

Robust Stability of Inverse LQ Regulator for Neutral Systems with Time-Varying Delay

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Abstract—A method to construct an inverse LQ regulator of neutral systems with time-varying delay is known. Using it, the feedback gain is calculated with a solution of a finite dimensional LMI, and the feedback law is implemented without real time integral operation. In this paper, it is shown that the regulator obtained by the above method has good robustness property against some class of static nonlinear perturbations or dynamic linear perturbations as well as finite dimensional LQ regulators. The resulting system is evaluated with a numerical example.

I. INTRODUCTION

A system with time delay belongs to a class of infinite dimensional systems. To construct a linear quadratic (LQ) regulator of such a system, generally a solution of an infinite dimensional Riccati equation is needed, and the feedback law contains a real time integral operation which is not easy to implement [1], [2], [3]. On the other hand, a method to construct an inverse LQ regulator of systems with time delay of retarded type is known, where the designer cannot assign the weights in the cost functional previously, but he can construct it with a solution of a finite dimensional linear matrix inequality (LMI) and the resulting feedback law does not contain a real time integral operation [4]. Recently, this method of designing the inverse LQ regulator was extended to neutral systems with time-varying delay [5].

In this paper, it is shown that the inverse LQ regulator, constructed with the above method, of neutral systems with time-varying delay has good robustness property against some class of static nonlinear perturbations or dynamic linear perturbations in the input channel as well as finite dimensional LQ regulators. The technique used here is a natural extension of one by Safonov and Athans, who treated the robustness property of finite dimensional LQ regulator [6]. In Section 2, the problem is formulated. In Section 3, the robust stability is examined, and in Section 4, it is evaluated with a numerical example.

II. FORMULATION

As a plant, let us consider the following neutral system with a time-varying delay discussed in [7];

$$\begin{cases} \dot{x}(t) = A_0x(t) + A_1x(t-h(t)) + A_{-1}\dot{x}(t-L) \\ \quad + Bu(t), & t > 0, \\ x(t) = \phi(t), & t \in [-r, 0], \end{cases} \quad (1)$$

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where $x(t) \in \mathbb{R}^n$ is the state variable; $u(t) \in \mathbb{R}^m$ is the control input; A_0, A_1, A_{-1} and B are coefficient matrices with appropriate dimensions; $0 \leq L \in \mathbb{R}$ is a constant delay; $h(t) \in \mathbb{R}$ is a time-varying delay satisfying

$$0 \leq h(t) \leq h_u, \quad (2)$$

$$\dot{h}(t) \leq g < 1, \quad (3)$$

where h_u is a constant; r is defined to be $\max\{h_u, L\}$; g is defined to be the supremum of $\dot{h}(t)$; and the initial condition, $\phi(t)$, is a continuously differentiable initial function of $t \in [-r, 0]$.

Introducing an auxiliary variable $w(t) \in \mathbb{R}^n$, system (1) is rewritten as

$$\dot{w}(t) = A_0w(t) + A_1x(t-h(t)) + A_0A_{-1}x(t-L) + Bu(t), \quad (4)$$

$$x(t) = w(t) + A_{-1}x(t-L).$$

In order to construct an inverse LQ regulator, we assume that the plant parameters satisfy the following condition.

Condition 1: There exist

$$0 < P_0 = P_0^T \in \mathbb{R}^{n \times n},$$

$$0 < P_1 = P_1^T \in \mathbb{R}^{n \times n},$$

$$0 < P_2 = P_2^T \in \mathbb{R}^{n \times n},$$

$$0 < R = R^T \in \mathbb{R}^{m \times m}$$

such that

$$Q = \begin{bmatrix} Q_1 & \star & \star \\ Q_2 & Q_3 & \star \\ Q_4 & 0 & Q_5 \end{bmatrix} > 0 \quad (5)$$

