VRFT-Based Controller Design for Nonlinear Systems with Block-Oriented Model Structures

Jyh-Cheng Jeng, Yi-Wei Lin, and Min-Wei Lee

***Abstract*—This paper presents a novel data-based controller design method for nonlinear systems based on the VRFT design framework. The control system is designed by considering the block-oriented representations of the nonlinear system, including the Hammerstein, Wiener, and Hammerstein-Wiener structures. In the proposed method, identification of a complete dynamic model of the nonlinear system is not required, whereas only the static nonlinearity, or its inverse, has to be estimated. Furthermore, the nonlinearity estimation and the controller design are performed simultaneously without the needs of iterative procedures or nonlinear optimization. Simulation studies confirm the effectiveness of the proposed controller design method for nonlinear systems.**

I. Introduction

Most dynamical systems can be better represented by nonlinear models, which are able to describe the global behavior of the system over wide ranges of operating conditions. One of the most frequently studied classes of nonlinear models is the so-called block-oriented nonlinear model [1], which involves a cascade combination of a linear dynamic block and a nonlinear static (memoryless) one. Such model is related very closely to linear one and can be easily adapted to linear control techniques. Two typical block-oriented model structures are Hammerstein and Wiener models. In the Hammerstein structure, the linear dynamic element, $G(z)$, is preceded by the static nonlinearity, $f(\cdot)$. The order of connection is reversed in the Wiener structure. A more general model structure is the Hammerstein-Wiener structure in which the linear dynamic element is placed between two nonlinear static functions, $f_1(\cdot)$ and $f_2(\cdot)$. Fig. 1 shows the structures of these block-oriented models, in which u and y are process input and output, respectively, whereas v and w are inaccessible intermediate variables. These model structures have been successfully used to describe nonlinear systems in a number of practical applications.

In the last decades, a considerable amount of research has been carried out on modeling, identification, and control of the nonlinear systems using the block-oriented representations. Traditional control design approaches are often based on mathematical models that approximate the behavior of the physical process. There are two steps in the model-based controller design: an empirical model of the process is

Jyh-Cheng Jeng is with the Department of Chemical Engineering and Biotechnology, National Taipei University of Technology, Taipei 106, Taiwan (corresponding author: e-mail: jcjeng@ntut.edu.tw).

Yi-Wei Lin is with the Department of Chemical Engineering and Biotechnology, National Taipei University of Technology, Taipei 106, Taiwan.

Min-Wei Lee is with the China Steel Corporation, Kaohsiung 812, Taiwan.

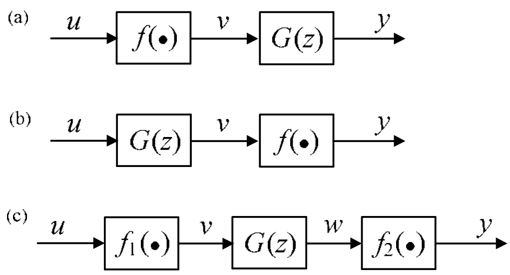

Figure 1. Schematic diagram of block-oriented nonlinear models: (a) Hammerstein, (b) Wiener , and (c) Hammerstein-Wiener models

identified first, which is subsequently used together with certain algorithms to design the controller. The identification process, however, usually relies on some prior assumptions such as model structure and order, which are often unavailable or subject to uncertainties. Hence, the complexity and modeling errors associated with such models increase the difficulty of the control design task, and may lead to degradation of control performance. In addition, because model identification and controller design are treated as two separated pieces of works, the identified model, depending on the identification technique used, may not contain adequate control-relevant information for controller design.

Data-based control design methods are very useful in many practical control applications, where obtaining a suitable model is a very difficult task. The virtual reference feedback tuning (VRFT) method [2] allows to directly design controllers using a set of process input-output data, without resorting to any process model. Most existing results on the VRFT design are however restricted to linear systems. Campi and Savaresi [3] explored the extension of VRFT to nonlinear systems, which however requires iterative procedure. Adaptive version of the VRFT design [4] was proposed for adaptive PID controller parameter tuning. Unlike the linear VRFT, the extended versions of VRFT to nonlinear systems are not one-shot methods so that a significant advantage of VRFT method is lost. This study aims to design controllers for nonlinear Hammerstein, Wiener, and Hammerstein-Wiener systems based on the one-shot (noniterative) VRFT design framework. A simple linearizing control scheme for these block-oriented systems using the inverse of nonlinearity and a PID controller is adopted. The control scheme results in an equivalent linear control system that enables the application of VRFT design method. The proposed method has two distinctive features: (1) identification of a complete block-oriented model of the nonlinear system is not required, whereas only the static nonlinearity, or its inverse, has to be estimated; (2) the nonlinearity estimation and the PID controller design are performed simultaneously without the needs of iterative procedures or nonlinear optimization.

II. Controller Design for Hammerstein Systems

The control scheme for Hammerstein system is as shown in Fig. 2(a) where $f^{-1}(\cdot)$ and $G_C(z)$ denote the (estimated) inverse of nonlinearity and the controller, respectively. This linearizing control scheme results in an equivalent linear control system shown in Fig. 2(b).

