Adaptive and Performance-Driven PID Control System Design for Discrete-time Systems with a Parallel Feedforward Compensator Designed via FRIT

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***Abstract*— This paper deals with a design problem of an adaptive and performance-driven PID control with an adaptive Neural Network (NN) feedforward control for discrete-time systems with a parallel feedforward compensator (PFC). By adopting PFC design scheme based on FRIT approach, which can design PFC only using an input/output experimental data, a performance-driven adaptive PID control will be proposed. The effectiveness of the proposed PFC design and adaptive and performance-driven PID control method will be confirmed through numerical simulations for an uncertain discrete-time system.**

I. INTRODUCTION

PID control is one of the most common control schemes and it has been applied to many industrial process and mechanical systems. However, in the case where there are some changes of system properties, it is difficult to maintain the desired control performance and stability during operation because most PID parameter tuning is done off line. Therefore, a great deal of attention has been focused on auto-tuning and adaptive PID methods [1], [2], [3]. Recently, an auto-tuning and an adaptive PID control strategies based on the almost strictly positive real (ASPR) property of the controlled system have been proposed [4], [5]. These adaptive PID schemes based on the ASPR property of the system can guarantee the asymptotic stability of the resulting PID control system. Unfortunately, the ASPR conditions are very severe restrictions for practical applications of the adaptive PID control. To overcome this problem, an introduction of the parallel feedforward compensator (PFC) has been proposed [4], [6]. This method fulfills the ASPR conditions of augmented system, which consists of the plant and the PFC, by designing the PFC accordingly. Although several methods have been proposed with respect to the design scheme of such a PFC, most of them need a priori informations of the controlled plant in order to design the PFC. To obtain *a priori* informations of the plant, we need to derive the system model or do experiment several times. This is time-consuming task and becomes a problem when considering the time and costs. From this reason, recently, PFC design method via fictitious reference iterative tuning (FRIT) approach has been proposed for continuous time system [7], [8] and for discrete time systems [13]. FRIT method can optimizes controller parameters for the uncertain plant from only one shot experimental input/output data without using the plant model [9]. By applying FRIT method to PFC design, PFC parameters could be optimized without using *a priori* informations [10]. However, the affects from the introduced PFC resulted in degradation of the control performance in the output tracking control because the adaptive controller is designed for the augmented system with the PFC. To overcome this problem, a method introducing a feedforward input generated by an adaptive Neural Network (NN) has been proposed concerning to output feedback controls [11], [12]. By using this method, one can attenuate affects from the PFC. Unfortunately however, one cannot maintain ASPRness if there are some changes in the plant.

In this paper, we propose an adaptive and performance-driven PID control system design with an adaptive NN feedforward control for discrete-time systems with a PFC. In addition, the effectiveness of the proposed method will be confirmed through a numerical simulation.

II. CONTROL SYSTEM DESIGN [13]

A. Problem Statement

Consider a SISO discrete-time system $G(z)$ expressed as

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{b}u(k) \\ y(k) &= \boldsymbol{c}^T\boldsymbol{x}(k) \end{aligned} \tag{1}$$

where $\boldsymbol{x}(k) \in \boldsymbol{R}^n$ is a state vector, $u(k) \in \boldsymbol{R}$ and $y(k) \in \boldsymbol{R}$ are the input and the output of the system, respectively.

Suppose that the reference signal $y_r(k)$ which the output $y(k)$ is required to track are generated by the following exosystem:

$$\begin{aligned} \boldsymbol{\omega}(k+1) &= A_\omega \boldsymbol{\omega}(k) \\ y_r(k) &= \boldsymbol{c}_\omega^T \boldsymbol{\omega}(k) \end{aligned} \tag{2}$$

For the system (1) and reference signal $y_r(t)$ given in (2), we impose the following assumptions.

Assumption 1: There exist an ideal state $\boldsymbol{x}^*(k)$ and an ideal input $v^*(k)$ which attain perfect tracking such that

$$\begin{aligned} \boldsymbol{x}^*(k+1) &= \boldsymbol{A}\boldsymbol{x}^*(k) + \boldsymbol{b}v^*(k) \\ y(k) &= \boldsymbol{c}^T\boldsymbol{x}^*(k) \equiv y_r(k) \end{aligned} \tag{3}$$

and they are given by functions of $\boldsymbol{\omega}(k)$ such as $\boldsymbol{x}^*(k) = \boldsymbol{\pi}(\boldsymbol{\omega}(k))$ and $v^*(k) = c(\boldsymbol{\omega}(k))$.