where

$$Q_1 = -A_0^T P_0 - P_0 A_0 - P_1 - P_2 + P_0 B R^{-1} B^T P_0,$$

$$Q_2 = -A_{-1}^T (A_0^T P_0 + P_1 + P_2),$$

$$Q_3 = P_2 - A_{-1}^T (P_1 + P_2) A_{-1},$$

$$Q_4 = -A_1^T P_0,$$

$$Q_5 = (1-g)P_1,$$

and \star denotes a symmetric block in the matrix. ■

Using solutions P_0 and R of the above inequality (5), a feedback law is constructed as

$$u(t) = -R^{-1} B^T P_0 w(t) \quad (6)$$

which gives a closed-loop system

$$\begin{aligned}\dot{w}(t) &= (A_0 - BR^{-1}B^T P_0)w(t) + A_1x(t - h(t)) \\ &\quad + A_0A_{-1}x(t - L), \\ x(t) &= w(t) + A_{-1}x(t - L).\end{aligned}\quad (7)$$

The next lemma was shown in [5].

Lemma 1: Under Condition 1, $\hat{Q}(t)$ satisfies

$$\hat{Q}(t) = \begin{bmatrix} Q_1 & \star & \star \\ Q_2 & Q_3 & \star \\ Q_4 & 0 & \hat{Q}_5(t) \end{bmatrix} > 0 \quad (8)$$

where

$$\hat{Q}_5(t) = (1 - \dot{h}(t))P_1.$$

By using Lemma 1, the following lemmas about the asymptotic stability and the optimality of the resulting closed-loop system were shown in [5].

Lemma 2: [Asymptotic Stability of Inverse LQ Regulator] Under Condition 1, the closed-loop system (7) is asymptotically stable. ■

Lemma 3: [Optimality of Inverse LQ Regulator] Under Condition 1, the feedback law (6) is the optimal control minimizing the cost functional

$$J = \int_0^\infty \left\{ z^T(t)\hat{Q}(t)z(t) + u^T(t)Ru(t) \right\} dt \quad (9)$$

where

$$z(t) = [w^T(t) \quad x^T(t - L) \quad x^T(t - h(t))]^T.$$

In this paper, it is shown that the closed-loop system has good robustness property against some class of static nonlinear perturbations or dynamic linear perturbations in the input channel as well as finite dimensional LQ regulators.

III. ROBUST STABILITY

The robust stability of ordinary LQ regulators for systems without time delay can be quantitatively characterized in terms of the classical notions of gain and phase margin [6]. It is widely known that the LQ regulators have

- 1) infinite gain margin, 50% gain reduction tolerance, and
- 2) at least $\pm 60^\circ$ phase margin.

In this section, it is shown that the inverse LQ regulator for neutral systems with time-varying delay has equal robust stability with the ordinary finite dimensional LQ regulator by employing the techniques by Safonov and Athans [6].

First, let us consider a situation in which a static nonlinear perturbation is inserted in the input channel of the closed-loop system. Then the control input is rewritten as

$$u(t) = f[v(t)] \quad (10)$$

$$v(t) = -R^{-1}B^T P_0 w(t) \quad (11)$$

where $f(v)$ is a nonlinear function which gives a unique solution to the closed-loop system. The following theorem then holds.

Theorem 1: Under Condition 1, if $f(v)$ satisfies

$$\frac{1}{2}v^T(t)Rv(t) \leq v^T(t)Rf(v) \quad (12)$$

$$f(0) = 0, \quad (13)$$

then the closed-loop system with $f(v)$ inserted is asymptotically stable.