It is assumed that the nonlinear static function can be represented by the B-spline series expansion as

$$v_k = f\left(u_k\right) = \sum_{i=1}^{r} a_i B_{u,i}\left(u_k\right) \tag{1}$$

where $B_{u,i}(\bullet)$ are the (known) B-splines with a knot sequence defined on the operating range of u, and a_i are unknown coefficients. The nonlinear function can also be represented using other basis functions, but the B-splines are used in this study because the B-spline series is a local basis which has more flexibility in signal (function) representation [5], compared with using a global basis such as polynomials and Laguerre functions. Because the steady-state gain of a block-oriented nonlinear system can be arbitrarily distributed in nonlinear and linear blocks, it is assumed, without loss of generality, that $a_1 = 1$. In addition, assume that the variables are defined as deviation variables based on a steady-state operating point, so that $f(0) = 0$. Therefore, when the third-order B-splines with 0 as one of its knots are used, the following relation holds, for some (known) s:

$$a_s B_{u,s}\left(0\right) + a_{s+1} B_{u,s+1}\left(0\right) = 0 \tag{2}$$

or

$$a_s = \alpha\, a_{s+1} \quad \text{where} \quad \alpha = -\frac{B_{u,s+1}(0)}{B_{u,s}(0)} \tag{3}$$

Consequently, the inaccessible intermediate variable v_k can be expressed as

$$\begin{aligned} v_k &= B_{u,1}\left(u_k\right) + \sum_{i=2}^{s-1} a_i B_{u,i}\left(u_k\right) \\ &\quad + a_{s+1}\left[\alpha B_{u,s}\left(u_k\right) + B_{u,s+1}\left(u_k\right)\right] + \sum_{i=s+2}^{r} a_i B_{u,i}\left(u_k\right) \end{aligned} \tag{4}$$

The controller G_C is a PID controller with discrete transfer function given by

$$G_C(z) = K_P + \frac{K_I}{1-z^{-1}} + K_D\left(1-z^{-1}\right) \tag{5}$$

where K_P, K_I, and K_D are the PID parameters. The problem of controller design is to determine the PID parameters and the unknown coefficients a_i $(i = 2,\ldots,r)$, from an N-point data set $\{u_k, y_k\}_{k=1\sim N}$ of observed input-output measurements.

This study applies the VRFT method to the equivalent linear control system shown in Fig. 2(b), so that a model-reference problem, as depicted in Fig. 3, is to be solved. In Fig. 3, the reference model $T(z)$ describes the desired behavior of the closed-loop system, which is typically specified by the following first-order dynamics:

$$T(z) = \frac{\left(1-A\right)z^{-1}}{1-Az^{-1}} \tag{6}$$

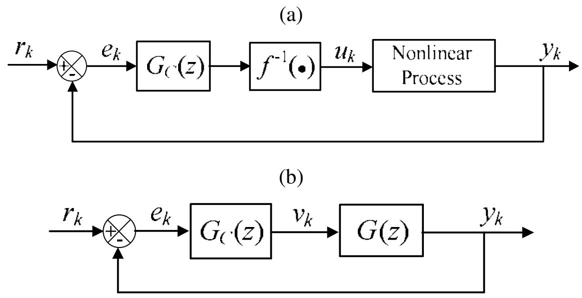

Figure 2. (a) Linearizing control scheme and (b) equivalent linear control system for nonlinear process with Hammerstein structure.

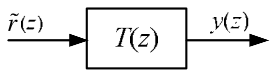

Figure 3. Reference model for controller design of Hammerstein system.

where A is a tuning parameter related to the speed of response. The design goal is to determine the PID parameters and the unknown coefficients a_i, such that the control system in Fig. 2(a) behaves as similarly as possible to the prespecified reference model $T(z)$. Based on the virtual reference signal $\tilde{r}(z) = T(z)^{-1}\, y(z)$, the virtual output of controller G_C can be calculated as

$$\begin{aligned} \tilde{v}(z) &= G_C(z)\left[\tilde{r}(z) - y(z)\right] \\ &= \left[K_P + \frac{K_I}{1-z^{-1}} + K_D\left(1-z^{-1}\right)\right]\frac{1-z^{-1}}{\left(1-A\right)z^{-1}}\, y(z) \end{aligned} \tag{7}$$

When the linear block is fed by v_k, it generates y_k. Therefore, a PID controller that shapes the closed-loop behavior to the reference model generates v_k when the error signal is given by $\tilde{r}_k - y_k$. The task of controller design then becomes minimizing the difference between v_k given in (4) and $\tilde{v}_k$ calculated from (7), or equivalently, minimizing the difference between $B_{u,1}(u_k)$ and x_k, where

$$\begin{aligned} x_k &= \tilde{v}_k - \sum_{i=2}^{s-1} a_i B_{u,i}\left(u_k\right) - a_{s+1}\left[\alpha B_{u,s}\left(u_k\right) + B_{u,s+1}\left(u_k\right)\right] \\ &\quad - \sum_{i=s+2}^{r} a_i B_{u,i}\left(u_k\right) \end{aligned} \tag{8}$$

Equation (8) can be written as $x_k = \boldsymbol{\psi}_k \boldsymbol{\theta}$, where

$$\begin{aligned} \boldsymbol{\psi}_k &= \left[\psi_{P,k} \quad \psi_{I,k} \quad \psi_{D,k} \quad -B_{u,2}\left(u_k\right) \cdots -B_{u,s-1}\left(u_k\right)\right. \\ &\quad \left. -\left(\alpha B_{u,s}\left(u_k\right) + B_{u,s+1}\left(u_k\right)\right) \; -B_{u,s+2}\left(u_k\right) \cdots -B_{u,r}\left(u_k\right)\right] \\ \boldsymbol{\theta} &= \left[K_P \quad K_I \quad K_D \quad a_2 \cdots a_{s-1} \quad a_{s+1} \quad a_{s+2} \cdots a_r\right]^T \\ \psi_{P,k} &= \frac{1}{1-A}\left(y_{k+1} - y_k\right); \quad \psi_{I,k} = \frac{1}{1-A}\, y_{k+1} \\ \psi_{D,k} &= \frac{1}{1-A}\left(y_{k+1} - 2y_k + y_{k-1}\right) \end{aligned} \tag{9}$$