Assumption 2: For the system (1), there exists a PFC of arbitrary order n_m:

$$\begin{aligned} \boldsymbol{x}_f(k+1) &= \boldsymbol{A}_f\boldsymbol{x}_f(k) + \boldsymbol{b}_f u(k) \\ y_f(k) &= \boldsymbol{c}_f^T\boldsymbol{x}_f(k) + d_f u(k) \end{aligned} \tag{4}$$

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such that the resulting augmented system with the PFC:

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{b}u(k) \\ \boldsymbol{x}_f(k+1) &= \boldsymbol{A}_f\boldsymbol{x}_f(k) + \boldsymbol{b}_f u(k) \\ y_a(k) &= y(k) + y_f(k) \\ &= \boldsymbol{c}^T\boldsymbol{x}(k) + \boldsymbol{c}_f^T\boldsymbol{x}(k) + d_f u(k) \end{aligned} \tag{5}$$

is ASPR.

B. Adaptive PID Control System Design

Under these assumptions, an adaptive PID control system design with an adaptive NN feedforward control can be designed as follows for discrete time systems with a PFC which is introduced so as to render the resulting augmented control system ASPR [13].

First, consider approximation of the ideal forward input $v^*(k)$ by a radial basis function (RBF) NN. $v^*(k)$ is approximated by the form of RBF NN as

$$v_{nn}(k) = W^T S(\boldsymbol{\omega}(k)) \tag{6}$$

where $W = [w_1, \cdots, w_l]^T \in \boldsymbol{R}^l$ is the weight vector, l is the number of NN nodes (weight number) and $S(\boldsymbol{\omega}) = [s_1(\boldsymbol{\omega}), \cdots, s_l(\boldsymbol{\omega})]^T$ is the radial basis function vector. This basis function vector $S(\boldsymbol{\omega})$ is generally designed by the Gaussian functions such as

$$s_i(\boldsymbol{\omega}) = \exp\left[\frac{-(\boldsymbol{\omega}-\boldsymbol{\mu}_i)^T(\boldsymbol{\omega}-\boldsymbol{\mu}_i)}{\eta_i^2}\right] \qquad i = 1,\ 2,\ \cdots,\ l \tag{7}$$

where $\boldsymbol{\mu}_i = [\mu_{i1}, \cdots, \mu_{iq}]^T$ is the center of the receptive field and η_i is the width of the Gaussian function.

Under Assumption 1, it has been clarified that, for a sufficiently large l and a compact set $\Omega_\omega \subset \boldsymbol{R}^q$, there exists an ideal constant weight vector W^* such that [14]

$$W^* := \arg\min_{W\in\boldsymbol{R}^l}\{\sup_{\boldsymbol{\omega}\in\Omega_\omega} |v^* - W^T S(\boldsymbol{\omega})|\} \tag{8}$$

and thus the ideal input $v^*(k)$ can be approximated by

$$v^*(k) = W^{*T}S(\boldsymbol{\omega}) + \varepsilon(\boldsymbol{\omega}),\ |\varepsilon(\boldsymbol{\omega})| \le \varepsilon^* \tag{9}$$

where $\varepsilon(\boldsymbol{\omega})$ is an approximation error.

Finally, an adaptive PID with adaptive NN feedforward input is designed as follows.

$$u(k) = -\tilde{\boldsymbol{\theta}}^T(k)\tilde{\boldsymbol{z}}(k) + \hat{W}^T(k)S(\boldsymbol{\omega}(k)) \tag{10}$$

where

$$\begin{aligned} \tilde{\boldsymbol{\theta}}^T(k) &= \left[\tilde{\theta}_p(k), \tilde{\theta}_i(k), \tilde{\theta}_d(k)\right] \\ \tilde{\boldsymbol{z}}(k) &= \left[\hat{e}_a(k), \bar{e}_{ai}(k-1), -\frac{1}{T}\bar{e}_a(k-1)\right]^T \end{aligned} \tag{11}$$

$$\hat{e}_a(k) = y(k) + \boldsymbol{c}^T\bar{\boldsymbol{x}}_f(k) - y_r(k) \tag{12}$$

$$\bar{e}_{ai}(k) = \bar{e}_{ai}(k-1) + T\bar{e}_a(k) \tag{13}$$

$$\bar{e}_a(k) = \bar{y}_a(k) - y_r(k) \tag{14}$$

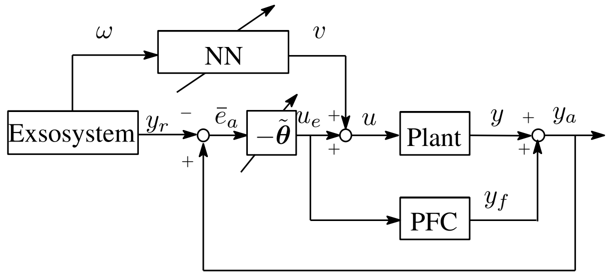


Fig. 1. Adaptive PID control system with adaptive NN feedforward

Here, $\bar{y}_a(k) = y(k) + \bar{y}_f(k)$ and $\bar{y}_f(k)$ is a PFC output with the input $u_e(k) = -\tilde{\boldsymbol{\theta}}^T(k)\tilde{\boldsymbol{z}}(k)$ and T is a sampling period. The parameter adjusting laws are given by