Proof: First, integrating (12) for any $\tau \geq 0$ and using (10),

$$\int_0^\tau \{2v^T(t)Ru(t) - v^T(t)Rv(t)\} dt \geq 0. \quad (14)$$

Next, we employ the following functional $V(t)$ in [5].

$$V(t) = V_0(t) + V_1(t) + V_2(t) \quad (15)$$

$$V_0(t) = w^T(t)P_0w(t) \quad (16)$$

$$V_1(t) = \int_{t-h(t)}^t x^T(\theta)P_1x(\theta)d\theta \quad (17)$$

$$V_2(t) = \int_{t-L}^t x^T(\theta)P_2x(\theta)d\theta. \quad (18)$$

Differentiating $V(t)$ by the time t along the solutions to (7), we have

$$\begin{aligned}\dot{V}_0(t) &= \dot{w}^T(t)P_0w(t) + w^T(t)P_0\dot{w}(t) \\ &= \{A_0w(t) + A_1x(t - h(t)) \\ &\quad + A_0A_{-1}x(t - L) + Bu(t)\}^T P_0w(t) \\ &\quad + w^T(t)P_0\{A_0w(t) + A_1x(t - h(t)) \\ &\quad + A_0A_{-1}x(t - L) + Bu(t)\} \\ &= z^T(t) \begin{bmatrix} A_0^T P_0 + P_0 A_0 & \star & \star \\ A_{-1}^T A_0^T P_0 & 0 & \star \\ A_1^T P_0 & 0 & 0 \end{bmatrix} z(t) \\ &\quad + u^T(t)B^T P_0w(t) + w^T(t)P_0Bu(t) \\ &= z^T(t) \times \\ &\quad \begin{bmatrix} A_0^T P_0 + P_0 A_0 - P_0 B R^{-1} B^T P_0 & \star & \star \\ A_{-1}^T A_0^T P_0 & 0 & \star \\ A_1^T P_0 & 0 & 0 \end{bmatrix} z(t) \\ &\quad + w^T(t)P_0 B R^{-1} B^T P_0 w(t) \\ &\quad + u^T(t)B^T P_0 w(t) + w^T(t)P_0 B u(t), \end{aligned}\quad (19)$$

$$\begin{aligned}\dot{V}_1(t) &= x^T(t)P_1x(t) \\ &\quad - (1 - \dot{h}(t))x^T(t - h(t))P_1x(t - h(t)) \\ &= \{w(t) + A_{-1}x(t - L)\}^T P_1\{w(t) + A_{-1}x(t - L)\} \\ &\quad - (1 - \dot{h}(t))x^T(t - h(t))P_1x(t - h(t)) \\ &= z^T(t) \times \\ &\quad \begin{bmatrix} P_1 & \star & \star \\ A_{-1}^T P_1 & A_{-1}^T P_1 A_{-1} & \star \\ 0 & 0 & -(1 - \dot{h}(t))P_1 \end{bmatrix} z(t), \end{aligned}\quad (20)$$

$$\dot{V}_2(t) = x^T(t)P_2x(t) - x^T(t - L)P_2x(t - L)$$

$$\begin{aligned}
&= \{w(t) + A_{-1}x(t-L)\}^T P_2 \{w(t) + A_{-1}x(t-L)\} \\
&\quad - x^T(t-L) P_2 x(t-L) \\
&= z^T(t) \begin{bmatrix} P_2 & \star & \star \\ A_{-1}^T P_2 & A_{-1}^T P_2 A_{-1} - P_2 & \star \\ 0 & 0 & 0 \end{bmatrix} z(t). \quad (21)
\end{aligned}$$

So

$$\begin{aligned}
\dot{V}(t) &= \dot{V}_0(t) + \dot{V}_1(t) + \dot{V}_2(t) \\
&= -z^T(t) \hat{Q}(t) z(t) + w^T(t) P_0 B R^{-1} B^T P_0 w(t) \\
&\quad + u^T(t) B^T P_0 w(t) + w^T(t) P_0 B u(t). \quad (22)
\end{aligned}$$

In addition, by using (10) and (11), the above equation gives

$$\begin{aligned}
\dot{V}(t) &= -z^T(t) \hat{Q}(t) z(t) + v^T(t) R v(t) \\
&\quad - v^T(t) R u(t) - u^T(t) R v(t) \\
&= -z^T(t) \hat{Q}(t) z(t) \\
&\quad - \{2v^T(t) R u(t) - v^T(t) R v(t)\}. \quad (23)
\end{aligned}$$