Now, the parameter $\boldsymbol{\theta}$ is computed by solving the following minimization problem:

$$\min_{\boldsymbol{\theta}} \sum_{k=2}^{N-1}\left[B_{u,1}\left(u_k\right) - \boldsymbol{\psi}_k \boldsymbol{\theta}\right]^2 = \min_{\boldsymbol{\theta}} \left\|\boldsymbol{\varphi} - \boldsymbol{\Psi}\boldsymbol{\theta}\right\|_2^2 \tag{10}$$

where

$$\boldsymbol{\Psi} = \begin{bmatrix} \boldsymbol{\psi}_2^T & \boldsymbol{\psi}_3^T & \cdots & \boldsymbol{\psi}_{N-1}^T \end{bmatrix}^T$$
$$\boldsymbol{\varphi} = \begin{bmatrix} B_{u,1}(u_2) & B_{u,1}(u_3) & \cdots & B_{u,1}(u_{N-1}) \end{bmatrix}^T \tag{11}$$

The solution can be calculated using the least-squares technique given by

$$\hat{\boldsymbol{\theta}} = \left(\boldsymbol{\Psi}^T\boldsymbol{\Psi}\right)^{-1}\boldsymbol{\Psi}^T\boldsymbol{\varphi} \tag{12}$$

The PID parameters and estimates of the coefficients a_i ($i = 2,\ldots,s-1,s+1,\ldots,r$), can be obtained by partitioning the estimate $\hat{\boldsymbol{\theta}}$, according to the definition of $\boldsymbol{\theta}$ in (9). The coefficient a_s is then computed from (3). With the estimated nonlinear function, its inverse f^{-1} can be readily obtained.

III. Controller Design for Wiener Systems

The control scheme for Wiener system is as shown in Fig. 4(a), which results in an equivalent linear control system shown in Fig. 4(b). It is assumed that the inverse function of the static nonlinearity can be represented by the B-spline series expansion as

$$v_k = f^{-1}(y_k) = \sum_{i=1}^{r} a_i B_{y,i}(y_k) \tag{13}$$

where $B_{y,i}(\bullet)$ are the (known) B-splines with a knot sequence defined on the operating range of y. To meet the condition of $f^{-1}(0) = 0$, it is required, for some (known) s, that

$$a_s = \beta a_{s+1} \quad \text{where} \quad \beta = -\frac{B_{y,s+1}(0)}{B_{y,s}(0)} \tag{14}$$

Consequently, the inaccessible intermediate variable v_k can be expressed as

$$v_k = \sum_{i=1}^{s-1} a_i B_{y,i}(y_k) + a_{s+1}\left[\beta B_{y,s}(y_k) + B_{y,s+1}(y_k)\right] + \sum_{i=s+2}^{r} a_i B_{y,i}(y_k) \tag{15}$$

The problem of controller design is to determine the PID parameters and the unknown coefficients a_i ($i = 1,\ldots,r$) from an N-point input-output data set $\{u_k, y_k\}_{k=1\sim N}$.

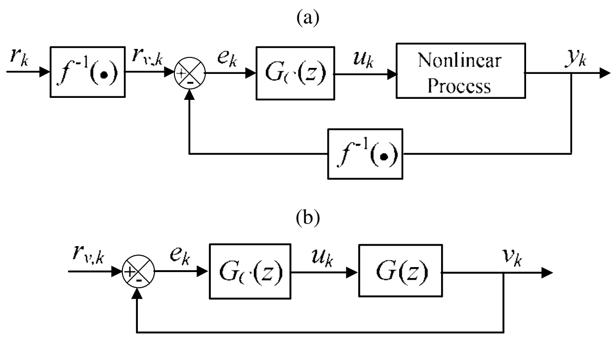

Figure 4. (a) Linearizing control scheme and (b) equivalent linear control system for nonlinear process with Wiener structure.

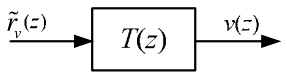

Figure 5. Reference model for controller design of Wiener system.

The VRFT method is applied to the equivalent linear control system shown in Fig. 4(b), so that a model-reference problem, as depicted in Fig. 5, is to be solved. The design goal is to determine the PID parameters and the unknown coefficients a_i, such that the control system in Fig. 4(a) behaves as similarly as possible to the prespecified reference model $T(z)$. Based on the virtual reference signal $\tilde{r}_v(z) = T(z)^{-1}v(z)$, the virtual output of controller G_C is calculated as

$$\begin{aligned} \tilde{u}(z) &= G_C(z)\left[\tilde{r}_v(z) - v(z)\right] \\ &= \left[K_P + \frac{K_I}{1-z^{-1}} + K_D\left(1-z^{-1}\right)\right]\frac{1-z^{-1}}{(1-A)z^{-1}}v(z) \end{aligned} \tag{16}$$