$$\begin{aligned} \tilde{\boldsymbol{\theta}}(k) &= \tilde{\boldsymbol{\theta}}(k-1) + \Gamma\tilde{\boldsymbol{z}}(k)\bar{e}_a(k) - \sigma\tilde{\boldsymbol{\theta}}(k) \\ &= \bar{\sigma}\tilde{\boldsymbol{\theta}}(k-1) + \bar{\sigma}\Gamma\tilde{\boldsymbol{z}}(k)\bar{e}_a(k) \\ \bar{\sigma} &= \frac{1}{1+\sigma},\ \sigma > 0,\ \Gamma = \Gamma^T > 0 \end{aligned} \tag{15}$$

$$\begin{aligned} \hat{W}(k) &= \bar{\sigma}_n\hat{W}(k-1) - \bar{\sigma}_n\Gamma_n S(\boldsymbol{\omega}(k))\bar{e}_a(k) \\ \bar{\sigma}_n &= \frac{1}{1+\sigma_n},\ \sigma_n > 0,\ \Gamma_n = \Gamma_n^T > 0 \end{aligned} \tag{16}$$

with design parameters Γ, Γ_n, σ and σ_n. In this case, the augmented output error $\bar{e}_a(t)$ can be obtained from (5), (10) and (15) as

$$\bar{e}_a(k) = \frac{\hat{e}_a(k) - \bar{\sigma}d_f\tilde{\boldsymbol{\theta}}^T(k-1)\tilde{\boldsymbol{z}}(k)}{1 + \bar{\sigma}d_f\tilde{\boldsymbol{z}}^T\Gamma\tilde{\boldsymbol{z}}(k)} \tag{17}$$

by using available signals. This means that the proposed adaptive PID controller can be designed without causality problems. Fig. 1. shows the block diagram of the control system. Then for this control system, the following theorem is known [13].

Theorem 1: Under the assumptions 1 and 2, with the control input (10), all the signals in the control system are bounded.

Remark 1: In the case where there is no NN feedforward input term, one can not obtain the perfect output tracking result of $y(t) \equiv y_r(t)$ because of the affects from the PFC output $y_f(t)$ even when the perfect output tracking $y_a(t) \equiv y_r(t)$ is achieved for augmented system with the PFC. The purpose of adding adaptive NN feedforward input term within the augmented system with a PFC is to achieve the perfect output tracking $y(t) \equiv y_r(t)$ ideally. If the $v(t)$ which is the output of the NN is equal to the ideal feedforward input $v^*(t)$ which can attain the perfect output tracking $y(t) \equiv y_r(t)$, one can easily confirm that $\bar{e}_a(t) \to 0$ leads $e(t) = y(t) - y_r(t) \to 0$. Thus by adding an adaptive NN feedforward input $v(t) = \hat{W}^T(k)S(\boldsymbol{\omega}(k))$, if the error $\bar{e}_a(t)$ becomes small then one can expect small tracking error e(t) accordingly.

III. PFC Design via FRIT Approach

In this section, we present a design method of a PFC which using only the input/output data of the controlled system.

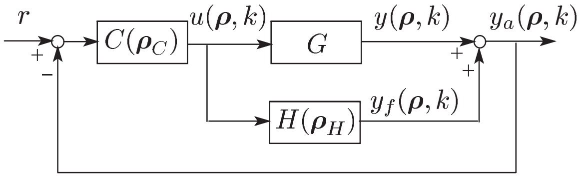


Fig. 2. Closed-loop systems

By this method, even if the system model of the controlled system is unknown, appropriate PFC can be designed [10].

A. Design Principle

Consider a closed-loop system for a single input/output system $G(z)$ with a controller $C(z,\boldsymbol{\rho}_C)$ and a PFC $H(z,\boldsymbol{\rho}_H)$, which are parameterized by $\boldsymbol{\rho} = [\boldsymbol{\rho}_C{}^T\ \boldsymbol{\rho}_H{}^T]^T$, as shown in Fig. 2. In the following, we use notations $C(\boldsymbol{\rho}_C)$ and $H(\boldsymbol{\rho}_H)$ instead of $C(z,\boldsymbol{\rho}_C)$ and $H(z,\boldsymbol{\rho}_H)$ for convenience.

Suppose that the controller and the PFC with the parameter $\boldsymbol{\rho} = [\boldsymbol{\rho}_C{}^T\ \boldsymbol{\rho}_H{}^T]^T$ are satisfies the following assumptions.

Assumption 3: $H(\boldsymbol{\rho}_H) = 0$ with $\boldsymbol{\rho}_H = \mathbf{0}$.
Assumption 4: $C(\boldsymbol{\rho}_C) = \rho_{c1}$(constant) with $\boldsymbol{\rho}_C = [\rho_{C1}, 0, \cdots, 0]^T$.

