\boldsymbol{U}_i$ or FEM simulations results as each simulation with different crack scenario. A matrix $\boldsymbol{P}$ stores the crack parameter set $\boldsymbol{P}_i$ of all simulations, which is in our study the crack's length.

The main purpose of POD is to propose a set $\boldsymbol{\Phi}$ of orthogonal vectors called POD basis vectors resembling the snapshot matrix $\mathbf{U}$ in an optimal way, by exploiting the expected correlation between the snapshots. represented by the linear relationship:

$$U = \Phi \cdot A \quad (2)$$

A is the matrix collecting the coefficients of the new basis combination known as amplitude matrix. Referring to the orthogonality of $\boldsymbol{\Phi}$ it can be computed from:

$$A = \Phi^T \cdot U \quad (3)$$

Optimal basis vectors are defined by the proper orthogonal decomposition (POD) [17,18]:

$$\Phi = U \cdot V \cdot \Lambda^{-1/2} \quad (4)$$

Where matrix $\boldsymbol{V}$ stores the corresponding normalized eigenvectors to the covariance matrix $\mathbf{C}$, and $\boldsymbol{\Lambda}$ is a diagonal matrix storing the eigenvalues of the same matrix:

$$C = U^T \cdot U \quad (5)$$

Due to the optimality of the new system $\boldsymbol{\Phi}$ constructed as a POD basis, a low dimensional approximation $\widehat{\boldsymbol{\Phi}}$ of high accuracy is extracted from it. This is known as the truncation of the POD basis and is accomplished by preserving only K knowing that K is very smaller than S columns of $\boldsymbol{\Phi}$ (POD directions) that correspond to the largest eigenvalues; consequently, according to Eq. (3), a simple matrix multiplication straight forwardly generates the K ×M matrix of amplitudes $\widehat{\mathbf{A}}$ consistent with the new truncated POD basis. Finally, a low-dimensional approximation of the snapshot matrix, or of a single snapshot, can be expressed as a linear transformation of amplitudes, given by:

$$\widehat{A} = \widehat{\Phi}^T \cdot U \quad (6)$$

Then,

$$U = \widehat{\Phi} \cdot \widehat{A} \quad (7)$$

Using RBF interpolation we can generalize different sets of parameters, not included in the initial selection $\boldsymbol{P}$. The amplitudes matrix $\boldsymbol{A}$ is defined by the combination of nonlinear interpolation functions of the parameter vector $\boldsymbol{P}$, gathered in the matrix $\boldsymbol{G}$. The unknown coefficients of this combination are gathered in a matrix $\boldsymbol{B}$:

$$A = B \cdot G \quad (8)$$

The interpolation functions are expressed by [19]:

$$g_i = g_i(|P - P_i|) = \frac{1}{\sqrt{|P-P_i|^2 + c^2}} \quad (9)$$

$\mathbf{P}_i$ is the parameter corresponding $\mathbf{U}_i$ (for i=1,2,…,S). The argument of the i-th RBF is the distance $|P - P_i|$ between its current parameter p and the reference parameter $\mathbf{P}_i$. $\boldsymbol{c}$ is known as the RBF smoothing factor. As a consequence of the normalization of vector $\boldsymbol{P}$, $\boldsymbol{c}$ is defined within this range 0 to 1. In general, larger values of $\boldsymbol{c}$ give better interpolation quality.

After the coefficient matrix $\boldsymbol{B}$ is evaluated, a low-dimensional model of (8) can be put in vector form:

$$a(P) = B \cdot g(P) \quad (10)$$

The equation (7) can be expressed as approximation of the snapshot $\boldsymbol{u}$ corresponding to a new parameter vector $\boldsymbol{P}$ by defining the amplitude vector as a function of parameters:

$$u(P) = \widehat{\Phi} \cdot a(P) \quad (11)$$

This model will now be known as the trained POD-RBF network; it is completely capable of reproducing the unknown boundary displacement field of the structure that corresponds to any crack parameters $\boldsymbol{P}$. It must be noted that extrapolation outside the range of $\boldsymbol{P}$ possibly lead to poor precision of the model. And if the knot points $\boldsymbol{P}_i$ or some of them are relatively close one to the other, the matrix $\boldsymbol{G}$ could be singular, which can be circumvented by reducing the $\boldsymbol{c}$ value.