When the linear block is fed by u_k, it generates v_k. Therefore, a PID controller that shapes the closed-loop behavior to the reference model generates u_k when the error signal is given by $\tilde{r}_{v,k} - v_k$. The task of controller design then becomes minimizing the difference between the measured u_k and the $\tilde{u}_k$ calculated from (16). Substituting (15) into (16) yields $\tilde{u}_k = \boldsymbol{\psi}_k\boldsymbol{\theta}$, where

$$\begin{aligned} \boldsymbol{\psi}_k = \big[& \psi_{P1,k} \;\; \psi_{I1,k} \;\; \psi_{D1,k} \cdots \psi_{P(s-1),k} \;\; \psi_{I(s-1),k} \;\; \psi_{D(s-1),k} \\ & \beta\psi_{Ps,k} + \psi_{P(s+1),k} \;\; \beta\psi_{Is,k} + \psi_{I(s+1),k} \;\; \beta\psi_{Ds,k} + \psi_{D(s+1),k} \\ & \psi_{P(s+2),k} \;\; \psi_{I(s+2),k} \;\; \psi_{D(s+2),k} \cdots \psi_{Pr,k} \;\; \psi_{Ir,k} \;\; \psi_{Dr,k} \big] \\ \boldsymbol{\theta} = \big[& K_P a_1 \;\; K_I a_1 \;\; K_D a_1 \cdots K_P a_{s-1} \;\; K_I a_{s-1} \;\; K_D a_{s-1} \\ & K_P a_{s+1} \;\; K_I a_{s+1} \;\; K_D a_{s+1} \\ & K_P a_{s+2} \;\; K_I a_{s+2} \;\; K_D a_{s+2} \cdots K_P a_r \;\; K_I a_r \;\; K_D a_r \big]^T \\ \psi_{Pi,k} &= \frac{1}{1-A}\left[B_{y,i}(y_{k+1}) - B_{y,i}(y_k)\right] \\ \psi_{Ii,k} &= \frac{1}{1-A}B_{y,i}(y_{k+1}) \\ \psi_{Di,k} &= \frac{1}{1-A}\left[B_{y,i}(y_{k+1}) - 2B_{y,i}(y_k) + B_{y,i}(y_{k-1})\right] \end{aligned} \tag{17}$$

The parameter $\boldsymbol{\theta}$ is computed by solving the following minimization problem:

$$\min_{\boldsymbol{\theta}} \sum_{k=2}^{N-1}\left[u_k - \boldsymbol{\psi}_k\boldsymbol{\theta}\right]^2 = \min_{\boldsymbol{\theta}} \left\|\boldsymbol{\varphi} - \boldsymbol{\Psi}\boldsymbol{\theta}\right\|_2^2 \tag{18}$$

where

$$\boldsymbol{\Psi} = \begin{bmatrix} \boldsymbol{\psi}_2^T & \boldsymbol{\psi}_3^T & \cdots & \boldsymbol{\psi}_{N-1}^T \end{bmatrix}^T$$
$$\boldsymbol{\varphi} = \begin{bmatrix} u_2 & u_3 & \cdots & u_{N-1} \end{bmatrix}^T \tag{19}$$

The solution can be calculated using the least-squares technique given by

$$\hat{\boldsymbol{\theta}} = \left(\boldsymbol{\Psi}^T\boldsymbol{\Psi}\right)^{-1}\boldsymbol{\Psi}^T\boldsymbol{\varphi} \tag{20}$$

The problem is how to obtain the PID parameters and estimates of the unknown coefficients a_i ($i = 1,\ldots,s-1,s+1,\ldots,r$) from the estimate $\hat{\boldsymbol{\theta}}$ in (20). Define

$$\boldsymbol{\Theta} = \begin{bmatrix} K_P a_1 & \cdots & K_D a_{s-1} & K_D a_{s+1} & \cdots & K_D a_r \\ K_I a_1 & \cdots & K_D a_{s-1} & K_D a_{s+1} & \cdots & K_D a_r \\ K_D a_1 & \cdots & K_D a_{s-1} & K_D a_{s+1} & \cdots & K_D a_r \end{bmatrix} = \mathbf{K}\cdot\mathbf{a}^T \tag{21}$$

where

$$\mathbf{K}=\begin{bmatrix}K_P & K_I & K_D\end{bmatrix}^T$$
$$\mathbf{a}=\begin{bmatrix}a_1 & \cdots & a_{s-1} & a_{s+1} & \cdots & a_r\end{bmatrix}^T \tag{22}$$

An estimate $\hat{\mathbf{\Theta}}$ of the matrix $\mathbf{\Theta}$ can then be obtained from the estimate $\hat{\mathbf{\theta}}$. Let the economy-size SVD of $\hat{\mathbf{\Theta}}$ be given by

$$\hat{\mathbf{\Theta}}=\begin{bmatrix}U_1 & U_2 & U_3\end{bmatrix}\begin{bmatrix}\sigma_1 & 0 & 0\\ 0 & \sigma_2 & 0\\ 0 & 0 & \sigma_3\end{bmatrix}\begin{bmatrix}V_1 & V_2 & V_3\end{bmatrix}^T \tag{23}$$

Then, the closest, in the 2-norm sense, estimates of the parameter vectors **K** and **a** can be computed as [6]

$$\hat{\mathbf{K}}=U_1\,;\quad \hat{\mathbf{a}}=\sigma_1 V_1 \tag{24}$$

The coefficient a_s is then computed from (14). Note that this SVD-based algorithm intrinsically delivers estimates that satisfy the uniqueness condition $\|\hat{\mathbf{K}}\|_2 = 1$.

