MULTIVARIABLE MD-PID CONTROL SYSTEM DESIGN METHOD AND CONTINUOUS SYSTEM MODELING

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***Abstract*— In this paper, we propose a multivariable Model Driven (MD)-PID control system which is composed of MD-PID control systems with PD feedback and inverted decoupling, discuss properties of the control system and also introduce a simple continuous system modeling by using CMA-ES and SIMULINK model, numerical example of a TITO process shows the practically effectiveness.**

I. INTRODUCTION

From the operational point of view, single-loop PID control systems have been widely used for regulatory control systems in Distributed Control System (DCS) even though most industrial processes are basically multivariable and coupling systems. In order to control the entire processes optimally, multivariable control methods, such as model predictive control (MPC) and decoupling control are well known. For the MPC systems with slow sampling rate, there have been raised some issues by Hugo[1], such as difficulties with operation, high maintenance cost, and lack of flexibility, slow regulation for unknown disturbance and low availability of the MPC system. Recently inverted decoupling control [2], [3], [4], [5] has been attracted attention to instead of normal decoupling control. The normal decoupling control is complex to deal with for the anti-reset-windup action and the auto/manual mode, however, the inverted decoupling control is easy to deal with those. But stability problem due to using feedback of input signals is remained for the inverted decoupling.

We have already proposed a Model Driven (MD) PID control[6], [7], [8], [9] which can be obtained a good control performance for a wide class of controlled processes. As continuous-time system modeling for the process is intuitive and easy to understand for process operators and engineers, we have also tried a continuous-time MIMO modeling by using of the CMA-ES (Covariance Matrix Adaptation Evolution Strategy) by Hansen [10] and SIMULINK model. Where the CMA-ES is a stochastic parameter optimization method of non-linear functions proposed by N.Hansen and SIMULINK is a trademarks of The MathWorks, Inc.

We planned an easy handling multivariable control system for DCS systems with good control performance which combine Model-Driven PID control systems with inverted decoupling. As we have obtained good results through some simulation studies, in this paper, we propose the new multivariable MD-PID control system for DCS system and a continuous-time MIMO modeling by using of the CMA-ES.

II. MULTIVARIABLE MD-PID CONTROL SYSTEM

Fig. 1 shows a block diagram of the Multivariable MD-PID control system with inverted decoupler $D_f(s)$ and PD feedback $F_{diag}(s)$.

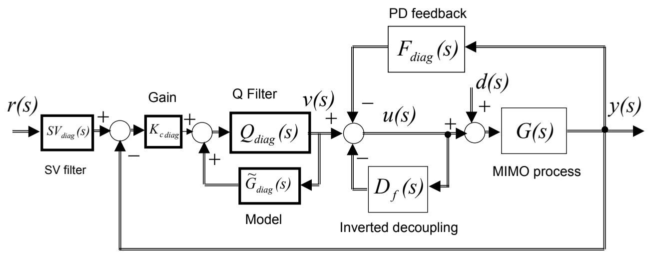

Fig. 1. Multivariable MD-PID control system

where $G(s)$ is a n-inputs and n-outputs controlled process and $r(s)$, $y(s)$, $v(s)$, $u(s)$ and $d(s)$ are n-vectors of set point, process output, internal control output, process input and disturbance, respectively. The controlled process $G(s)$ is expressed by a transfer function matrix with n-inputs and n-outputs as in (1)

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) & \dots & G_{1n}(s) \\ G_{21}(s) & G_{21}(s) & \dots & G_{2n}(s) \\ \vdots & \vdots & \ddots & \vdots \\ G_{n1}(s) & G_{n2}(s) & \dots & G_{nn}(s) \end{bmatrix} \quad (1)$$

For the sake of simplicity, by using the Relative Gain Array (RGA) measure introduced by Bristol [11], the pairing is set to (y_i, u_i) $(i = 1, 2, , n)$. Then $G_{diag}(s)$ of a diagonal matrix of $G(s)$ is shown in (2)

$$G_{diag}(s) \quad = \quad diag(G_{11}(s), G_{22}(s), \cdots, G_{nn}(s)) \quad (2)$$

According to the inverted decoupling by Garrido [3], $D_f(s)$ becomes as in (3). And (4) shows element-wise expression.

$$D_f(s) = G_{diag}^{-1}(s)G(s) - I \quad (3)$$

$$D_f(s) = \begin{bmatrix} 0 & \frac{G_{12}(s)}{G_{11}(s)} & \cdots & \frac{G_{1n}(s)}{G_{11}(s)} \\ \frac{G_{21}(s)}{G_{22}(s)} & 0 & \cdots & \frac{G_{2n}(s)}{G_{22}(s)} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{G_{n1}(s)}{G_{nn}(s)} & \frac{G_{n2}(s)}{G_{nn}(s)} & \cdots & 0 \end{bmatrix} \quad (4)$$

In order to operate $D_f(s)$ stable, the following three conditions by Garrido et al. [3] are necessary.