Integrating both sides from 0 to τ ,

$$\begin{aligned}
\int_0^\tau \dot{V}(t) dt &= - \int_0^\tau z^T(t) \hat{Q}(t) z(t) dt \\
&\quad - \int_0^\tau \{2v^T(t) R u(t) - v^T(t) R v(t)\} dt, \quad (24)
\end{aligned}$$

then

$$\begin{aligned}
V(0) &= V(\tau) + \int_0^\tau z^T(t) \hat{Q}(t) z(t) dt \\
&\quad + \int_0^\tau \{2v^T(t) R u(t) - v^T(t) R v(t)\} dt. \quad (25)
\end{aligned}$$

In the right hand side of the above equation, the first term is $V(\tau) > 0 \quad \forall \tau > 0$, and the third term is a positive value from (14). Therefore we have

$$V(0) \geq \int_0^\tau z^T(t) \hat{Q}(t) z(t) dt. \quad (26)$$

Although the left hand side is a constant decided by the initial value, the right hand side is monotone increasing from Lemma 1. This means that the limit of the right hand side of (26) exists as $\tau \rightarrow \infty$. So we have

$$V(0) \geq \int_0^\infty z^T(t) \hat{Q}(t) z(t) dt, \quad (27)$$

which means that

$$\lim_{t \rightarrow \infty} z^T(t) \hat{Q}(t) z(t) = 0, \quad (28)$$

and

$$\lim_{t \rightarrow \infty} x(t) = 0 \quad (29)$$

is obtained. \blacksquare

Next, let us consider a situation in which a stable dynamic linear perturbation is inserted in the input channel. Using a

stable transfer function matrix $\Xi(s)$, the control input is then rewritten as

$$u(t) = \int_0^t \xi(t-\tau) v(\tau) d\tau \quad (30)$$

$$v(t) = -R^{-1} B^T P_0 w(t), \quad (31)$$

where $\xi(t)$ is the impulse response matrix of $\Xi(s)$. The following theorem then holds.

Theorem 2: Under Condition 1, if $\Xi(s)$ satisfies

$$\Xi^*(j\omega) R + R \Xi(j\omega) - R \geq 0 \quad \forall \omega \in \mathbb{R}, \quad (32)$$

then the closed-loop system with $\Xi(s)$ inserted is asymptotically stable. $\Xi^*(\cdot)$ denotes here the complex-conjugate of $\Xi^T(\cdot)$.

The following lemma by Okada and Ikeda [8] gives the key to prove the above theorem.

Lemma 4: If inequality (32) is satisfied, then the following inequality holds:

$$\int_0^\tau \{2v^T(t) R u(t) - v^T(t) R v(t)\} dt \geq 0 \quad (33)$$

$$\forall \tau > 0.$$

Using this lemma, Theorem 2 can be proved.

Proof of Theorem 2: From Lemma 4, (33) holds. Using similar arguments to the proof of Theorem 1, we know equation (29). Hence the closed-loop system with stable $\Xi(s)$ inserted is asymptotically stable. \blacksquare

In addition to the above discussion, as a restricted class, we consider the following assumptions.

Assumption 1: Each input channel is mutually independent.

Assumption 2: The matrix R of the cost functional (9) is diagonal.

Under Assumption 1, a static nonlinear perturbation $f(v)$ is rewritten as

$$f(v) = \begin{bmatrix} f_1(v_1) \\ f_2(v_2) \\ \vdots \\ f_m(v_m) \end{bmatrix}, \quad (34)$$

where v_i is the i -th element of the vector v . Moreover, under Assumption 2, for the each element v_i of inequality (12),

$$\frac{1}{2} v_i^T(t) R v_i(t) \leq v_i^T(t) R f_i(v_i)$$

can be reduced to

$$\frac{1}{2} v_i^2(t) \leq v_i(t) f_i(v_i), \quad (35)$$

$$f_i(0) = 0. \quad (36)$$

As a special case, if f_i is a linear function, the above inequality is then equivalent to

$$f_i(v_i) = \beta_i v_i(t), \quad \beta_i \geq \frac{1}{2}. \quad (37)$$

This means that even though the perturbation of the control input reduces the original feedback gain to 50%, the perturbation does not destabilize the closed-loop system.