IV. Controller Design for Hammerstein-Wiener Systems

The control scheme for the Hammerstein-Wiener system is as shown in Fig. 6(a), which results in an equivalent linear control system shown in Fig. 6(b). The aforementioned algorithms for Hammerstein and Wiener systems can be combined to design the controller for a Hammerstein-Wiener system. It is assumed that the input nonlinear static function and the inverse of the output nonlinear static function can be represented by the B-spline series expansion as

$$v_k = f_1(u_k)=\sum_{i=1}^{r_1} a_{1,i}B_{u,i}(u_k) \tag{25}$$

$$w_k = f_2^{-1}(y_k)=\sum_{i=1}^{r_2} a_{2,i}B_{y,i}(y_k) \tag{26}$$

It is assumed, without loss of generality, that $a_{1,1} = 1$. To assure $f_1(0) = 0$ and $f_2^{-1}(0) = 0$, it is required respectively, for some (known) s_1 and s_2, that

$$a_{1,s_1}=\alpha\, a_{1,s_1+1};\quad a_{2,s_2}=\beta\, a_{2,s_2+1} \tag{27}$$

where

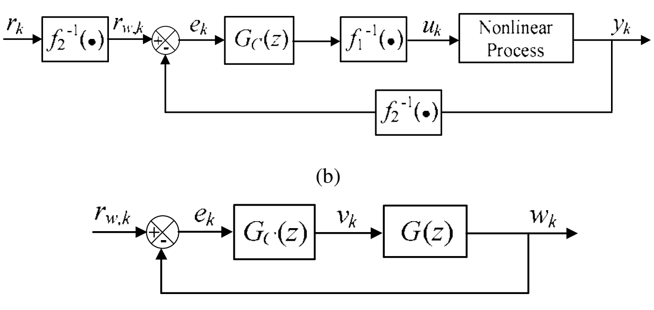

Figure 6. (a) Linearizing control scheme and (b) equivalent linear control system for nonlinear process with Hammerstein-Wiener structure.

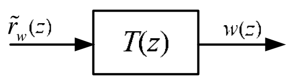

Figure 7. Reference model for controller design of Hammerstein-Wiener system.

$$\alpha=-\frac{B_{u,s_1+1}(0)}{B_{u,s_1}(0)};\quad \beta=-\frac{B_{y,s_2+1}(0)}{B_{y,s_2}(0)} \tag{28}$$

The problem of controller design is to determine the PID parameters and the unknown coefficients $a_{1,i}$ ($i = 2,\ldots,r_1$) and $a_{2,i}$ ($i = 1,\ldots,r_2$) from an N-point input-output data set $\{u_k, y_k\}_{k=1\sim N}$.

The VRFT method is applied to the equivalent linear control system shown in Fig. 6(b), so that a model-reference problem, as depicted in Fig. 7, is to be solved. The design goal is to determine the PID parameters and the unknown coefficients, $a_{1,i}$ and $a_{2,i}$, such that the control system in Fig. 6(a) behaves as similarly as possible to the prespecified reference model $T(z)$. Based on the virtual reference signal $\tilde{r}_w(z)=T(z)^{-1}w(z)$, the virtual output of controller G_C is calculated as

$$\begin{aligned}\tilde{v}(z)&=G_C(z)\left[\tilde{r}_w(z)-w(z)\right]\\ &=\left[K_P+\frac{K_I}{1-z^{-1}}+K_D\left(1-z^{-1}\right)\right]\frac{1-z^{-1}}{(1-A)z^{-1}}w(z)\end{aligned} \tag{29}$$

The task of controller design then becomes minimizing the difference between the v_k given in (25) and the $\tilde{v}_k$ calculated from (29), or equivalently, minimizing the difference between $B_{u,1}(u_k)$ and x_k, where

$$\begin{aligned}x_k=\tilde{v}_k-\sum_{i=2}^{s_1-1}a_{1,i}B_{u,i}(u_k)-a_{1,s_1+1}\left[\alpha B_{u,s_1}(u_k)+B_{u,s_1+1}(u_k)\right]\\ -\sum_{i=s_1+2}^{r_1}a_{1,i}B_{u,i}(u_k)\end{aligned} \tag{30}$$

Substituting (26) into (29), x_k given in (30) can be written as $x_k=\mathbf{\psi}_k\mathbf{\theta}$, where

$$\begin{aligned}\mathbf{\psi}_k=\big[&\psi_{P1,k}\ \psi_{I1,k}\ \psi_{D1,k}\cdots\psi_{P(s_2-1),k}\ \psi_{I(s_2-1),k}\ \psi_{D(s_2-1),k}\\ &\beta\psi_{Ps_2,k}+\psi_{P(s_2+1),k}\ \ \beta\psi_{Is_2,k}+\psi_{I(s_2+1),k}\ \ \beta\psi_{Ds_2,k}+\psi_{D(s_2+1),k}\\ &\psi_{P(s_2+2),k}\ \psi_{I(s_2+2),k}\ \psi_{D(s_2+2),k}\cdots\psi_{Pr_2,k}\ \psi_{Ir_2,k}\ \psi_{Dr_2,k}\\ &-B_{u,2}(u_k)\cdots-B_{u,s_1-1}(u_k)\ -\left(\alpha B_{u,s_1}(u_k)+B_{u,s_1+1}(u_k)\right)\\ &-B_{u,s_1+2}(u_k)\ \cdots\ -B_{u,r_1}(u_k)\big]\end{aligned} \tag{31}$$