- Properness of each element in (5),

$$\lim_{s\to\infty}\left|\frac{G_{ik}(s)}{G_{ii}(s)}\right| < \infty \quad i,k = 1,2,\ldots,n,\ k \neq i \tag{5}$$

- non-causal time delay,
- following equation in (6) has no pole in right half plane, which means no unstable pole.

$$|I + D_f(s)| = 0 \tag{6}$$

By using the inverted decoupling $D_f(s)$, the transfer function matrix from $v(s)$ to $y(s)$ becomes as $G_{diag}(s)$. So the PD feedback $F_{diag}(s)$ in (7) can be designed in order to be matching to (9) and (10) as first order delay systems with dead time proposed by Shigemasa et al. [11] in element-wisely as shown in appendix. The PD feedback $F_{diag}(s)$ is set in (7),

$$F_{diag}(s) = diag(F_{11}(s), F_{22}(s), \cdots, F_{nn}(s)) \tag{7}$$

where a PD feedback of ii loop $F_{ii}(s)$ is expressed as in (8)

$$F_{ii}(s) = K_{fii}\frac{1+T_{fii}s}{1+\kappa_{ii}T_{fii}s} i = 1,2,,,,n \tag{8}$$

Matching equations (9) and (10) are as follows.

$$(I + G_{diag}(s)F_{diag}(s))^{-1}G_{diag}(s) = G_{F_{diag}}(s) \tag{9}$$

$$G_{F\,diag}(s) \cong K_{F\,diag}(I + sT_{F\,diag})^{-1}exp(-sL_{F\,diag}) \tag{10}$$

where $K_{F\,diag}$, $T_{F\ diag}$ and $L_{F\ diag}$ are shown following diagonal matrix.

$$K_{F\ diag} = diag(K_{F\ 11}, K_{F\ 22}, \cdots, K_{F\ nn}) \tag{11}$$
$$T_{F\ diag} = diag(T_{F\ 11}, T_{F\ 22}, \cdots, K_{T\ nn}) \tag{12}$$
$$L_{F\ diag} = diag(L_{F\ 11}, L_{F\ 22}, \cdots, K_{L\ nn}) \tag{13}$$

As the PD loops designed to first order delay systems with dead time, main control system is IMC (Internal Model Control) by Morari et al. [12]. So let the gain matrix K_{cdiag} and the internal model $\tilde{G}_{diag}(s)$ are set in (14) and (15) respectively.

$$K_{cdiag} = diag(K_{c1}, K_{c2}, \cdots, K_{cn}) \tag{14}$$

$$\tilde{G}_{diag}(s) = (I + sT_{c\ diag})^{-1}exp(-sL_{c\ diag}) \tag{15}$$

where $T_{c\ diag}$ and $L_{c\ diag}$ are diagonal matrices as in (16) and (17) respectively.

$$T_{c\ diag} = diag(T_{c\ 11}, T_{c\ 22}, \cdots, T_{c\ nn}) \tag{16}$$

$$L_{c\ diag} = diag(L_{c\ 11}, L_{c\ 22}, \cdots, L_{c\ nn}) \tag{17}$$

Two degrees of freedom Qfilter $Q_{diag}(s)$ of IMC can be set as in (18)

$$Q_{diag}(s) = (I + sA_{diag}T_{c\ diag})(I + sT_{c\ diag}) \cdot (I + s\Lambda_{diag}T_{c\ diag})^{-2} \tag{18}$$

The set point filter $SV_{diag}(s)$ of IMC is set as in (19)

$$SV_{diag}(s) = (I + s\Lambda_{diag}T_{c\ diag})(I + sA_{diag}T_{c\ diag})^{-1} \tag{19}$$

where Λ_{diag} in (20) and A_{diag} in (21) are diagonal matrices which work for tuning of each loop.