III. Identification Algorithm

A. Genetic Algorithms

The genetic algorithm is a common optimization method that belongs to the class of evolutionary algorithms; it is widely used for different kinds of optimization problems in last decade [20].

In a genetic algorithm a population of feasible solutions called also individuals is randomly generated in the research domain, and evolve toward better solution in an iterative process inspired from the natural evolution, each individual has a set of properties or chromosomes represented in binary encoding or other encoding, and they are allowed to reproduce and cross among themselves in order to obtain solutions with better fitness. The fitness is the objective function value, as the objective function and the constraints play the role of the environment. The best individuals in the term of best fitness are given the higher probability of being chosen as parent to new feasible solutions, where the characteristics of the parents are combined in a way of exchanging chromosomes parts, leading to two new designs. Then a possibility of mutation is imposed on the resulting individuals, which is randomly changing the digits inside a randomly selected chromosome. These basic operators are used to pass to the next iteration, containing the next generation of the same size and with better fitness. This process is continued until stopping criterion is satisfied, commonly until a maximum number of generations is reached, or a satisfactory fitness value has been achieved [21].

B. Structure of the identification algorithm

1. Creation of a starting population of N individuals; Created in real encoding in a random generation. Each individual represent the crack length (s).
2. Evaluation of every individual; by introducing the proposed parameters to the trained POD-RBF network that produce a corresponding boundary displacement vector $u(P)$, and evaluate the fitness by calculating the error between the resulting vector and the measured deformation field cause by the real crack parameter $u(P_0)$ expressed as:

$$\begin{cases} F(P) = \frac{\|u(P_0) - u(P)\|^2}{\|u(P_0)\|^2} \\ F(P_{optimal}) = \min[F(P)] \end{cases} \quad (12)$$

3. If the maximum number of generations or a fitness level is reached, the algorithm is terminated, else continue.
4. Rank the population according their fitness. Then select a proportion for breeding a new generation. The top ranked are more likely to be selected.
5. Performance of the crossover operation to produce new individuals.
6. Mutation of a specified percentage of the resulting population.
7. Replacement of the old population by new generation and go to step 2.

IV. Results and Discussions

A. Problem description

A plane strain plate containing a single crack is considered, the 40 x 40 mm square plate with material constants: the Poisson's ratio ν = 0.3 and shear modulus G=1×105. The Plate subjected to a traction load (1mm displacement is imposed on both up and down sides) is simulated using FE commercial code ABAQUS where the external boundaries are discretized by means of 80 elements per edge to finally collect the displacement of the border's 320 nodes, for the construction of the reduced model, from 13 applications of crack length ***s*** belongs to the range 0 (no crack) to 12 mm. The identification method is implemented in MATLAB. Through a series of experiments, the following genetic parameters are chosen based on the accuracy of results: Population size: 100, Crossover rate: 0.8, Mutation rate: 0.01.

B. Optimized parameter

In the case of the elasticity, the crack is represented by a line segment. Thus, the parameter to be optimized is its length. As, in this study the position and the orientation angle in the plate are fixed, in the center of the plate by 0° angle, and the crack's size is considered.

C. Sensors

Since the proposed method relays on the nodal displacement, all boundary nodes are considered as sensor point. We consider data from sensors by obtaining the displacement results from the nodes chosen as sensors point. Figure 1 depicts an example of the controlled plate using 8 deformation sensors.
For the placement of the sensors, we found that better results are acquired when they are dispersed uniformly on both left and right sides, respecting the same distance between every sensor point.