$$\begin{aligned}\mathbf{\theta}=\big[&K_Pa_{2,1}\ K_Ia_{2,1}\ K_Da_{2,1}\cdots K_Pa_{2,s_2-1}\ K_Ia_{2,s_2-1}\ K_Da_{2,s_2-1}\\ &K_Pa_{2,s_2+1}\ K_Ia_{2,s_2+1}\ K_Da_{2,s_2+1}\\ &K_Pa_{2,s_2+2}\ K_Ia_{2,s_2+2}\ K_Da_{2,s_2+2}\cdots K_Pa_{2,r_2}\ K_Ia_{2,r_2}\ K_Da_{2,r_2}\\ &a_{1,2}\cdots a_{1,s_1-1}\ \ a_{1,s_1+1}\ \ a_{1,s_1+2}\cdots a_{1,r_1}\big]^T\end{aligned}$$

and $\psi_{Pi,k}$, $\psi_{Ii,k}$, and $\psi_{Di,k}$ are those given in (17). The parameter $\mathbf{\theta}$ is computed by solving the following minimization problem:

$$\min_{\mathbf{\theta}}\sum_{k=2}^{N-1}\left[B_{u,1}(u_k)-\mathbf{\psi}_k\mathbf{\theta}\right]^2=\min_{\mathbf{\theta}}\left\|\mathbf{\varphi}-\mathbf{\Psi}\mathbf{\theta}\right\|_2^2 \tag{32}$$

where

$$\mathbf{\Psi}=\begin{bmatrix}\mathbf{\psi}_2^{\ T} & \mathbf{\psi}_3^{\ T} & \cdots & \mathbf{\psi}_{N-1}^{\ T}\end{bmatrix}^T$$
$$\mathbf{\varphi}=\begin{bmatrix}B_{u,1}(u_2) & B_{u,1}(u_3) & \cdots & B_{u,1}(u_{N-1})\end{bmatrix}^T \tag{33}$$

The solution can be calculated using the least-squares technique given by

$$\hat{\boldsymbol{\theta}} = \left(\boldsymbol{\Psi}^T\boldsymbol{\Psi}\right)^{-1}\boldsymbol{\Psi}^T\boldsymbol{\varphi} \tag{34}$$

The estimates of the coefficients $a_{1,i}$ ($i=2,\ldots,s_1-1,s_1+1,\ldots,r_1$), can be directly obtained from the estimate $\hat{\boldsymbol{\theta}}$, according to the definition of $\boldsymbol{\theta}$ in (31). The PID parameters and estimates of the coefficients $a_{2,i}$ ($i=1,\ldots,s_2-1,s_2+1,\ldots,r_2$), can be obtained by the aforementioned SVD-based algorithm. The coefficient $a_{1,s1}$ and $a_{2,s2}$ are then computed from (27).

V. Simulation Examples

A. Example 1: Hammerstein System

Consider the following Hammerstein system:

$$f(u) = 1.5\left(1-e^{-0.5u}\right)|u|; \quad G(z) = \frac{0.008z^{-1}+0.0071z^{-2}}{1-1.683z^{-1}+0.698z^{-2}} \tag{35}$$

To design the control system by the proposed method, a set of 400 data points have been obtained (sampling time = 0.3) by feeding the system with uniformly random steps distributed in the range of [–2 3], as shown in Fig. 8. By specifying the tuning parameter of $A = 0.942$, the resulting PID parameters are $K_P = -0.222$, $K_I = -0.012$, and $K_D = -0.601$, and the estimated nonlinear function is plotted in Fig. 9. The estimated nonlinear function is scaled (by a factor of –5.01) to compare with the actual nonlinearity, as shown in Fig. 9. The result shows that the system nonlinearity was accurately estimated. For the purpose of comparison, the conventional PID controller was also designed based on the (linear) VRFT method using the same process data and tuning parameter, which resulted in $K_P = 0.737$, $K_I = 0.036$, and $K_D = 1.838$. Fig. 10 shows the closed-loop responses of the proposed nonlinear control system and the conventional PID control system to successive set-point changes. The control system designed by the proposed method reproduces the desired reference model with high accuracy for the set-point changes, whereas the conventional PID control system considerably deviates from the reference model for each set-point change.

B. Example 2: pH Neutralization Process

The pH neutralization process is important in various chemical processes such as wastewater treatment processes, biochemical processes, and polymerization processes. The proposed controller design methods for Wiener and Hammerstein-Wiener systems were applied to a pH process [7], which involves the neutralization of acetic acid (AcH), propionic acid (PrH), and sodium hydroxide (NaOH) in a single tank. Without buffering, the process exhibits a high degree of nonlinearity, causing the difficulty in control. The governing equations are

$$\begin{aligned}
\frac{dC_{\mathrm{AcH}}}{dt} &= \frac{1}{V}\left[q_a C_{0\mathrm{AcH}} - (q_a+q_b)C_{\mathrm{AcH}}\right] \\
\frac{dC_{\mathrm{PrH}}}{dt} &= \frac{1}{V}\left[q_a C_{0\mathrm{PrH}} - (q_a+q_b)C_{\mathrm{PrH}}\right] \\
\frac{dC_{\mathrm{NaOH}}}{dt} &= \frac{1}{V}\left[q_b C_{0\mathrm{NaOH}} - (q_a+q_b)C_{\mathrm{NaOH}}\right] \\
\frac{C_{\mathrm{AcH}}}{1+10^{\mathrm{pK_{AcH}-pH}}} &+ \frac{C_{\mathrm{PrH}}}{1+10^{\mathrm{pK_{PrH}-pH}}} + 10^{\mathrm{pH}-14} - C_{\mathrm{NaOH}} - 10^{-\mathrm{pH}} = 0
\end{aligned} \tag{36}$$