$$\Lambda_{diag} = diag(\lambda_{11}, \lambda_{22}, \cdots, \lambda_{nn}) \tag{20}$$

$$A_{diag} = diag(\alpha_{11}, \alpha_{22}, \cdots, \alpha_{nn}) \tag{21}$$

Based on IMC [12], K_{cdiag}, T_{cdiag} and L_{cdiag} are set to as (22) , (23) and (24) respectively.

$$K_{cdiag} = (K_{F\ diag})^{-1} \tag{22}$$

$$T_{cdiag} = T_{F\ diag} \tag{23}$$

$$L_{cdiag} = L_{F\ diag} \tag{24}$$

The process output $y(s)$, the process input $u(s)$ and the internal control output $v(s)$ are expressed as in (23) , (24) and (25) respectively.

$$\begin{aligned} y(s) = {} & (I + s\Lambda_{diag}T_{c\ diag})^{-1}exp(-sL_{C\ diag})r(s) \\ & +[I - (I + sA_{diag}T_{c\ diag})(I + sT_{c\ diag}) \\ & \cdot(I + s\Lambda_{diag}T_{c\ diag})^{-2}exp(-sL_{c\ diag})] \\ & \cdot G_{F\ diag}(s)G_{diag}(s)^{-1}G(s)d(s) \end{aligned} \tag{25}$$

$$u(s) = (I + D_f(s))^{-1}(v(s) - F_{diag}(s)y(s)) \tag{26}$$

$$\begin{aligned} v(s) = {} & (I + sT_{c\ diag})(I + s\Lambda_{diag}T_{c\ diag})K_{cdiag}r(s) \\ & -(I + sA_{diag}T_{c\ diag})(I + s\Lambda_{diag}T_{c\ diag})^{-2} \\ & \cdot exp(-sL_{c\ diag})G_{diag}(s)^{-1}G(s)d(s) \end{aligned} \tag{27}$$

III. TITO PROCESS MODELING

Continuous-time system modeling for the process is intuitive and easy to understand, we used a very simple continuous-time MIMO modeling form input data and output data by using of the CMA-ES and SIMULINK model. Outline of the modeling is shown in Fig. 2.

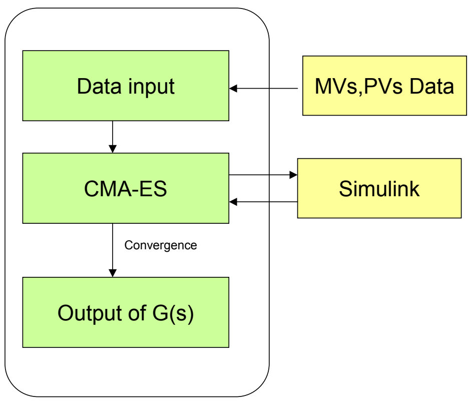

Fig. 2. Modeling approach

A TITO (Two Input and Two Output) process [13] tested is shown in (28).

$$G(s)=\begin{bmatrix} \frac{4.05exp(-27s)}{1+50s} & \frac{1.77exp(-28s)}{1+60s} \\ \frac{5.39exp(-18s)}{1+50s} & \frac{5.27exp(-14s)}{1+60s} \end{bmatrix} \quad (28)$$

Firstly we got input data (u_1,u_2) and output data (y_1,y_2) from step responses of the TITO process.

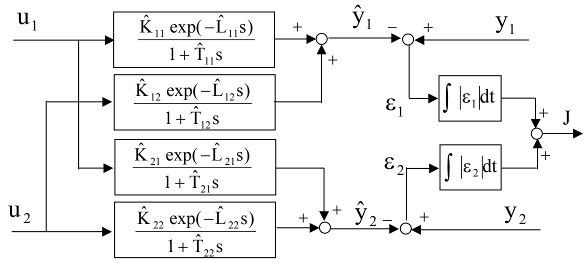

Fig. 3. TITO Block diagram

In order to identify the TITO process, a continuous TITO model in SIMULINK model with 12 parameters as shown in Fig. 3 are used. Fig. 4 shows convergence profile of objective function J of IAE in Fig. 3.