In this case, the closed-loop system from r to the augmented output $y_a(\boldsymbol{\rho})$ with a controller $C(\boldsymbol{\rho}_C)$ and a PFC $H(\boldsymbol{\rho}_H)$ can be expressed by

$$G_{ac}(\boldsymbol{\rho}) = \frac{(G + H(\boldsymbol{\rho}_H))\,C(\boldsymbol{\rho}_C)}{1 + (G + H(\boldsymbol{\rho}_H))\,C(\boldsymbol{\rho}_C)} \tag{18}$$

Here, we assume that one can obtain an input/output data set $\{u_0(k), y_0(k)\}$ for appropriate controller $C(\boldsymbol{\rho}_{C0})$ and PFC $H(\boldsymbol{\rho}_{H0})$ with parameters $\boldsymbol{\rho}_0 = [\boldsymbol{\rho}_{C0}{}^T\ \boldsymbol{\rho}_{H0}{}^T]^T$. Under this statement, the objective here is to obtain a PFC which renders the augmented system with the PFC ASPR.

To this end, we first consider a desired SPR system:

$$y_{SPR} = G_{SPR} r \tag{19}$$

and then consider to find a parameter $\boldsymbol{\rho} = [\boldsymbol{\rho}_C{}^T\ \boldsymbol{\rho}_H{}^T]^T$ which minimizes the error between SPR model output y_{SPR} and the obtained augmented system's output $y_a(\boldsymbol{\rho},k)$. That is, to find a parameter $\boldsymbol{\rho} = [\boldsymbol{\rho}_C{}^T\ \boldsymbol{\rho}_H{}^T]^T$ which minimize the following performance function:

$$J(\boldsymbol{\rho}) = \sum_{k=0}^{N} \left(y_a(\boldsymbol{\rho},k) - y_{SPR}\right)^2 \tag{20}$$

is objective. However, this performance function cannot be obtained directly, because the plant model $G(z)$ is unknown. Therefore we adopt FRIT approach to the parameter tuning.

B. PFC Parameter Tuning by FRIT Approach

In order to achieve the objective of PFC design by using an input/output data set $\{u_0(k), y_0(k)\}$, here FRIT approach is considered.

For the closed-loop system given in Fig. 2, let's consider a signal $r^*(\boldsymbol{\rho},k)$ which satisfies the following relation for any parameter vector $\boldsymbol{\rho} = [\boldsymbol{\rho}_C{}^T\ \boldsymbol{\rho}_H{}^T]^T$:

$$C(\boldsymbol{\rho}_C)\left(r^*(\boldsymbol{\rho},k) - y_{a0}(\boldsymbol{\rho},k)\right) = u_0(k) \tag{21}$$

where $y_{a0}(\boldsymbol{\rho},k)$ is the augmented output with the PFC output $H(\boldsymbol{\rho}_H)u_0(k)$:

$$y_{a0}(\boldsymbol{\rho},k) = y_0(k) + H(\boldsymbol{\rho}_H)u_0(k) \tag{22}$$

This leads

$$\begin{aligned} r^*(\boldsymbol{\rho},k) &= C(\boldsymbol{\rho}_C)^{-1}u_0(k) + y_{a0}(\boldsymbol{\rho},k) \\ &= C(\boldsymbol{\rho}_C)^{-1}u_0(k) + y_0(k) + H(\boldsymbol{\rho}_H)u_0(k) \end{aligned} \tag{23}$$

$r^*(\boldsymbol{\rho},k)$ obtained from (23) is called 'fictitious reference signal'. Taking this signal $r^*(\boldsymbol{\rho},k)$ as a reference signal, the control system in Fig.1 with controller and PFC of any parameter vector $\boldsymbol{\rho}$ gives the input $u_0(k)$ and the output $y_0(k)$.

Now, impose the following assumption.

Assumption 5: There exists an ideal parameter vector $\boldsymbol{\rho}_d = [\boldsymbol{\rho}_{Cd}^T\ \boldsymbol{\rho}_{Hd}^T]^T$ with $\boldsymbol{\rho}_{Cd} = [K^*, 0, \cdots, 0]$ such that the obtained closed-loop system with the controller $C(\boldsymbol{\rho}_{Cd}) = K^*$ and the PFC $H(\boldsymbol{\rho}_{Hd})$can be expressed by the given SPR model G_{SPR}. That is,

$$G_{SPR} = \frac{(G + H(\boldsymbol{\rho}_{Hd}))\,K^*}{1 + (G + H(\boldsymbol{\rho}_{Hd}))\,K^*} \tag{24}$$

Under Assumption 3, we have the following relation.

