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Decentralized Variable Gain Robust Controllers for a Class of Uncertain Large-Scale Interconnected Systems with State Delays

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***Abstract*— In this paper, we propose a decentralized variable gain robust controller which achieves not only robust stability but also satisfactory transient performance for a class of uncertain large-scale interconnected systems with state delays. The proposed decentralized robust controller consists of a fixed feedback gain controller and a variable one tuned by parameter adjustment laws. In this paper, it is shown that sufficient conditions for the existence of the proposed decentralized variable gain robust control system are given in terms of LMIs. Finally, a simple illustrative example is shown.**

I. INTRODUCTION

Robust controllers for uncertain systems have been the focus of much attention in feedback control. A great many results have been obtained on the problems of robust stability analysis and robust stabilization (e.g. [1] and references therein). Besides, several variable gain robust state feedback controllers for uncertain systems have also been suggested (e.g. [2], [3]). In the work of [2], a robust controller with adaptation mechanism has been presented and the adaptive robust controller is tuned on-line based on the information about parameter uncertainties. Additionally, Oya and Hagino have proposed robust controllers with adaptive compensation inputs which acieve not only robust stability but also satisfactory transient response[3].

On the other hand, due to the complication of systems because of the rapid development of modern industry, decentralized control problems for large-scale interconnected systems have been widely studied[4]. Thus decentralized robust control of uncertain large-scale interconnected systems has also attracted the attention of many researchers (e.g [5], [6], [7]). In Mao and Lin[6] for large-scale interconnected systems with unmodelled interaction, the aggregative derivation are tracked by using a model following technique with on-line improvement, and a sufficient condition for which the overall system when controlled by the completely decentralized control is asymptotically stable has been established. Gong[7] has proposed decentralized robust controllers which guarantee robust stability with prescribed degree of exponential convergence. Mukaidani et al.[8], [9] have also proposed decentralized guaranteed cost controllers for uncertain large-scale interconnected systems. In addition, we have suggested a decentralized variable gain robust controller which achieves not only robust stability but also satisfactory transient behavior for a class of uncertain large-scale interconnected systems[10].

In this paper, on the basis of the existing result[10], we propose a decentralized variable gain robust controller for a class of uncertain large-scale interconnected systems with state delays. For the uncertain large-scale interconnected system with state delays, uncertainties and interactions satisfy the matching condition. The proposed decentralized robust control input is composed of a state feedback with a fixed feedback gain matrix designed by using nominal subsystem and an error signal feedback with a fixed compensation gain matrix and a variable one determined by a parameter adjustment law. This paper is organized as follows. notations and useful lemmas which are used in this paper are shown in Section II, and in Section III, the class of uncertain large-scale interconnected systems with state delays which are considered in this paper is introduced. The main results are presented in Section IV, i.e. LMI-based sufficient conditions for the existence of the proposed decentralized variable gain robust controller are presented. Finally, a simple illustrative example is included.

II. NOTATIONS AND LEMMAS

In this section, we introduce notations, and useful and well-known lemmas (see [11], [12] for details) which are used in this paper as well as the existing work[13].

In the paper, the following notations are used. For a matrix $\mathcal{X}$, the inverse of matrix $\mathcal{X}$ and the transpose of one are denoted by $\mathcal{X}^{-1}$ and $\mathcal{X}^T$, respectively. Additionally $H_e\{\mathcal{X}\}$ and I_n mean $\mathcal{X} + \mathcal{X}^T$ and n-dimensional identity matrix, respectively, and the notation $\mathrm{diag}\,(\mathcal{X}_1, \cdots, \mathcal{X}_\mathcal{M})$ represents a block diagonal matrix composed of matrices $\mathcal{X}_i$ for $i = 1, \cdots, \mathcal{M}$. For real symmetric matrices $\mathcal{X}$ and $\mathcal{Y}$, $\mathcal{X} > \mathcal{Y}$ (resp. $\mathcal{X} \geq \mathcal{Y}$) means that $\mathcal{X} - \mathcal{Y}$ is positive (resp. nonnegative) definite matrix. For a vector $\alpha \in \mathbb{R}^n$, $||\alpha||$ denotes standard Euclidian norm and for a matrix $\mathcal{X}$, $||\mathcal{X}||$ represents its induced norm. The symbols "$\triangleq$" and "$\star$" mean equality by definition and symmetric blocks in matrix inequalities, respectively.