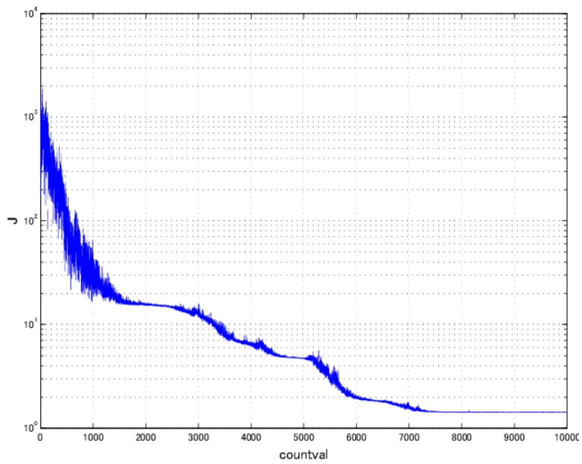

Fig. 4. Convergence profile of the objective function J

Fig. 5 shows comparison of the TITO model responses (Magenta line) by using the converged parameters and the output data(Blue line) and shows good convergence.

The TITO model $\tilde{G}(s)$ converged is shown in (29). Errors on gains, time constants and dead times are almost negligible compared to (28).

$$\tilde{G}(s)=\begin{bmatrix} \frac{4.050exp(-27.28s)}{1+50.08s} & \frac{1.770exp(-28.27s)}{1+60.11s} \\ \frac{5.390exp(-18.29s)}{1+50.08s} & \frac{5.270exp(-14.31s)}{1+60.07s} \end{bmatrix} \quad (29)$$

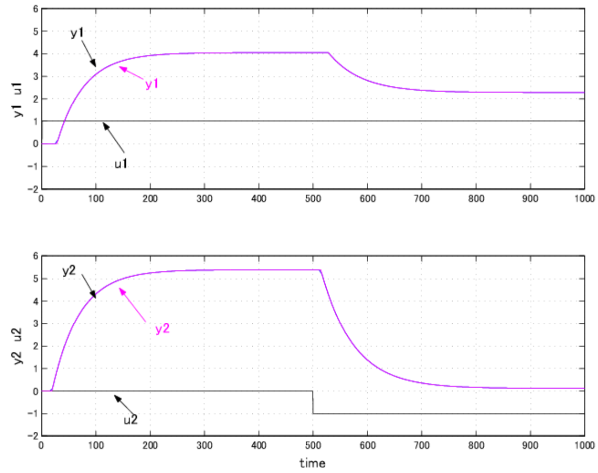

Fig. 5. Comparison of the TITO model responses (Magenta line) by using the converged parameters and the output data(Blue line)

IV. Design of Multivariable MD-PID control system

The RGA measure as in (30) shows the pairing of (y_1,u_1) and (y_2,u_2).

$$RGA = G(0)\circ(G(0)^T)^{-1} = \begin{bmatrix} 1.8083 & -0.8083 \\ -0.8083 & 1.8083 \end{bmatrix} \quad (30)$$

Based on (3) or (4), the inverted decoupling $D_f(s)$ for the TITO process becomes to (31).

$$D_f(s)=\begin{bmatrix} 0 & \frac{1.771+50s}{4.051+60s}exp(-s) \\ \frac{5.39}{5.27}\frac{1+60s}{1+50s}exp(-4s) & 0 \end{bmatrix} \quad (31)$$

Based on (6), (32) is obtained.

$$G_{11}(s)G_{22}(s)-G_{12}(s)G_{21}(s)= \frac{21.3435exp(-41s)-9.5403exp(-46s)}{(1+50s)(1+60s)}=0 \quad (32)$$

As (33) is obtained from (32), the inverted decoupling $D_f(s)$ has no unstable pole as in (33)

$$exp(5s)=\frac{9.5403}{21.3435}<1 \quad (33)$$

So $G_{diag}(s)$ becomes as in (34) from the pairing information.

$$G_{diag}(s)=\begin{bmatrix} \frac{4.05exp(-27s)}{1+50s} & 0 \\ 0 & \frac{5.27exp(-14s)}{1+60s} \end{bmatrix} \quad (34)$$

Table 1 shows the design results of the multivariable MD-PID control system with inverted decoupling. Case1 is using $\lambda_{ii}=1$, $\alpha_{ii}=1$ without PD feedback, Case2 is using active λ_{ii}, α_{ii} as in Table 1 without PD feedback and Case3 is using $\lambda_{ii}=1$, $\alpha_{ii}=1$ with PD feedback as in Table 1.