$$\begin{aligned} y_0(k) &= y_{a0}(\boldsymbol{\rho}_d,k) - H(\boldsymbol{\rho}_{Hd})u_0(k) \\ &= G_{SPR}\left(K^{*-1}u_0(k) + y_0(k) + H(\boldsymbol{\rho}_{Hd})\,u_0(k)\right) \\ &\quad -H(\boldsymbol{\rho}_{Hd})u_0(k) \end{aligned} \tag{25}$$

From this relation in (25), we define a virtual output $\tilde{y}(\bar{\boldsymbol{\rho}},k)$ for the system with a controller and a PFC with a parameter $\bar{\boldsymbol{\rho}}$ as follows by using the input/output data set $\{u_0(k)$,$y_0(k)\}$.

$$\begin{aligned} \tilde{y}(\bar{\boldsymbol{\rho}},k) &= G_{SPR}(C(\bar{\boldsymbol{\rho}}_C)^{-1}u_0(k) + y_0(k) + H(\bar{\boldsymbol{\rho}}_H)\,u_0(k)) \\ &\quad -H(\boldsymbol{\rho}_H)u_0(k) \\ &\quad \bar{\boldsymbol{\rho}}_C = [\bar{\rho}_{c1}, 0, \cdots, 0]^T \end{aligned} \tag{26}$$

Then, consider minimizing the following performance function:

$$J_F(\bar{\boldsymbol{\rho}}) = \sum_{k=0}^{N} \left(\tilde{y}(\bar{\boldsymbol{\rho}},k) - y_0(k)\right)^2 \tag{27}$$

The obtained optimal $\boldsymbol{\rho}_d$ by this FRIT approach can be expected to guarantee the minimization of the performance function given in (20) [10].

Now, consider a typical PFC:$H(z)$ given as the following nth compensator.

$$H(z) = \frac{b_0 z^n + b_1 z^{n-1} + \cdots + b_n}{z^n + a_1 z^{n-1} + \cdots + a_n} \tag{28}$$

Here, we approximate this PFC with n_mth FIR model as follows:

$$H(z) = f_0 + f_1 z^{-1} + f_2 z^{-2} + \cdots + f_{n_m} z^{-n_m} \tag{29}$$

Then the virtual output $\tilde{y}(\bar{\boldsymbol{\rho}}, k)$ can be represented as

$$\tilde{y}(\bar{\boldsymbol{\rho}}, k) = \boldsymbol{\xi}^T \bar{\boldsymbol{\rho}} + G_{SPR} y_0(k) \tag{30}$$

where $\boldsymbol{\xi} = [\xi_0 \ \xi_1 \ \xi_2 \ \cdots \ \xi_{n_m+1}]^T$, $\xi_0(k) = G_{SPR} u_0(k)$, $\xi_i(k) = (G_{SPR} - 1) u_0(k+1-i)$ and $\bar{\boldsymbol{\rho}} = [K^{*-1} \ f_0 \ f_1 \ \cdots \ f_{n_m}]^T$.

From (30), we obtain

$$\begin{bmatrix} \tilde{y}(\bar{\boldsymbol{\rho}}, 0) \\ \tilde{y}(\bar{\boldsymbol{\rho}}, 1) \\ \vdots \\ \tilde{y}(\bar{\boldsymbol{\rho}}, N) \end{bmatrix} = \begin{bmatrix} \xi_0(0) & \xi_1(0) & \cdots & \xi_{n_m+1}(0) \\ \xi_0(1) & \xi_1(1) & \cdots & \xi_{n_m+1}(1) \\ \vdots & & & \\ \xi_0(N) & \xi_1(N) & \cdots & \xi_{n_m+1}(N) \end{bmatrix} \bar{\boldsymbol{\rho}}$$
$$+ \left[y_{0_{SPR}}(0), y_{0_{SPR}}(1), \cdots, y_{0_{SPR}}(N) \right]^T$$
$$= \boldsymbol{\Phi} \bar{\boldsymbol{\rho}} + \boldsymbol{Y}_{0_{SPR}} \tag{31}$$

where $y_{0_{SPR}}(k) = G_{SPR} y_0(k)$ and $\boldsymbol{Y}_{0_{SPR}} = [y_{0_{SPR}}(0) \ y_{0_{SPR}}(1) \ \cdots \ y_{0_{SPR}}(N)]^T$.