Lemma 1: For arbitrary vectors α and β and the matrices $\mathcal{X}$ and $\mathcal{Y}$ which have appropriate dimensions, the following inequality holds.

$$2\alpha^T \mathcal{X}\Delta(t)\mathcal{Y}\beta \leq 2\left\|\mathcal{X}^T\alpha\right\|\left\|\mathcal{Y}\beta\right\|$$

where $\Delta(t) \in \mathbb{R}^{s\times t}$ is a time-varying matrix and it satisfies the relation $\left\|\Delta(t)\right\| \leq 1.0$.

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$$\frac{d}{dt}x_i(t) = A_{ii}(t)x_i(t) + \sum_{\substack{j=1 \\ j\neq i}}^{N} A_{ij}(t)x_j(t) + \sum_{j=1}^{N} H_{ij}(t)x_j(t-h_{ij}) + B_iu_i(t) \tag{1}$$

$$\frac{d}{dt}e_i(t) = (A_{K_i} + B_iF_i)\,e_i(t) + B_i\Delta_{ii}(t)\mathcal{L}_ix_i(t) + B_i\sum_{\substack{j=1 \\ j\neq i}}^{N}(\mathcal{D}_{ij} + \Delta_{ij}(t)\mathcal{M}_{ij})x_j(t)$$
$$+ B_i\sum_{j=1}^{N}(\mathcal{E}_{ij} + \Delta_{hij}(t)\mathcal{N}_{ij})x_j(t-h_{ij}) + B_i\mathcal{G}_i(x_i, e_i, t)e_i(t) \tag{8}$$

Lemma 2: (Schur complement) For a given constant real symmetric matrix Ξ, the following items are equivalent.

(i) $\Xi = \begin{pmatrix} \Xi_{11} & \Xi_{12} \\ \Xi_{12}^T & \Xi_{22} \end{pmatrix} > 0$

(ii) $\Xi_{11} > 0$ and $\Xi_{22} - \Xi_{12}^T\Xi_{11}^{-1}\Xi_{12} > 0$

(iii) $\Xi_{22} > 0$ and $\Xi_{11} - \Xi_{12}\Xi_{22}^{-1}\Xi_{12}^T > 0$

III. Problem Formulation

Consider the uncertain large-scale interconnected system with state delays composed of N subsystems described as (1). In (1), $x_i(t) \in \mathbb{R}^{n_i}$ and $u_i(t) \in \mathbb{R}^{m_i}$ $(i = 1, \cdots, N)$ are the vectors of the state and the control input for the i-th subsystem, respectively and $x(t) = \left(x_1^T(t), \cdots, x_N^T(t)\right)^T$ is the state of the overall system. The matrices $A_{ii}(t)$, $A_{ij}(t)$ and $H_{ij}(t)$ are given by

$$\begin{aligned} A_{ii}(t) &= A_{ii} + B_i\Delta_{ii}(t)\mathcal{L}_{ii} \\ A_{ij}(t) &= B_i\mathcal{D}_{ij} + B_i\Delta_{ij}(t)\mathcal{M}_{ij} \\ H_{ij}(t) &= B_i\mathcal{E}_{ij} + B_i\Delta_{h_{ij}}(t)\mathcal{N}_{ij} \end{aligned} \tag{2}$$

i.e. the uncertainties and the interaction terms satisfy the matching condition. In (2), the matrices $A_{ii} \in \mathbb{R}^{n_i\times n_i}$, $A_{ij} \in \mathbb{R}^{n_i\times n_j}$ and $B_i \in \mathbb{R}^{n_i\times m_i}$ denote the nominal system matrix and the nominal input one. Additionally, the matrices $\mathcal{L}_{ii}$, $\mathcal{D}_{ij}$, $\mathcal{M}_{ij}$, $\mathcal{E}_{ij}$ and $\mathcal{N}_{ij}$ with appropriate dimensions mean the structure of uncertainties, interactions and state delays. Besides, matrices $\Delta_{ii}(t) \in \mathbb{R}^{m_i\times p_i}$, $\Delta_{ij}(t) \in \mathbb{R}^{m_i\times q_{ij}}$ and $\Delta_{h_{ij}}(t) \in \mathbb{R}^{m_i\times r_{ij}}$ denote unknown parameters satisfying the relations $\|\Delta_{ii}(t)\| \le 1.0$, $\|\Delta_{ij}(t)\| \le 1.0$ and $\|\Delta_{h_{ij}}(t)\| \le 1.0$ respectively.