On the other hand, under Assumption 1, the matrix $\Xi(j\omega)$ is rewritten as

$$\Xi(j\omega) = \begin{bmatrix} \Xi_1(j\omega) & & 0 \\ & \ddots & \\ 0 & & \Xi_m(j\omega) \end{bmatrix}. \quad (38)$$

In addition, under Assumption 2, inequality (32) is reduced to

$$2 \times \Re[\Xi_i(j\omega)] \geq 1 \quad (i = 1, 2, \dots, m). \quad (39)$$

In particular, if each element of $\Xi(j\omega)$ is replaced by

$$\Xi_i(j\omega) = e^{j\phi_i(\omega)} \quad (i = 1, 2, \dots, m) \quad (40)$$

where ϕ_i is a phase shift, then inequality (39) gives

$$e^{j\phi_i} + e^{-j\phi_i} \geq 1 \quad (i = 1, 2, \dots, m) \quad (41)$$

or

$$|\phi_i| \leq 60^\circ. \quad (42)$$

This indicates that the phase shift $-60^\circ \leq \phi_i \leq 60^\circ$ of the control input does not destabilize the closed-loop system.

Thus, it is shown that the inverse LQ regulator proposed in [5] has the good robustness properties as well as the ordinary finite dimensional LQ regulator.

IV. DESIGN PROCEDURE

To construct the inverse LQ regulator, it is necessary to solve the bilinear matrix inequality (BMI) (5) of Condition 1. So a design procedure is proposed in [5] where a solution of BMI (5) can be found as a solution of some LMIs. In this section, we show an alternative method, which uses modified LMIs to obtain a solution of BMI (5), to construct the inverse LQ regulator. Here we assume that parameters of the system (4) satisfy the following condition.

Condition 2: Given scalars $0 < a \in \mathbb{R}$ and $1 < b \in \mathbb{R}$. There exist

$$\begin{aligned} 0 < S &= S^T \in \mathbb{R}^{n \times n} \\ 0 < T &= T^T \in \mathbb{R}^{m \times m} \end{aligned}$$

such that

$$\Psi = \begin{bmatrix} \Psi_1 & \star & \star \\ \Psi_2 & \Psi_3 & \star \\ \Psi_4 & 0 & \Psi_5 \end{bmatrix} > 0 \quad (43)$$

$$\tilde{\Psi} = \begin{bmatrix} cS & \star \\ SA_{-1}^T & S \end{bmatrix} > 0 \quad (44)$$

where

$$\begin{aligned} \Psi_1 &= -SA_0^T - A_0S - (a+b)S + BTB^T, \\ \Psi_2 &= -SA_{-1}^T \{A_0^T + (a+b)I\}, \\ \Psi_3 &= (b-1)S, \\ \Psi_4 &= -SA_1^T, \\ \Psi_5 &= a(1-g)S, \end{aligned}$$

and $c = (a+b)^{-1}$. ■

Now the following lemma holds.

Lemma 5: Using the solutions S and T satisfying Condition 2, the matrices P_0 , P_1 , P_2 and R can be calculated as

$$\begin{aligned} P_0 &= S^{-1}, \\ P_1 &= aS^{-1}, \\ P_2 &= bS^{-1}, \\ R &= T^{-1}, \end{aligned}$$

where these solutions are simultaneously satisfying Condition 1. ■

Proof: Applying Schur Complement, the LMI (44) and

$$\tilde{\Psi}_3 = S - SA_{-1}^T(a+b)S^{-1}A_{-1}S > 0 \quad (45)$$

are equivalent. Using the above inequality and LMI (43),

$$\Psi_0 = \Psi + \text{block-diag} \left\{ 0, \tilde{\Psi}_3, 0 \right\} > 0 \quad (46)$$