TABLE I

SUMMARY OF PARAMETERS DESIGNED FOR G_{11} AND G_{22}

	Case1	Case1	Case2	Case2	Case3	Case3
	G_{11}	G_{22}	G_{11}	G_{22}	G_{11}	G_{22}
K_c	1/4.05	1/5.27	1/4.05	1/5.27	1/2.519	1/2.274
T_c	50	60	50	60	22.199	16.263
L_c	27	14	27	14	28.98	15.178
λ	1	1	0.7	0.5	1	1
α	1	1	1	0.8	1	1
K_f	0	0	0	0	0.15	0.3
T_f	0	0	0	0	9.648	5.025
κ	0.1	0.1	0.1	0.1	0.1	0.1
K	4.05	5.27	4.05	5.27	2.519	2.274
T	50	60	50	60	22.199	16.263
L	27	14	27	14	28.982	15.178

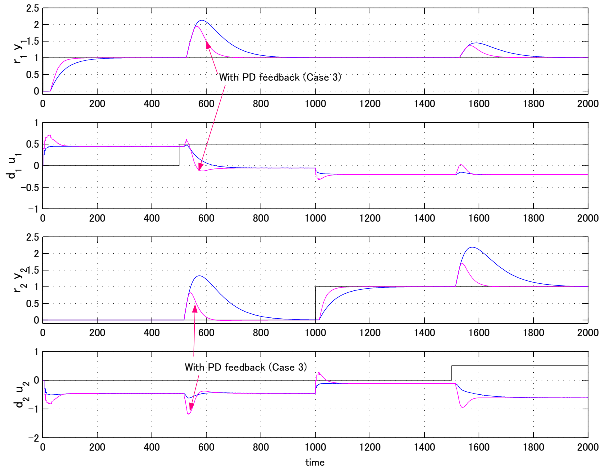

Fig. 6. A multivariable MD-PID control system with inverted decoupling for MIMO process (Case1 and Case3)

Fig. 6 shows the comparison of the responses of multivariable MD-PID control system of case1 and case3. In the simulation, set-point $r_1(t)$ was changed from 0 to 1 at t = 1 while $r_2(t) = 0$, disturbance $d_1(t)$ was applied from 0 to 0.5 at t=500sec while $d_2(t) = 0$, set-point $r_2(t)$ was changed from 0 to 1 at t = 1000sec while $r_1(t) = 1$ and disturbance $d_2(t)$ was applied from 0 to 0.5 at t = 1500sec while $d_1(t) = 0.5$. The case3 using PD feedback shows good control performances, such as quick disturbance regulation and quick set-point tracking.

Fig. 7 shows the comparison of the responses of multivariable MD-PID control system of case2 and case3. the case3 using PD feedback shows good control performances, such as quick disturbance figuration and quick set-point tracking. So the PD feedback is effective way to regulate disturbance quickly and to track to set-point quickly under using inverted decoupling.

We investigated the influence to the other loop on saturation of process input under the multivariable MD-PID control system using the PD feedback and inverted decoupling at case 3. ARW(anti-reset Windup) operation was set to 0.4 for

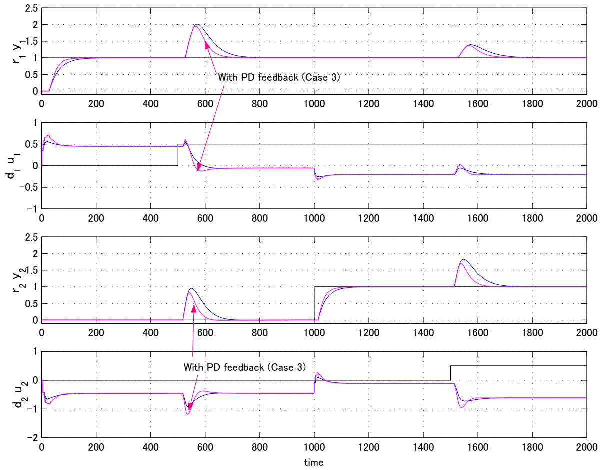

Fig. 7. A multivariable MD-PID control system with inverted decoupling for MIMO process (Case2 and Case3)

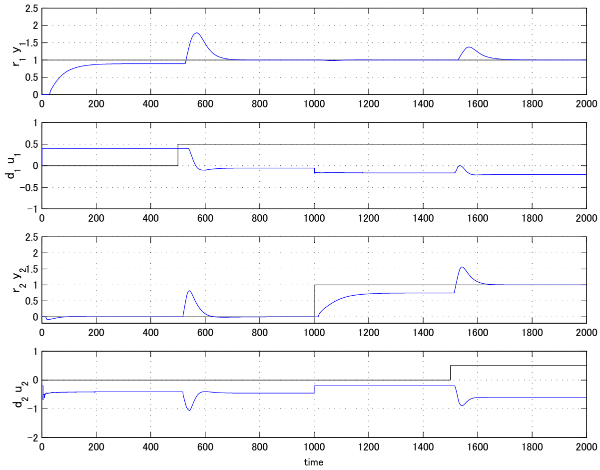

Fig. 8. Responses of the multivariable MD-PID control system using the inverted decoupling and the PD feedback of case 3 under saturation of process inputs

$u_1(t)$ and 0.1 for $u_2(t)$. multivariable MD-PID control system with inverted decoupling and PD feedback can be seen the good control structure that does not affect the other loops by using only normal ARW operation.