Now, the nominal subsystem which is obtained by ignoring uncertainties and interactions in (1), is shown as

$$\frac{d}{dt}\overline{x}_i(t) = A_{ii}\overline{x}_i(t) + B_i\overline{u}_i(t). \tag{3}$$

In (3), $\overline{x}_i(t) \in \mathbb{R}^{n_i}$ and $\overline{u}_i(t) \in \mathbb{R}^{m_i}$ are the vectors of the state and the control input for the i-th nominal subsystem, respectively.

First of all, we adopt the standard linear quadratic control problem for the i-th nominal subsystem of (3) so as to generate the desired trajectory in time response for the uncertain i-th subsystem of (1) systematically. Note that some other design method for deriving the desirable response can also be adopted (e.g. pole placement). It is widely known that the optimal control input for the i-th nominal subsystem of (3) can be obtained as

$$\begin{aligned} \overline{u}_i(t) &= K_i\overline{x}_i(t) \\ K_i &\stackrel{\triangle}{=} -\mathcal{R}_i^{-1}B_i^T\mathcal{X}_i \end{aligned} \tag{4}$$

In (4), $\mathcal{X}_i \in \mathbb{R}^{n_i\times n_i}$ is a symmetric positive define matrix which satisfies the algebraic Riccati equation

$$H_e\left\{A_{ii}^T\mathcal{X}_i\right\} - \mathcal{X}_iB_i\mathcal{R}_i^{-1}B_i^T\mathcal{X}_i + \mathcal{Q}_i = 0 \tag{5}$$

where the weighting matrices $\mathcal{Q}_i \in \mathbb{R}^{n_i\times n_i}$ and $\mathcal{R}_i \in \mathbb{R}^{m_i\times m_i}$ are positive definite and are determined in advance so that the desirable transient behavior is achieved.

Based on the existing result[3], let us introduce error vectors $e_i(t) \stackrel{\triangle}{=} x_i(t) - \overline{x}_i(t)$. Besides, using the feedbackgain matrix $K_i \in \mathbb{R}^{m_i\times n_i}$ of (4) for the i-th subsystem of (1), we consider the following control input.

$$u_i(t) \stackrel{\triangle}{=} K_ix_i(t) + v_i(t) \tag{6}$$

where $v_i(t) \in \mathbb{R}^{m_i}$ is the compensation input[3] defined as

$$v_i(t) \stackrel{\triangle}{=} F_ie_i(t) + \mathcal{G}_i(x_i, e_i, t)e_i(t). \tag{7}$$

In (7), $F_i \in \mathbb{R}^{m_i\times n_i}$ and $\mathcal{G}_i(x_i, e_i, t) \in \mathbb{R}^{m_i\times n_i}$ are the fixed compensation gain matrix and the variable one for the i-th subsystem of (1). From (1) – (3), (6) and (7), the uncertain error subsystem of (8) is derived. In (8), $A_{K_i} \in \mathbb{R}^{n_i\times n_i}$ is the stable matrix described as $A_{K_i} = A_{ii} + B_iK_i$.

From the above discussion, our design objective in this paper is to determine the decentralized variable gain robust control input of (6) such that the resultant overall system achieves not only robust stability but also satisfactory transient behavior. That is to design the fixed compensation gain matrix $F_i \in \mathbb{R}^{m_i\times n_i}$ and the variable one $\mathcal{G}_i(x_i, e_i, t) \in \mathbb{R}^{m_i\times n_i}$ such that the overall error system composed of N error subsystems of (8) is asymptotically stable.

IV. Decentralized Variable Gain Controllers

The following theorem shows a sufficient condition for the existence of the proposed decentralized control system.

Theorem 1: Consider the uncertain error subsystem of (8) and the control input of (6).