Moreover, by defining $\boldsymbol{Y}_0 = [y_0(0) \ y_0(1) \ \cdots \ y_0(N)]^T$, the performance function $J_F(\bar{\boldsymbol{\rho}})$ can be represented as

$$\begin{aligned} J_F(\bar{\boldsymbol{\rho}}) &= \sum_{k=0}^{N} \left(\tilde{y}(\bar{\boldsymbol{\rho}}, k) - y_0(k) \right)^2 \\ &= \| \boldsymbol{\Phi} \bar{\boldsymbol{\rho}} + \boldsymbol{Y}_{0_{SPR}} - \boldsymbol{Y}_0 \|^2 \\ &= \left\| \boldsymbol{\Phi} \bar{\boldsymbol{\rho}} - \bar{\boldsymbol{Y}} \right\|^2 \end{aligned} \tag{32}$$

where $\bar{\boldsymbol{Y}} = \boldsymbol{Y}_0 - \boldsymbol{Y}_{0_{SPR}}$. Then, the optimal $\boldsymbol{\rho}_d$ which minimize the performance function $J_F(\bar{\boldsymbol{\rho}})$ can be obtained by

$$\boldsymbol{\rho}_d = \left(\boldsymbol{\Phi}^T \boldsymbol{\Phi} \right)^{-1} \boldsymbol{\Phi}^T \bar{\boldsymbol{Y}} \tag{33}$$

IV. Performance-driven Adaptive Control System Design

ASPR conditions of augmented system might not be hold when the system have changed during the operation. In this case, the control performances would deteriorate and the system would be unstable in the worst case. Conversely, by monitoring the control performance, it is possible to detect a change of the system. If it is determined that the system has changed, by redesigning the PFC, the control system can be kept to be stable. In the following, we will propose a performance-driven PID control system design for discrete time systems.

A. The Robustness of The Designed PFC

For preparation, the robustness of the designed PFC should be considered. The designed PFC must be supposed to render the augmented system with the PFC ASPR if the performance function was minimized. Unfortunately however, the obtained closed loop system with the designed parameter vector $\bar{\boldsymbol{\rho}}$ does not perfectly match to the ideal SPR model G_{SPR} in a practical sense. The resulting augmented system G_a with the PFC $H(\bar{\boldsymbol{\rho}}_H)$ can be represented by

$$G_a = G + H(\bar{\boldsymbol{\rho}}_H) = G^*_{ASPR}(1 + \Delta) \tag{34}$$

where G^*_{ASPR} is the ideal ASPR model given by $G^*_{ASPR} = G + H(\boldsymbol{\rho}_{Hd})$, and Δ_H and Δ are defined as follows:

$$\Delta_H = 1 - H(\bar{\boldsymbol{\rho}}_H)^{-1} H(\boldsymbol{\rho}_{Hd}), \tag{35}$$
$$\Delta = {G^*_{ASPR}}^{-1} H(\bar{\boldsymbol{\rho}}_H) \Delta_H \tag{36}$$

For the ASPR-ness of the augmented system (34) with a mismatch Δ has been investigated as in the following theorem [6].

Theorem 2: The augmented system (34) is ASPR if (a) G^*_{ASPR} is ASPR. (b) $\Delta \in RH_\infty$. (c) $\|\Delta\|_\infty < 1.0$.

It is apparent that the conditions (a) and (b) in Theorem 2 are satisfied for the obtained augmented system (34). Thus, if the mismatch Δ between the ideal PFC $H(\boldsymbol{\rho}_{Hd})$ and the obtained PFC $H(\bar{\boldsymbol{\rho}}_H)$ are sufficiently small, then the resulting augmented system is ASPR even if the designed parameter vector $\bar{\boldsymbol{\rho}}$ does not perfectly match to the ideal parameter vector $\boldsymbol{\rho}_d$.

The performance function (27) can be evaluated as follows.

$$J_F(\bar{\boldsymbol{\rho}}) \leq {\delta_H}^2 {\delta_{PH}}^2 \sum_{k=0}^{N} u_0(k)^2 \tag{37}$$

where, $\delta_H = \|\Delta_H\|_\infty$ and $\delta_{PH} = \left\| \left(G_{SPR}(n) - 1 \right) H(\bar{\boldsymbol{\rho}}_H, n) \right\|_\infty$. Then, by defining $\beta_u = \sum_{k=0}^{N} u_0(k)^2$, we have

$${\delta_H}^2 \geq \frac{J_F(\bar{\boldsymbol{\rho}})}{{\delta_{PH}}^2 \beta_u} \tag{38}$$

On the other hand, from (36), $\|\Delta\|_\infty$ can be evaluated by

$$\|\Delta\|_\infty = \|{G^*_{ASPR}}^{-1} H(\bar{\boldsymbol{\rho}}_H) \Delta_H\|_\infty \leq \delta_{GH} \delta_H \tag{39}$$

where $\delta_{GH} = \|{G^*_{ASPR}}^{-1} H(\bar{\boldsymbol{\rho}}_H)\|_\infty$. This means that $\delta_{GH}\delta_H < 1.0$ is necessary condition to be ASPR. Now, from (38), define the lower limit of δ_H as

$$\bar{\delta}_H = \sqrt{\frac{J_F(\bar{\boldsymbol{\rho}})}{{\delta_{PH}}^2 \beta_u}} \tag{40}$$

Then, at least $\delta_{GH}\bar{\delta}_H < 1.0$ have to be fulfilled to be $\delta_{GH}\delta_H < 1.0$. The condition $\delta_{GH}\bar{\delta}_H < 1.0$ is the necessary condition that to fulfill the condition (c) of Theorem 2.