We have already obtained good results for other MIMO processes; such as a 3*3 Tyreus distillation column and a 4*4 HVAC process shown in the paper [2] by J.Garrido et al., but we would like to omit them because those are the same in the methodological approach.

V. CONCLUSIONS AND FUTURE WORKS

The paper introduced a new multivariable model-driven (MD) PID control system using inverted decoupling and PD feedback, the control properties and simple continuous time modeling approach by using CMA-ES and SIMULINK model and a TITO process application. The multivariable

MD-PID control system with inverted decoupling and PD feedback show good control performance. Data driven tuning approach without modeling for MD-PID control system was presented by Shigemasa et al. [16], [17]. As the multivariable MD-PID control system can work at DCS with with fast control period, a large scale multivariable control system can be developed at DCS in the bottom-up style.

APPENDIX

By applying the partial model matching approach by Kitamori [14], [15] to (7) $\sim$ (10) , (35) for ii loop can be obtained.

$$G_{ii}(s)^{-1} + F_{ii}(s) = \frac{(1+T_{ii}s)exp(L_{ii}s)}{K_{ii}} \quad (35)$$

where $G_{ii}(s)^{-1}$ can be expanded as denominator series of $G_{ii}(s)$ as in (36),

$$G_{ii}(s)^{-1} = g_{ii0} + g_{ii1}s + g_{ii2}s^2 + g_{ii3}s^3 + g_{ii4}s^4 + \quad (36)$$

$F_{ii}(s)$ and $(1+T_{ii}s)exp(L_{ii}s)/K_{ii}$ can be expanded as in (37) and (38) by taking a well-known Taylor series expansion method for each transfer functions.

$$F_{ii}(s) = K_{Fii} + K_{Fii}T_{Fii}(1-\kappa_{ii})s - K_{Fii}T_{Fii}^2\kappa_{ii}(1-\kappa_{ii})s^2 + K_{Fii}T_{Fii}^3\kappa_{ii}^2(1-\kappa_{ii})s^3 - \cdots \quad (37)$$

$$\frac{(1+T_{ii}s)exp(L_{ii}s)}{K_{ii}} = \frac{1}{K_{ii}} + \frac{T_{ii}+L_{ii}}{K_{ii}}s + \frac{T_{ii}L_{ii}+L_{ii}^2/2}{K_{ii}}s^2 + \frac{T_{ii}L_{ii}^2/2+L_{ii}^3/6}{K_{ii}}s^3 + \cdots \quad (38)$$

By substituting (36), (37) and (38) to (35) and matching the low-order coefficients at powers of s between both side, nonlinear equations as shown in (39) $\sim$ (43) are obtained.

$$\frac{1}{K_{ii}} = g_{ii0} + K_{Fii} \quad (39)$$

$$\frac{T_{ii}+L_{ii}}{K_{ii}} = g_{ii1} + K_{Fii}T_{Fii}(1-\kappa_{ii}) \quad (40)$$

$$\frac{T_{ii}L_{ii}+L_{ii}^2/2}{K_{ii}} = g_{ii2} - K_{Fii}T_{Fii}^2(1-\kappa_{ii})\kappa_{ii} \quad (41)$$

$$\frac{T_{ii}L_{ii}^2/2+L_{ii}^3/6}{K_{ii}} = g_{ii3} + K_{Fii}T_{Fii}^3(1-\kappa_{ii})\kappa_{ii}^2 \quad (42)$$

$$\frac{T_{ii}L_{ii}^3/6+L_{ii}^4/24}{K_{ii}} = g_{ii4} - K_{Fii}T_{Fii}^4(1-\kappa_{ii})\kappa_{ii}^3 \quad (43)$$

By solving these equations of (39) $\sim$ (43), optimal PD feedback parameters K_{Fii}, T_{Fii} and the first order delay system with dead time K_{ii}, T_{ii} and L_{ii} can be obtained.

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