If the LMIs of (9) – (12) are feasible, by using symmetric positive definite matrices $\mathcal{Y}_i \in \mathbb{R}^{n_i\times n_i}$, $\mathcal{Y}_{ij} \in \mathbb{R}^{n_i\times n_i}$,

$$\mathcal{G}_i(x_i,e_i,t) \triangleq \begin{cases} -\dfrac{\|B_i^T\mathcal{P}_ie_i(t)\|\|\mathcal{L}_{ii}x_i(t)\| + 2\left\{\epsilon_i(N-1)+\delta_iN\right\}\|B_i^T\mathcal{P}_ie_i(t)\|^2}{\|B_i^T\mathcal{P}_ie_i(t)\|^2}B_i^T\mathcal{P}_i \\ \mathcal{G}_i(x_i,e_i,t_\epsilon) \end{cases} \quad (13)$$

$$\mathit{\Lambda}_i(\mathcal{Y}_i) = \begin{pmatrix} \mathcal{Y}_i\mathcal{D}_{1i}^T & \mathcal{Y}_i\mathcal{M}_{1i}^T & \cdots & \mathcal{Y}_i\mathcal{D}_{i-1\ i}^T & \mathcal{Y}_i\mathcal{M}_{i-1\ i}^T & \mathcal{Y}_i\mathcal{D}_{i+1\ i}^T & \mathcal{Y}_i\mathcal{M}_{i+1\ i}^T & \cdots & \mathcal{Y}_i\mathcal{D}_{Ni}^T & \mathcal{Y}_i\mathcal{M}_{Ni}^T \end{pmatrix} \quad (14)$$

$$\Theta(\mathcal{Y}_i) = \begin{pmatrix} \mathcal{Y}_i & \mathcal{Y}_i & \cdots & \mathcal{Y}_i \end{pmatrix} \quad (15)$$

$$\Omega(\mathcal{Y}_i) = \mathrm{diag}\begin{pmatrix} -\mathcal{Y}_{1i} & -\mathcal{Y}_{2i} & \cdots & -\mathcal{Y}_{Ni} \end{pmatrix} \quad (16)$$

$$\mathit{\Psi}_i = \begin{pmatrix} \mathcal{D}_{1i}^T & \mathcal{M}_{1i}^T & \cdots & \mathcal{D}_{i-1\ i}^T & \mathcal{M}_{i-1\ i}^T & \mathcal{D}_{i+1\ i}^T & \mathcal{M}_{i+1\ i}^T & \cdots & \mathcal{D}_{Ni}^T & \mathcal{M}_{Ni}^T \end{pmatrix} \quad (17)$$

$$\mathit{\Gamma}_i(\epsilon_i) = \mathrm{diag}\left(-\epsilon_1 I_{m_1},\ \epsilon_1 I_{q_{1i}},\ \cdots,\ \epsilon_{i-1}I_{m_{i-1}}, \epsilon_{i-1}I_{q_{i-1i}},\ \epsilon_{i+1}I_{m_{i+1}}, \epsilon_i I_{q_{i+1i}},\ \cdots,\ \epsilon_N I_{m_N},\ \epsilon_N I_{q_{Ni}}\right) \quad (18)$$

