A frequency domain PID controller design method for integrating processes with dead-time in modified Smith predictor

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***Abstract*— In this paper a new PID controller design method in a modified Smith predictor configuration is proposed for integrating processes with dead time. A two-degree-of freedom control scheme has been considered where first a controller is designed to achieve the desired set-point response and then the load-disturbance rejecting controller is computed. Both the controllers are designed using the direct synthesis approach in frequency domain. The obtained Smith predictor controllers are converted into PID form by frequency response matching of the respective controllers at two low frequency points. The efficacy of the method is demonstrated through simulation of examples taken from the literature and the performance is compared with some of the prevailed methods.**

I. Introduction

The Proportional-Integral-Derivative (PID) controllers with its variants are widely accepted in the process industry due to its simplicity, easy applicability and capability of controlling processes having different dynamics [1]. The Smith predictor control scheme [2] is specially designed to work for processes with dead-time where the controller is designed with consideration of the delay-free part of the process. For integrating processes with dead-time the set-point performance of the Smith predictor is good but load-disturbance rejection is poor [3,4]. Astrom et al. [5] proposed a modified Smith predictor scheme with two-degree of freedom to consider both the set-point and load-disturbance performances separately. Numerous modified Smith predictor schemes for integrating processes with dead-time have been proposed in the literature to achieve better closed-loop performance [6], [7], [8], [9], [10].

Rao et al. [9] proposed modified Smith predictor for low order integrating processes with dead-time where set-point controller is designed using the direct synthesis approach and load-disturbance controller is designed using optimal gain and phase margin approach and set-point weighting is also considered for improving the set-point response. Uma et al. [10] extended this work where set-point controller is PID with lag filter and disturbance rejection controller is designed as PD with lead-lag structure and the two controllers are designed using direct synthesis approach. For improvement of the set-point response set-point weighting is used and for improvement of the load-disturbance response an additional first order filter is used in the feedback path.

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In this paper a PID control scheme in a two-degree-of-freedom Smith predictor configuration has been proposed for integrating processes with dead time. Considering the desired set-point and load-disturbance models the set-point controller and the load-disturbance controller, respectively, are obtained through the direct synthesis approach. These controllers are further approximated to PID form using frequency response matching at two low frequency points. A simple and useful criterion has been provided for selection of these low frequency points. The method involves solution of linear algebraic equations. The effectiveness of the proposed method is demonstrated through simulation of various examples taken from the literature.

The rest of the paper is organized as follows. The design method is discussed in detail in Section 2 and demonstrated through simulation of various examples in Section 3. Conclusion is given in Section 4.

II. The Design Method

The modified Smith predictor proposed by Astrom [5] as shown in Figure 1 is considered where, $G_P(s)$ is the process, $G_m(s)$ is delay-free part of the process model, L is the time delay, $G_c(s)$ is the set-point controller, $G_d(s)$ is the load-disturbance controller, r is the input, d is the disturbance and y is the output.

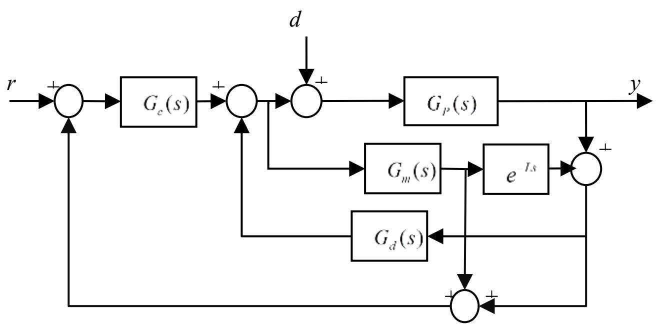

Figure 1: Modified Smith predictor.

For perfect modeling of the process, i.e., $G_P(s) = G_m(s)e^{-Ls}$, the transfer function of set-point response from r to y may be written as

$$G_{r,y}(s) = \frac{G_c(s)G_m(s)e^{-Ls}}{1 + G_c(s)G_m(s)} \tag{1}$$

and transfer function for load-disturbance response from d to y may be written as

$$G_{d,y}(s)=\frac{[1+G_c(s)G_m(s)-G_c(s)G_m(s)e^{-Ls}]G_m(s)e^{-Ls}}{[1+G_c(s)G_m(s)][1+G_d(s)G_m(s)e^{-Ls}]} \quad (2)$$

From the Equations (1) and (2), it can be seen that the set-point response depends only on the controller $G_c(s)$ whereas the load-disturbance response depends on both the controllers $G_c(s)$ and $G_d(s)$. Hence, the controller $G_c(s)$ may be designed first independent of $G_d(s)$ and then the controller $G_d(s)$ will be designed to achieve improved load-disturbance response. The design method is based on direct-synthesis approach for set-point as well as load disturbance responses for which the desired closed-loop transfer functions for set-point response and load disturbance response are selected as $M_{r,y}(s)$ and $M_{d,y}(s)$, respectively. The desired reference model transfer functions for set-point as well as load-disturbance responses are required to include non-minimum phase zero of plant and dead-time, if any. Further, for the choice of the desired closed-loop transfer function $M_{d,y}(s)$, it is required to have one zero at origin for rejection of load-disturbance.