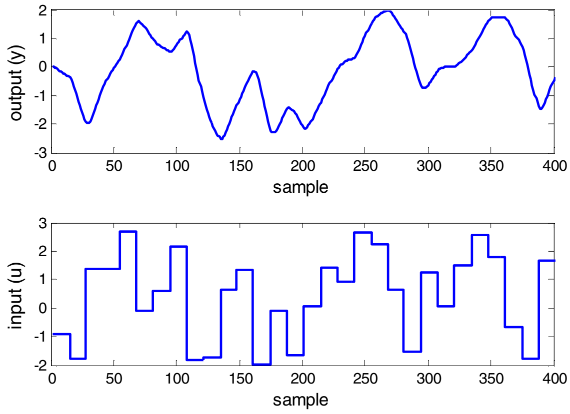

Figure 8. Process input-output data for example 1.

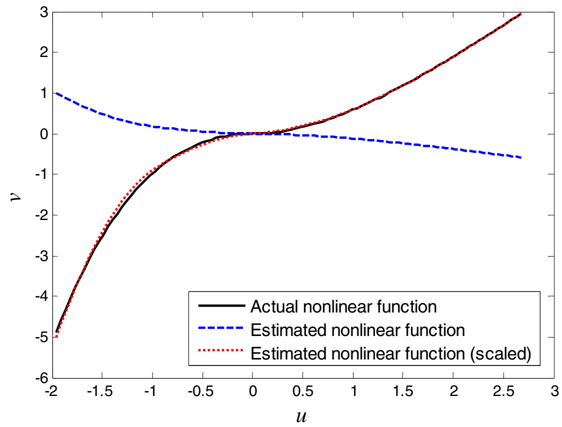

Figure 9. Process nonlinearity for example 1.

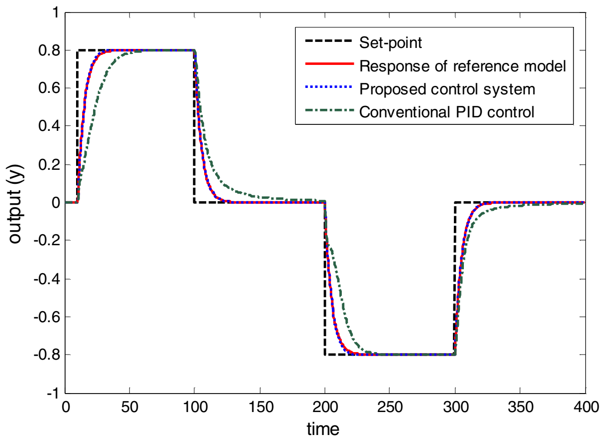

Figure 10. Closed-loop response for example 1.

where q_a and q_b are the flow rates of acidic and alkaline streams, V is the tank volume, and C denotes the concentration. The nominal operating conditions are $q_a = 0.0142$ L/s, $V = 1.0$ L, $C_{0\mathrm{AcH}} = 1$ M, $C_{0\mathrm{PrH}} = 1$ M, $C_{0\mathrm{NaOH}} = 2$ M, $\mathrm{pK_{AcH}} = 4.75$, $\mathrm{pK_{PrH}} = 4.87$. The objective is to control the pH of the effluent solution by manipulating the base flow rate q_b.

At the steady-state of $q_b = 0.0142$ L/s and pH = 9.407, a uniform random signal, with a maximum amplitude of ±50% of the nominal value, was introduced to the base flow q_b and the resulting pH was measured with a sampling time of 0.1 s, as shown in Fig. 11. A set of 1000 data points were used for the proposed controller design, with a tuning parameter $A = 0.905$.

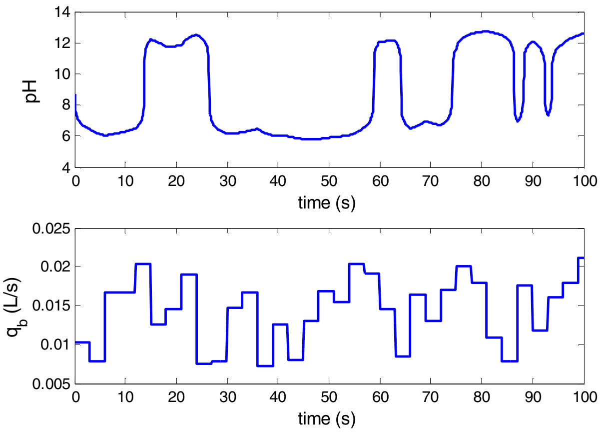

Figure 11. Process input-output data for the pH neutralization process.