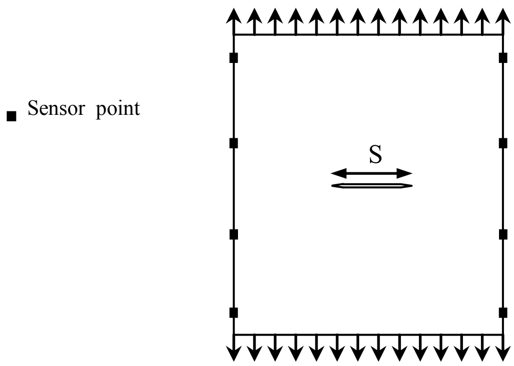

Figure 1. Geometric parameter of the crack.

D. Model reduction

The reduced model provides very precise results as shown in the next two figures. Where the Figure 2 demonstrates the identical results of the FEM and the POD models, and the Figure 3 display the error magnitude which is the absolute value of the deference between the two results, where the major error is in the order of 0.0001 mm.

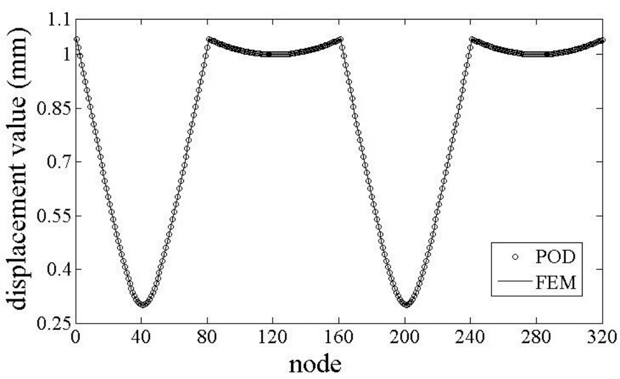

Figure 2. Boundary displacement field comparison calculated by FEM and POD.

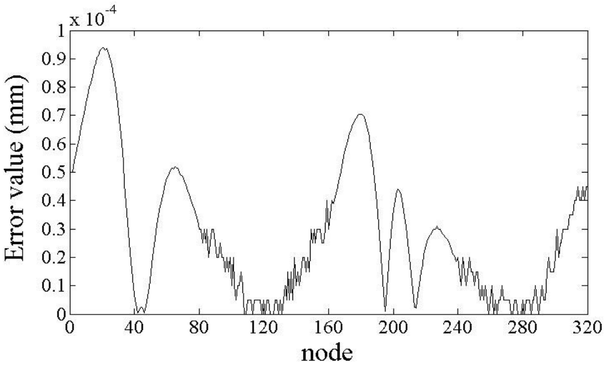

Figure 3. Efficiency of the proper orthogonal model.

E. Crack size estimation

The inverse problem solved by GA minimizing the cost function which is the deference between border's displacement of the plate caused by the crack we want to estimate its size and the one proposed randomly by GA using a Population size equal 100. Table.1 presents the normalized crack size estimation results for different application by means of sensor's number. Sensors number represents the quantity of sensors used knowing that it is spread on the left and right side of the plate. The first experience uses the data of all the nodes, later smaller sensor number is considered.

TABLE I. NORMALIZED RESULTS AND SOLUTION INFORMATION

Number of sensors	generations	Best fitness	Normalized result of size
230	3	2,5514E-06	0,9999784
24	2	1,022E-06	1,0000473
12	2	4,4396E-06	1,0002098
8	2	3,1268E-07	0,9999664
6	2	9,8813E-07	0,9999212
4	5	4,9845E-06	0,9995465

The approach presents could estimate the crack size in a precession of 10^{-06} of cost function, presenting high accuracy, even with a very low number of sensors, and shows that the boundary deformation is very practical data for crack identification problem.