For the design of the controllers, respective transfer functions will be equated as given by

$$G_{r,y}(s)=M_{r,y}(s) \quad (3)$$

and

$$G_{d,y}(s)=M_{d,y}(s) \quad (4)$$

which gives the controllers as

$$G_c(s)=\frac{M_{r,y}(s)}{G_m(s)e^{-Ls}-M_{r,y}(s)G_m(s)} \quad (5)$$

and

$$G_d(s)=\frac{1}{G_m(s)e^{-Ls}}\left[\frac{(1+G_c(s)G_m(s)-G_c(s)G_m(s)e^{-Ls})}{[(1+G_c(s)G_m(s))]M_{d,y}(s)}-1\right] \quad (6)$$

From Equations (5) and (6) it is observed that the two controllers may not be practically implementable. Hence, PID controllers $G_c^{PID}(s)$ and $G_d^{PID}(s)$ are to be computed from the controllers $G_c(s)$ and $G_d(s)$, respectively, with the following form

$$K_P+\frac{K_I}{s}+K_D s \quad (7)$$

where, K_P, K_I and K_D are proportional, integral and derivative gains, respectively. First, the set-point controller $G_c^{PID}(s)$ is determined and then, the load-disturbance controller $G_d^{PID}(s)$ is evaluated. To get the $G_c^{PID}(s)$ from $G_c(s)$, the frequency responses of the two controllers are matched and may be written as

$$G_c^{PID}(s)\big|_{s=j\omega}\cong G_c(s)\big|_{s=j\omega} \quad (8)$$

where, the left-hand side (LHS) expression is equivalent with the right-hand side (RHS) expression in terms of frequency response. We may write

$$G_{cr}^{PID}(\omega)+jG_{ci}^{PID}(\omega)=G_{cr}(\omega)+jG_{ci}(\omega) \quad (9)$$

where,

$$G_c^{PID}(s)\big|_{s=j\omega}=G_{cr}^{PID}(\omega)+jG_{ci}^{PID}(\omega)$$

$$\text{and } G_c(s)\big|_{s=j\omega}=G_{cr}(\omega)+jG_{ci}(\omega)$$

Separating the real and the imaginary parts in Equation (9), one may write

$$G_{cr}^{PID}(\omega)\cong G_{cr}(\omega) \text{ and } G_{ci}^{PID}(\omega)\cong G_{ci}(\omega) \quad (10)$$

In order to have the equivalence of two real functions, $G_{cr}(\omega)$ and $G_{ci}(\omega)$ with their approximants $G_{cr}^{PID}(\omega)$ and $G_{ci}^{PID}(\omega)$, respectively one may equate appropriate number of initial few terms of the corresponding Taylor series expansions about $\omega=0$. Thus, to accomplish approximate matching of the LHS functions in Equation (9) with the corresponding functions on the RHS, initial N derivatives of the corresponding functions are equated at $\omega=0$ to give

$$\frac{d^k}{d\omega^k}\left[G_{cr}^{PID}(\omega)\right]\bigg|_{\omega=0}=\frac{d^k}{d\omega^k}\left[G_{cr}(\omega)\right]\bigg|_{\omega=0} \quad (11)$$

$$\frac{d^k}{d\omega^k}\left[G_{ci}^{PID}(\omega)\right]\bigg|_{\omega=0}=\frac{d^k}{d\omega^k}\left[G_{ci}(\omega)\right]\bigg|_{\omega=0} \quad (12)$$

where $k\in[0,N-1]$

The above derivative relations can be simplified to the following algebraic relations using the divided difference calculus as shown in [11]

$$G_{cr}^{PID}(\omega)\big|_{\omega=\omega_k}=G_{cr}(\omega)\big|_{\omega=\omega_k};\quad k\in[0,N-1] \quad (13)$$

$$\text{and, } G_{ci}^{PID}(\omega)\big|_{\omega=\omega_k}=G_{ci}(\omega)\big|_{\omega=\omega_k};\quad k\in[0,N-1] \quad (14)$$

where ω_k are small positive values around $\omega=0$.