By modeling the pH process as a Wiener system, the resulting PID parameters are $K_P = -0.999$, $K_I = -0.003$, and $K_D = 0.0474$, and the estimated inverse of the nonlinearity is plotted in Fig. 12. By modeling the pH process as a Hammerstein-Wiener system, the resulting PID parameters are $K_P = -0.999$, $K_I = -0.0028$, and $K_D = 0.0445$, and the estimated input nonlinearity, f_1, and inverse of the output nonlinearity, f_2^{-1}, are plotted in Fig. 13. For the purpose of comparison, the conventional PID controller was also designed based on the (linear) VRFT method using the same process data and tuning parameter, which resulted in $K_P = 6.4{*}10^{-4}$, $K_I = 1.95{*}10^{-5}$, and $K_D = -3.0{*}10^{-4}$. Fig. 14 compares the closed-loop performance of the proposed nonlinear control systems and the conventional PID control system to successive set-point changes. Both of the nonlinear control systems exhibit better and more consistent responses than the PID control system for these set-point changes because they effectively compensate the process nonlinearity. However, the conventional PID controller shows different dynamic behaviors for the different set-point values due to the nonlinearity. The nonlinear control system designed based on the Hammerstein-Wiener modeling shows the best control performance because more degree of freedom is used for modeling and control of the process.

VI. Conclusion

In this paper, VRFT-based methods for controller design of nonlinear systems modeled by the Hammerstein, Wiener, and Hammerstein-Wiener structures have been presented. The PID controller parameters and process nonlinearities are simultaneously obtained based directly on a set of plant data. This is in sharp contrast to the model-based design methods that require identifying an approximate process model, which is often difficult and subject to modeling errors, before controller design. In the proposed algorithms, combining the B-splines representation of the nonlinearity with the VRFT framework allows putting the system in linear regressor form, so that least-squares techniques can be used to design the controller, which avoids implementation problems due to computational limitations. The superiority of the proposed nonlinear control design over linear control has been illustrated using a Hammerstein system and a pH neutralization process.

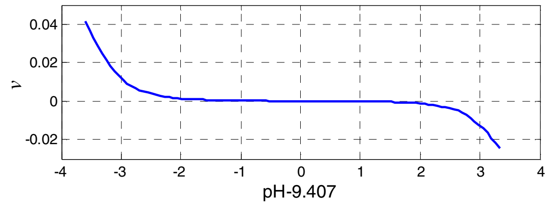

Figure 12. Process nonlinearity using Wiener modeling.

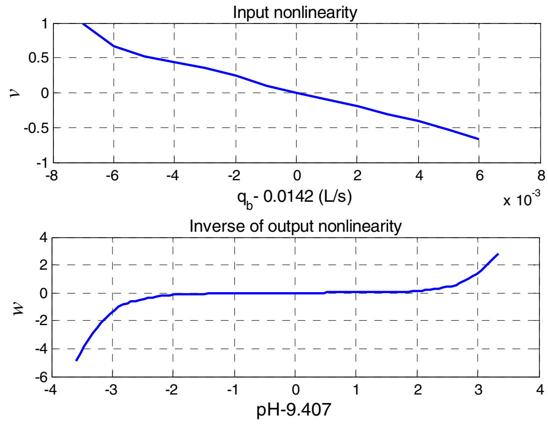

Figure 13. Process nonlinearities using Hammerstein-Wiener modeling.

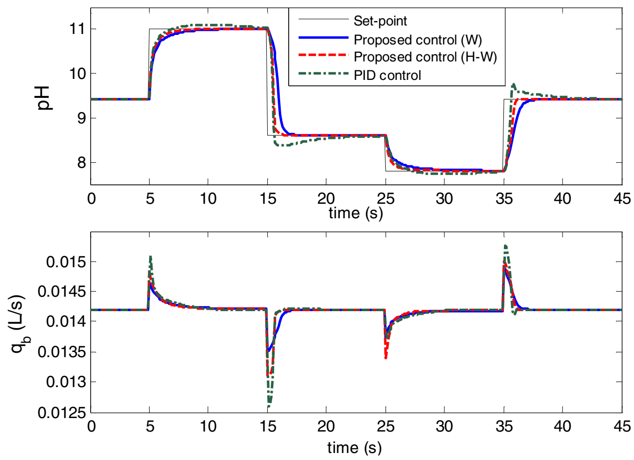

Figure 14. Closed-loop response for the pH neutralization process.

References

[1] R. Pearson, and M. Pottmann, "Gray-box identification of block-oriented nonlinear models," *J. Process Control*, vol. 10, pp. 301–315, 2000.

[2] M. C. Campi, A. Lecchini, and S. M. Savaresi, "Virtual reference feedback tuning: a direct approach for the design of feedback controllers," *Automatica*, vol. 38, pp. 1337–1346, 2002.

[3] M. C. Campi and S. M. Savaresi, "Direct nonlinear control design: the virtual reference feedback tuning (VRFT) approach," *IEEE Trans. Automatic Control*, vol. 51, pp. 14–27, 2006.

[4] X. Yang, Y. Li, Y. Kansha, and M. S. Chiu, "Enhanced VRFT design of adaptive PID controller," *Chem. Eng. Sci.*, vol. 76, pp. 66–72, 2012.

[5] J. C. Jeng, "Closed-loop identification of multivariable systems, using B-spline series expansions for step responses," *Ind. Eng. Chem. Res.*, vol. 51, pp. 2376–2387, 2012.

[6] J. C. Gómez, and E. Baeyens, "Identification of block-oriented nonlinear systems using orthonormal bases," *J. Process Control*, vol. 14, pp. 685–697, 2004.

[7] M. C. Palancar, J. M. Aragon, and J. S. Torrecilla, "pH-control systems based on artificial neural networks," *Ind. Eng. Chem. Res.*, vol. 37, pp. 2729-2740, 1998.