F. Noise

In order to study the stability of the crack size identification algorithm to measurements uncertainty, some level of perturbation has been added to the exact input displacement from the 8 sensor point's example. The noise is modelled by the White Gaussian law with fixed standard deviations. Figures: 4, 5 and 6 shows the convergence to the results of three noise levels: 1%, 5% and 10% respectively, illustrating the performance of the

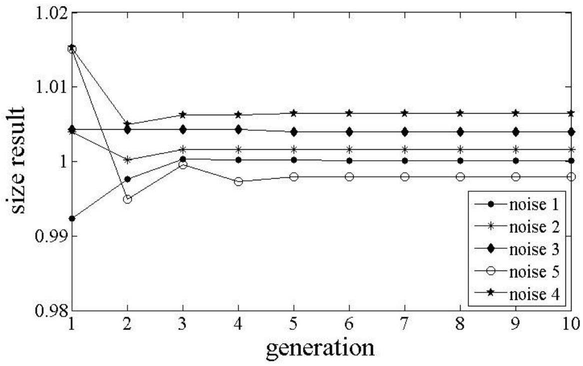

Figure 4. Crack size estimation in noise level 1%

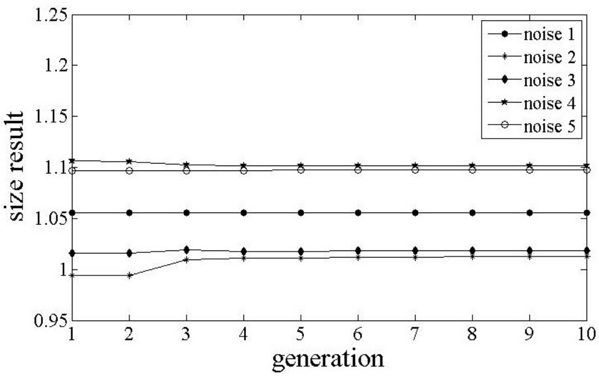

Figure 5. Crack size estimation in noise level 5%

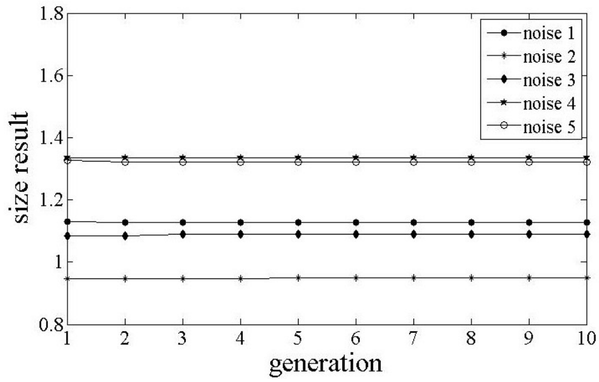

Figure 6. Crack size estimation in noise level 10%.

algorithm thought 5 applications in each level. A number of 10 generations is taken as a stopping criterion.

The variations obtained in the crack identity are in good agreement with the noise levels. It is noted that 1% of noise level does not affect the exactness of the results as shown in Figure 4. Giving an excellent average standard deviation equal to 0.003 at convergence, this is lower than the imposed perturbations rate of 0.01.

The results of noise level 5% is satisfactory, since the algorithm still can approximate the crack length as the average standard deviation for convergence is 0.042 which did not exceed 5%. Recognizing that, for the same configuration, the accuracy increases with the number of sensors. The perturbation for noise level 10% in the application of 8 sensors, the average standard deviation for convergence is high (0.163).

V. CONCLUSION

This numerical study has presented proper orthogonal decomposition as model reduction of cracked plate under traction after the finite element model of the structure was created for different crack lengths. Using the genetic algorithm for the optimization through boundary displacement data, the results have clearly shown that the developed algorithm is capable of predicting crack size accurately, and prove its effectiveness even with a very low number of sensors, and presented stability after a white noise in introduced to the input data, simulating measurement uncertainty.

The employment of the GA for the optimization task helps avoid imitations problems typical for the classical optimization methods. POD-RBF produced an accurate reduced model of the system, providing a low computational cost.

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