It is clear from Equations (13) and (14) that N values of ω give $2N$ linear algebraic equations with the unknown parameters. For 3 unknowns of the PID controller N is at least equal to 2 and for two low frequency points ω_0 and ω_1 the following expression is obtained.

$$\boldsymbol{A}\bar{x}=\bar{b} \quad (15)$$

where,

$$\boldsymbol{A}=\begin{bmatrix}1 & 0 & 0\\ 0 & -\frac{1}{\omega_0} & \omega_0\\ 1 & 0 & 0\\ 0 & -\frac{1}{\omega_1} & \omega_1\end{bmatrix};\quad \bar{x}=\begin{bmatrix}K_P & K_I & K_D\end{bmatrix}^T; \text{ and}$$

$$\bar{b}=\begin{bmatrix}G_{cr}(\omega_0) & G_{ci}(\omega_0) & G_{cr}(\omega_1) & G_{ci}(\omega_1)\end{bmatrix}^T$$

Directly from Equation (15), we get two values of K_P as:

$$K_{P1}=G_{cr}(\omega_0);\ K_{P2}=G_{cr}(\omega_1)$$

It is observed from various examples, that $K_{P1}\approx K_{P2}$ (as ω_0 and ω_1 are very close to each other) and we may take the value of K_P as any one of K_{P1} or K_{P2} or an average of these.

To evaluate K_I and K_D, Equation (15) may be simplified as

$$\boldsymbol{A}_1 \bar{x}_1 = \bar{b}_1 \quad (16)$$

where,

$$\boldsymbol{A}_1 = \begin{bmatrix} -\frac{1}{\omega_0} & \omega_0 \\ -\frac{1}{\omega_1} & \omega_1 \end{bmatrix}; \quad \bar{x}_1 = \begin{bmatrix} K_I & K_D \end{bmatrix}^T; \text{ and}$$

$$\bar{b}_1 = \begin{bmatrix} G_{ci}(\omega_0) & G_{ci}(\omega_1) \end{bmatrix}^T$$

Then, solution of Equation (16) determines K_I and K_D. Thus, the parameters of the set-point controller $G_c^{PID}(s)$ are evaluated.

Similarly, the PID controller $G_d^{PID}(s)$ for the desired load-disturbance response is obtained from $G_d(s)$ by frequency response matching of the two controllers as

$$G_d^{PID}(s)\Big|_{s=j\omega} \cong G_d(s)\Big|_{s=j\omega} \quad (17)$$

Following the procedure through the Equations (8) to (16) the parameters of the $G_d^{PID}(s)$ is determined. Thus, the two PID controllers for the set-point as well as the load-disturbance responses are evaluated.

Generally, the industrial processes show dominantly low-pass dynamics in the frequency responses. For such cases, the low frequency region is more important for steady state response. With this consideration both the frequency points are selected very small values. The small values of frequency points are chosen at around 1% of the bandwidth frequency of the desired reference model, where bandwidth may be treated as an indication to the effective range of frequency response. Such frequency points for matching have been observed through simulation to give good result for most of the processes.

For the purpose of frequency response matching Taylor series expansion around $\omega = 0$ and then divided difference calculus have been proposed. With this, computation has been simplified and reduced significantly. In this way, non-consideration of higher derivative terms in Taylor series expansion might be compensated by reduced computational error due to less amount of computation of the simpler algebraic equations.

Moreover, both the frequency points for matching being very small, the low frequency region ($\omega \to 0$) is emphasized which ensures the desired steady-state response.

With these considerations, the design procedure maintains the stability for the stable reference model and results in acceptable responses even for the perturbed process as shown in the simulation.

III. Simulation Results

Two examples of integrating processes taken from the literature are considered for simulation study. The processes, the reference models and the proposed controllers are shown in Table 1. The performance comparisons of the proposed controllers along with some of the prevailed methods [9], [10], [12] are shown in Table 2.

For Example 1 the unit step input is applied at time 0 for set-point response and a negative step input of magnitude 0.1 for load-disturbance response is applied at 100 sec. The process outputs and the controller outputs are shown in Figures 2 and 3, respectively. For robustness study +10% change in the dead-time is considered and the process outputs are shown in Figure 4.

In Example 2 the unit step input is applied at 0 sec for set-point response and for load-disturbance response a negative step input of magnitude 0.5 is applied at 20 sec. The process outputs and the controller outputs are shown in Figures 5 and 6, respectively. To show the robustness $+10\%$ changes in the process gain, the dead-time, the time-constant of the zero and a -10% change in the time-constant of the pole are considered simultaneously and the corresponding responses are shown in Figure 7.