Remark 2: Theorem 2 is the sufficient condition that the control system would be ASPR. Therefore, there exist ASPR control system which does not fulfill the conditions of Theorem 2. However, as an one of the standard, when it is $\delta_{GH}\bar{\delta}_H \geq 1.0$, we redesigning the PFC.

B. Performance Evaluation

We consider adopting the minimum variance performance-index (MV-index) which is common performance evaluation method [15].

The MV-index η_p is defined by

$$\eta_p(t) = \frac{\sigma_{MV}^2}{\sigma_y^2(t)} \tag{41}$$

σ_{MV}^2 is the minimum variance of the output error signal and σ_y^2 is a variance given later.

For this index, the inequality $0 \leq \eta_p \leq 1$ holds and a higher value of η_p indicates good control performance. In the proposed method, we calculate η_p at time t_m by using a data set $\{u^t(k),\, y^t(k)\}$ obtained in the time interval $[t_m - T,\ t_m]$ with a period of T_0. Thus, the updating time of η_p is given by $t_m = T + mT_0(m = 0, 1, \cdots)$ and then a variance σ_y^2 in the time interval $[t_m - T,\ t_m]$ is given by

$$\sigma_y^2(t) = \frac{\displaystyle\sum_{k=0}^{N}(y(k) - y_r(k))^2}{N} \tag{42}$$

here, N is the number of data set which obtained in the time interval $[t_m - T,\ t_m]$.

C. Performance-Driven PFC Design

According to the value of updated MV-Index η_p, we redesign the PFC as follows:

Step 1: Set an assessment criterion $\eta^*(0 \leq \eta^* \leq 1)$.
Step 2: Update MV-Index η_p at updating time t_m.
Step 3: If $\eta_p \geq \eta^*$, then assess that the control performance is good and go back Step 2 without redesigning of PFC. If $\eta_p < \eta^*$, then assess that the control performance is poor and go Step 4.
Step 4: Using a data set $\{u^t(k),\, y^t(k)\}$ obtained in the time interval $[t_m - T,\ t_m]$, redesign PFC by the FRIT method.
Step 5: Calculate $\delta_{GH}\bar{\delta}_H$ of old PFC and redesigned PFC and check the values of $\delta_{GH}\bar{\delta}_H$. When $\delta_{GH}\bar{\delta}_H$ for redesigned PFC is less than 1.0 and the redesigned one is lower value than old one then use redesigned PFC, otherwise use old one.

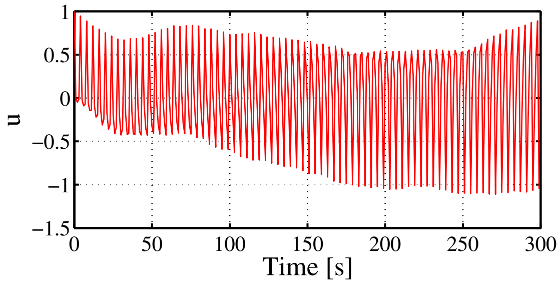
V. Simulation

To confirm the effectiveness of the proposed method, this section shows numerical simulation results.

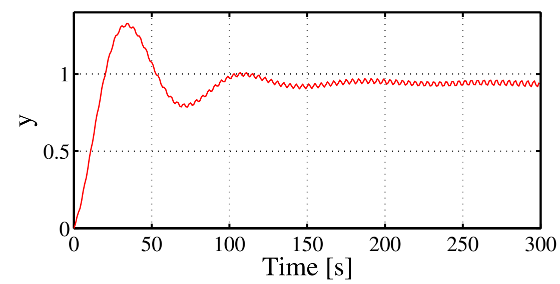
Let's consider a tracking control of the following SISO discrete-time system with the sampling period of 1.0 [s].

$$G(z) = \frac{0.027z^4 + 0.0065z^3 - 0.018z^2 - 2.5\times10^{-4}z - 9.7\times10^{-7}}{z^5 - 2.41z^4 + 1.83z^3 - 0.416z^2 - 0.0074z - 3.3\times10^{-5}} \tag{43}$$

We obtain an input/output data set $\{u_0(k), y_0(k)\}$ shown in Fig.3. Here, we considered the case where white noise is added to the output signal of the plant, and the power spectral



(a)Output of the plant



(b)Control input

Fig. 3. The input/output signals for the PFC design.