is obtained. Pre- and post-multiplying the matrix Ψ_0 by

$$S_b^{-1} = \text{block-diag} \{ S^{-1}, S^{-1}, S^{-1} \},$$

we have

$$S_b^{-1} \Psi_0 S_b^{-1} = Q,$$

so that $Q > 0$ from the inequality (46). ■

The solutions satisfying Condition 1 can be obtained to construct the feedback law

$$u(t) = -R^{-1}B^T P_0 w(t), \quad (47)$$

which yields the inverse LQ regulator. If Assumption 2 is to be held, it is significant to note that LMIs (43) and (44) must be solved as the matrix T is diagonal because the matrix R is calculated as T^{-1} .

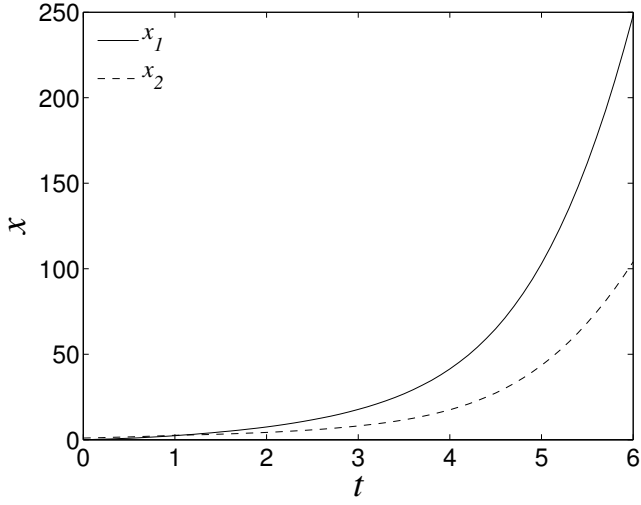


Fig. 1. Open-loop system

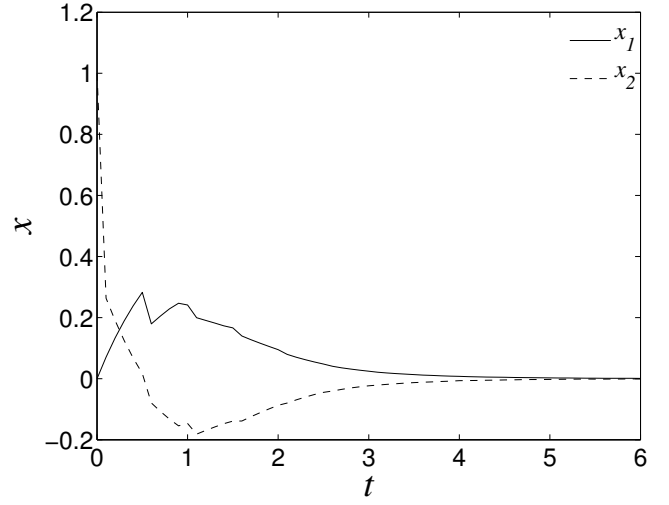


Fig. 3. Static nonlinear perturbation

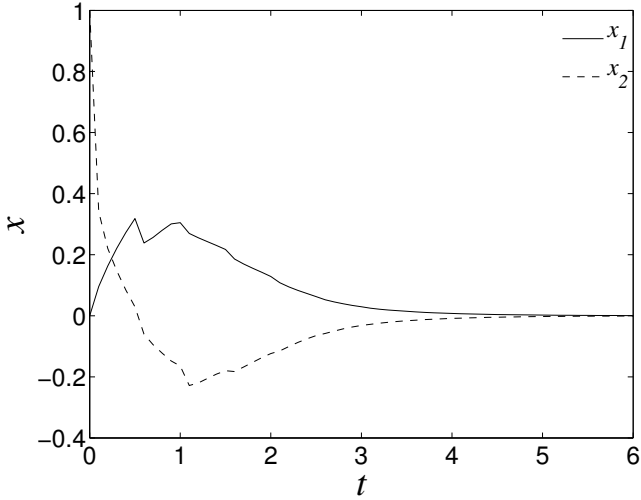


Fig. 2. Nominal closed-loop system

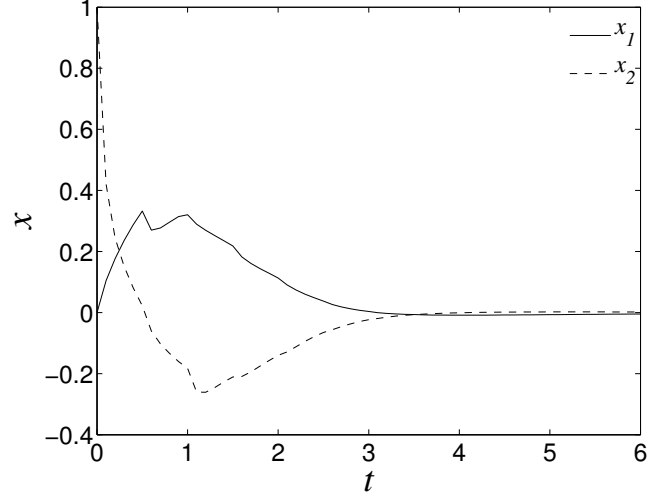


Fig. 4. Dynamic linear perturbation

V. NUMERICAL EXAMPLES

Let us consider a system with the following parameters

$$A_0 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, A_1 = \begin{bmatrix} 0.2 & 0.4 \\ 0.1 & 0.3 \end{bmatrix},$$

$$A_{-1} = \begin{bmatrix} 0.15 & 0.20 \\ 0.25 & 0.10 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

time-delays

$$L = 0.5, h(t) = 0.5 \sin(t) + 0.5,$$

and initial condition

$$\phi(t) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (-r \leq t \leq 0).$$

As the maximum of the time-delay, g is given as

$$g = \max_t \{ \dot{h}(t) \} = 0.5.$$

By using MATLAB to solve the LMIs (43) and (44), if we choose scalars as

$$a = 0.5, b = 1.1,$$

then solutions S and T are