It is to note in Table 1 that the process in the second example contains a non-minimum phase zero (that gives inverse response dynamics) which is considered in the corresponding reference models.

The proposed method gives P-controller as the set-point controller and PD controller as the load-disturbance controller for Example 1 and whereas for Example 2 both the controllers are obtained as PD-controller. Rao et al. [9] and Liu et al. [12] considered three controllers for Example 1 whereas for Example 2 Uma et al. [10] used four controllers.

It is observed from tables and figures that the proposed method gives favourably comparable performances in set-point as well as load-disturbance responses both for the nominal and the perturbed processes.

IV. Conclusion

A PID controller design method in a modified Smith predictor configuration has been proposed for integrating processes with dead time. A two-degree-of freedom control has been considered through direct synthesis approach from where PID controllers have been computed by approximate frequency response matching at two low frequency points. A simple and useful criterion has been provided for selection of these low frequency points. The design is involved with solution of linear algebraic equations. The method is applicable to high-order integrating processes in addition to low-order one without reduction of the process model. There is no requirement of approximation of the dead-time term e^{-Ls} by the Pade approximation or by the power series expansion. The design method is also applicable to integrating processes with non-minimum phase zeros as shown here. Efficacy of the method against mathematical simplicity along with low computational burden has been shown through some examples taken from the literature and comparing favourably with some of the prevailing methods.

Table 1: Processes, reference models and controllers.

Process	Nature of the Process	$M_{r,y}(s)$	$M_{d,y}(s)$	$G_c(s)$			$G_d(s)$		
				K_P	K_I	K_D	K_P	K_I	K_D
$G_1(s) = e^{-5s}/s$	Integrating plus dead-time (IPDT) process	$e^{-5s}/(s+1)$	$\frac{50se^{-5s}}{(4s+1)^2}$	1	0	0	0.12	0	0.19
$G_2(s) = \frac{0.547(-0.418s+1)e^{-0.1s}}{s(1.06s+1)}$	Integrating first order plus dead-time (IFOPDT) process with inverse response	$\frac{(-0.418s+1)e^{-0.1s}}{(0.5s+1)}$	$\frac{s(-0.418s+1)e^{-0.1s}}{(s+1)^2}$	2.01	0	2.11	1.02	0	0.18

Table 2: Performance comparison.

Process	Method	set-point		load-disturbance		IAE
		M_P	$t_s(s)$	y_{peak}	$t_s(s)$	
$G_1(s)$	Proposed	0	8.90	0.53	34.3	11.0
	Rao et al. [9]	0	19.9	0.53	62.2	14.06
	Liu et al. [12]	0	12.8	0.54	55.1	12.37
$G_1(s)$ perturbed	Proposed	7.6	27.0	0.54	46.5	12.85
	Rao et al.	2.5	30.8	0.54	62.7	15.42
	Liu et al.	7.4	30.7	0.54	55.1	13.84
$G_2(s)$	Proposed	0	5.27	0.18	9.8	2.75
	Uma et al. [10]	0.5	4.93	0.17	10.9	3.14
$G_2(s)$ perturbed	Proposed	0	5.3	0.20	9.0	2.65
	Uma et al.	0.5	5.5	0.19	7.7	3.14

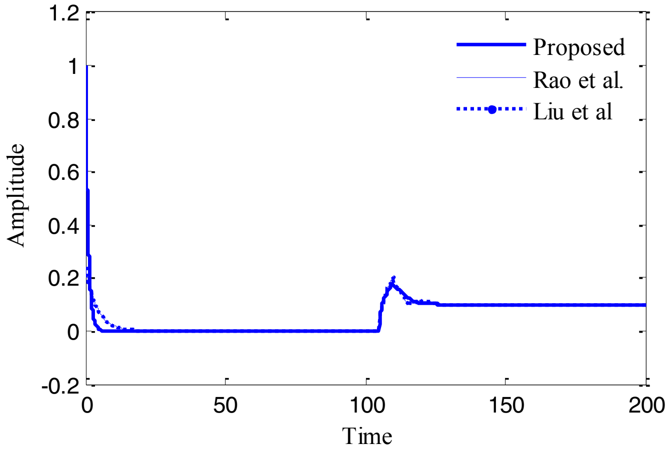

Figure 3: Controller output for Example 1.