density of white noise is 5.0×10^{-3}. Then, from (33), we designed a second order PFC:$H(\bar{\boldsymbol{\rho}})$ by using SPR model:

$$G_{SPR}(z) = \frac{15z - 13}{16z - 14} \tag{44}$$

and obtained

$$H(z, \bar{\boldsymbol{\rho}}) = 1.515 + 0.125z^{-1} + 0.0662z^{-2} \tag{45}$$

For the system with (45), the design parameters in the adaptive controller are set by $\boldsymbol{\omega} = 1,\ \sigma = 1.0\times10^{-3},\ \sigma_i = 1.0\times10^{-3},\ \sigma_n = 1.0\times10^{-5},\ \Gamma = \mathrm{diag}[\gamma_p, \gamma_i, \gamma_d],\ \gamma_p = 1,\ \gamma_i = 1\times10^{-4},\ \gamma_d = 1\times10^{-5}$, and $\Gamma_n = 5\times10^{-3}$. Also, the reference signal $y_r(k)$ is given by

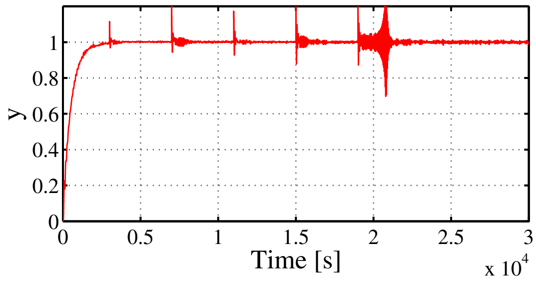
$$y_r(k) = \frac{1/500}{z - (1 - 1/500)}[r(k)],\ r(k) = 1 \tag{46}$$

where the notation $G(z)[r(k)]$ implies the output of the system $G(z)$ with an input $r(k)$. Here, we considered that controlled systems parameter changed as following.

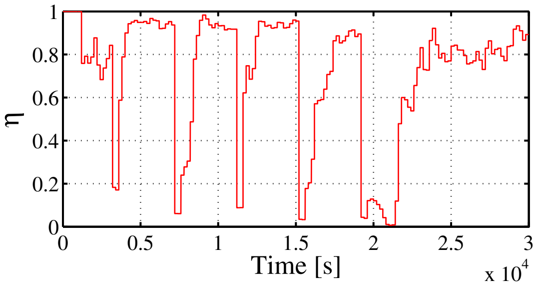
$$G(z) = \frac{b_0z^4 + b_1z^3 + b_2z^2 + b_3z + b_4}{z^5 + a_1z^4 + a_2z^3 + a_3z^2 + a_4z + a_5} \tag{47}$$

$$b_1 = \begin{cases} b_1 = 0.0065 & t \leq 3000 \\ b_{11} = b_1 \times 1.3 & 3000 < t \leq 7000 \\ b_{12} = b_{11} \times 1.4 & 7000 < t \leq 11000 \\ b_{13} = b_{12} \times 1.3 & 11000 < t \leq 15000 \\ b_{14} = b_{13} \times 1.4 & 15000 < t \leq 19000 \\ b_{15} = b_{14} \times 1.3 & 19000 < t \end{cases} \tag{48}$$

Also, we set the assessment criterion η^* as 0.7[3]. The simulation results are shown in Fig.4. We can see that even

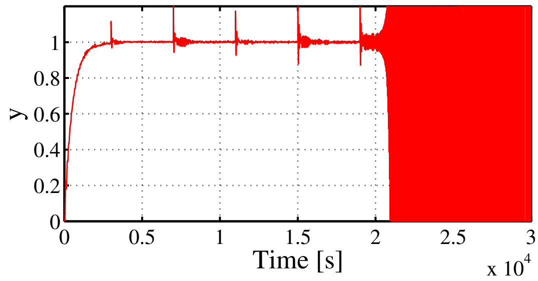


(a)Output of the plant

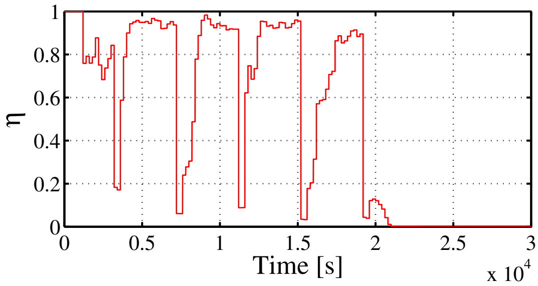


(b)Performance index

Fig. 4. Simulation results of performance driven PFC design and adaptive PID controller with adaptive NN.



(a)Output of the plant



(b)Performance index

Fig. 5. Simulation results by adaptive PID controller with adaptive NN without redesigning the PFC.

if the control system have changed, the PFC was redesigned and the control system keeping the stability.

Fig.5 shows the simulation results by adaptive PID controller with adaptive NN without redesigning the PFC. We can see that after the 5th change of controlled system, the system became out of control.

VI. Conclusions

In this paper, we proposed an adaptive and performance-driven PID control system design for discrete-time systems with PFC designed through FRIT approach. Even if the parameters of the system have changed during the operation drastically, the proposed system can hold the ASPRness of the augmented system by redesigning the PFC. In addition, the effectiveness of the proposed method has confirmed through a numerical simulation.

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