$$\begin{aligned}
\frac{d}{dt}\mathcal{V}_{e_i}(t) &= e_i^T(t)\left[H_e\left\{(A_{K_i}+B_iF_i)^T\mathcal{P}_i\right\}\right]e_i(t) + 2e_i^T(t)\mathcal{P}_iB_i\Delta_{ii}(t)\mathcal{L}_ix_i(t) + 2e_i^T(t)\mathcal{P}_iB_i\mathcal{G}_i(x_i,e_i,t)e_i(t) \\
&\quad + 2e_i^T(t)\mathcal{P}_iB_i\sum_{\substack{j=1\\j\neq i}}^{N}\left(\mathcal{D}_{ij}+\Delta_{ij}(t)\mathcal{M}_{ij}\right)\left(e_j(t)+\overline{x}_j(t)\right) \\
&\quad + 2e_i^T(t)\mathcal{P}_iB_i\sum_{j=1}^{N}\left(\mathcal{E}_{ij}+\Delta_{h_{ij}}(t)\mathcal{N}_{ij}\right)\left(e_j(t-h_{ij})+\overline{x}_j(t-h_{ij})\right) \\
&\quad + \sum_{j=1}^{N}\left(e_j^T(t)\mathcal{P}_{ij}e_j(t) - e_j^T(t-h_{ij})\mathcal{P}_{ij}e_j(t-h_{ij})\right) \\
&\leq e_i^T(t)\left[H_e\left\{(A_{K_i}+B_iF_i)^T\mathcal{P}_i\right\}\right]e_i(t) \\
&\quad + 2\|B_i^T\mathcal{P}_ie_i(t)\|\|\mathcal{L}_ix_i(t)\| + 2e_i^T(t)\mathcal{P}_iB_i\mathcal{G}_i(x_i,e_i,t)e_i(t) + 4\epsilon_i(N-1)\|B^T\mathcal{P}_ie_i(t)\|^2 + 4\delta_iN\|B^T\mathcal{P}_ie_i(t)\|^2 \\
&\quad + \frac{1}{\epsilon_i}\sum_{\substack{j=1\\j\neq i}}^{N}e_j^T(t)\left(\mathcal{D}_{ij}^T\mathcal{D}_{ij}+\mathcal{M}_{ij}^T\mathcal{M}_{ij}\right)e_j(t) + \frac{1}{\epsilon_i}\sum_{\substack{j=1\\j\neq i}}^{N}\overline{x}_j^T(t)\left(\mathcal{D}_{ij}^T\mathcal{D}_{ij}+\mathcal{M}_{ij}^T\mathcal{M}_{ij}\right)\overline{x}_j(t) \\
&\quad + \frac{1}{\delta_i}\sum_{j=1}^{N}e_j^T(t-h_{ij})\left(\mathcal{E}_{ij}^T\mathcal{E}_{ij}+\mathcal{N}_{ij}^T\mathcal{N}_{ij}\right)e_j(t-h_{ij}) + \frac{1}{\delta_i}\sum_{j=1}^{N}\overline{x}_j^T(t-h_{ij})\left(\mathcal{E}_{ij}^T\mathcal{E}_{ij}+\mathcal{N}_{ij}^T\mathcal{N}_{ij}\right)\overline{x}_j(t-h_{ij}) \\
&\quad + \sum_{j=1}^{N}\left\{e_j^T(t)\mathcal{P}_{ij}e_j(t) - e_j^T(t-h_{ij})\mathcal{P}_{ij}e_j(t-h_{ij})\right\}
\end{aligned} \quad (22)$$

$\mathcal{S}_i \in \mathbb{R}^{n_i\times n_i}$ and $\mathcal{S}_{ij} \in \mathbb{R}^{n_i\times n_i}$, matrices $\mathcal{W}_i \in \mathbb{R}^{m_i\times n_i}$ and positive constants ϵ_i and δ_i which satisfy the LMIs

$$\begin{pmatrix} H_e\{A_{K_i}\mathcal{Y}_i + B_i\mathcal{W}_i\} & \Theta(\mathcal{Y}_i) & \mathit{\Lambda}_i(\mathcal{Y}_i) \\ \star & \Omega(\mathcal{Y}_i) & 0 \\ \star & \star & -\mathit{\Gamma}_i(\epsilon_i) \end{pmatrix} < 0 \quad (9)$$

$$\begin{pmatrix} H_e\{\mathcal{S}_iA_{K_i}\} + \sum_{j=1}^{N}\mathcal{S}_{ji} & \mathit{\Psi}_i \\ \star & -\mathit{\Gamma}_i(\epsilon_i) \end{pmatrix} < 0 \quad (10)$$

$$\begin{pmatrix} -\mathcal{Y}_{ij} & \mathcal{Y}_{ij}\mathcal{E}_{ij}^T & \mathcal{Y}_{ij}\mathcal{N}_{ij}^T \\ \star & -\delta_iI_{m_i} & 0 \\ \star & \star & -\delta_iI_{r_{ij}} \end{pmatrix} < 0 \quad (11)$$

$$\begin{pmatrix} -\mathcal{S}_{ij} & \mathcal{E}_{ij}^T & \mathcal{N}_{ij}^T \\ \star & -\delta_iI_{m_i} & 0 \\ \star & \star & -\delta_iI_{r_{ij}} \end{pmatrix} < 0 \quad (12)$$