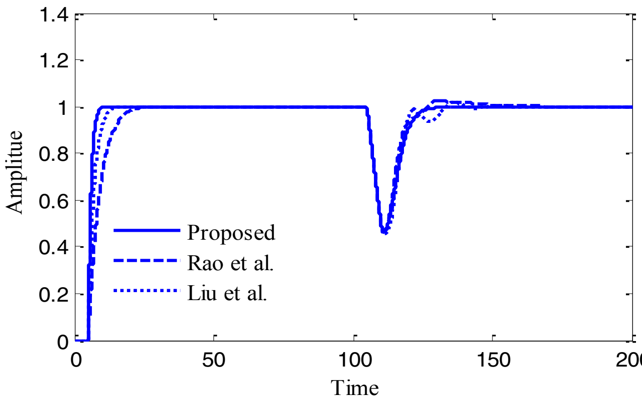

Figure 2: Process output for Example 1.

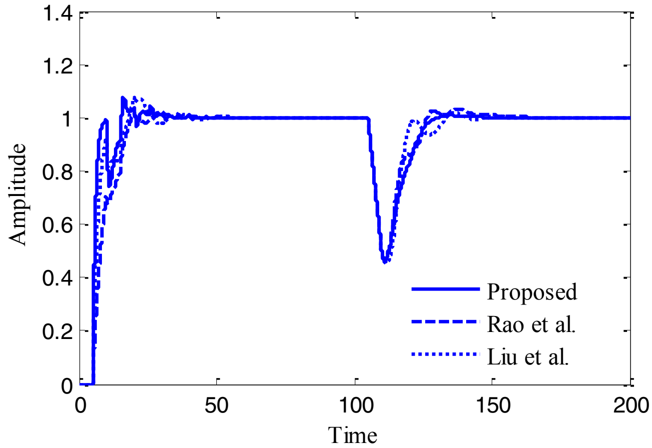

Figure 4: Process output for perturbed process for Example 1.

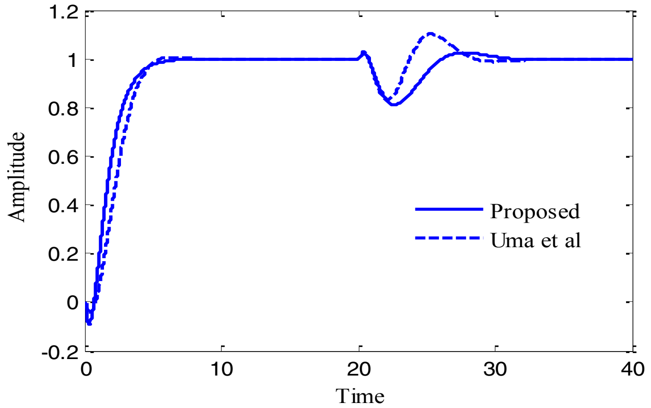

Figure 5: Process output for Example 2.

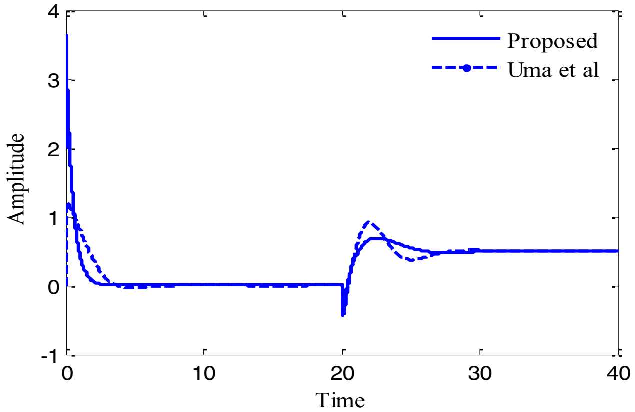

Figure 6: Controller output for Example 2.

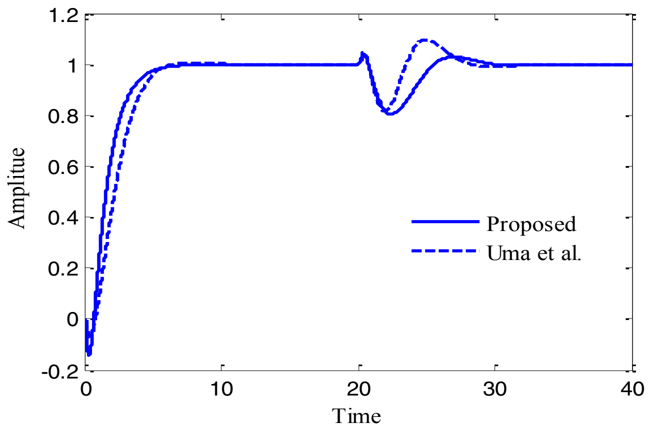

Figure 7: Process output for perturbed process for Example 2.

V. References

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