$$S = \begin{bmatrix} 23.2406 & -26.9483 \\ -26.9483 & 34.5140 \end{bmatrix}, T = 89.8673.$$

So

$$P_0 = \begin{bmatrix} 0.4546 & 0.3550 \\ 0.3550 & 0.3061 \end{bmatrix}, R = 0.0111$$

is obtained. Then the feedback law is constructed as

$$u(t) = - [31.9003 \quad 27.5113] w(t). \quad (48)$$

The time responses of the

- 1) open-loop system
- 2) nominal closed-loop system

3) closed-loop system with a static nonlinear perturbation

$$f(v) = \sin v(t) + 5v^3(t) + 4v(t) \quad (49)$$

4) closed-loop system with a dynamic linear perturbation

$$\Xi(s) = \frac{2s + 1}{3s + 0.5} \quad (50)$$

are shown in Figure from 1 to 4.

The open-loop system for free response is unstable as Fig. 1 shows. Fig. 2 shows the time response of the nominal closed-loop system. So we have an asymptotically stable system applying the feedback law (48) for the system. Moreover, the figures 3 and 4 indicate that the resulting system remains robustly stable when these perturbations are inserted to the control input.

VI. CONCLUSIONS

In this paper, it was shown that the inverse LQ regulator for systems with a time-varying delay had good robustness properties in terms of gain and phase margin, in which the closed-loop system had infinite gain margin, 50% gain reduction tolerance and $\pm 60^\circ$ phase margin. As a result, the inverse LQ regulator had good robust stability as well as the ordinary finite dimensional LQ regulator. Finally, the

robustness of the inverse LQ regulator was evaluated with a numerical example.

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