the fixed gain matrix $F_i \in \mathbb{R}^{m_i\times n_i}$ and the variable one $\mathcal{G}_i(x_i,e_i,t) \in \mathbb{R}^{m_i\times n_i}$ are determined as $F_i = \mathcal{W}_i\mathcal{Y}_i^{-1}$ and (13), respectively. In (9) – (12), matrices $\mathit{\Lambda}_i(\mathcal{Y}_i)$, $\Theta(\mathcal{Y}_i)$, $\Omega(\mathcal{Y}_i)$, $\mathit{\Psi}_i$ and $\mathit{\Gamma}_i(\epsilon_i)$ are given by (14) – (18), respectively. Besides, t_ϵ in (13) is given by $t_\epsilon = \lim_{\epsilon>0,\epsilon\to 0}(t-\epsilon)$[2].

Then the overall error system composed of the N error subsystems of (8) is robustly stable.

Proof : In order to prove **Theorem 1**, let us define the following quadratic function as a Lyapunov function candidate.

$$\mathcal{V}(t) \triangleq \sum_{i=1}^{N}\mathcal{V}_{e_i}(t) + \sum_{i=1}^{N}\mathcal{V}_{\overline{x}_i}(t) \quad (19)$$

where $\mathcal{V}_{e_i}(t)$ and $\mathcal{V}_{\overline{x}_i}(t)$ are given by

$$\mathcal{V}_{e_i}(t) \triangleq e_i^T(t)\mathcal{P}_ie_i(t) + \sum_{j=1}^{N}\int_{t-h_{ij}}^{t}e_j^T(\theta)\mathcal{P}_{ij}e_j(\theta)d\theta \quad (20)$$

$$\mathcal{V}_{\overline{x}_i}(t) \triangleq \overline{x}_i^T(t)\mathcal{S}_i\overline{x}_i(t) + \sum_{j=1}^{N}\int_{t-h_{ij}}^{t}\overline{x}_j^T(\theta)\mathcal{S}_{ij}\overline{x}_j(\theta)d\theta. \quad (21)$$

$$\frac{d}{dt}\mathcal{V}_{\overline{x}_i}(t) = \overline{x}_i^T(t)\left[H_e\left\{A_{K_i}^T\mathcal{S}_i\right\}\right]\overline{x}_i(t) + \sum_{j=1}^{N}\left\{\overline{x}_j^T(t)\mathcal{S}_{ij}\overline{x}_j(t) - \overline{x}_j^T(t-h_{ij})\mathcal{S}_{ij}\overline{x}_j(t-h_{ij})\right\} \tag{24}$$

$$\begin{aligned}\frac{d}{dt}\mathcal{V}_{e_i}(t) \le\ & e_i^T(t)\left[H_e\left\{\left(A_{K_i}+B_iF_i\right)^T\mathcal{P}_i\right\}\right]e_i(t) + \sum_{j=1}^{N}\left\{e_j^T(t)\mathcal{P}_{ij}e_j(t) - e_j^T(t-h_{ij})\mathcal{P}_{ij}e_j(t-h_{ij})\right\} \\ & + \frac{1}{\epsilon_i}\sum_{\substack{j=1\\j\ne i}}^{N} e_j^T(t)\left(\mathcal{D}_{ij}^T\mathcal{D}_{ij}+\mathcal{M}_{ij}^T\mathcal{M}_{ij}\right)e_j(t) + \frac{1}{\epsilon_i}\sum_{\substack{j=1\\j\ne i}}^{N}\overline{x}_j^T(t)\left(\mathcal{D}_{ij}^T\mathcal{D}_{ij}+\mathcal{M}_{ij}^T\mathcal{M}_{ij}\right)\overline{x}_j(t) \\ & + \frac{1}{\delta_i}\sum_{j=1}^{N} e_j^T(t-h_{ij})\left(\mathcal{E}_{ij}^T\mathcal{E}_{ij}+\mathcal{N}_{ij}^T\mathcal{N}_{ij}\right)e_j(t-h_{ij}) + \frac{1}{\delta_i}\sum_{j=1}^{N}\overline{x}_j^T(t-h_{ij})\left(\mathcal{E}_{ij}^T\mathcal{E}_{ij}+\mathcal{N}_{ij}^T\mathcal{N}_{ij}\right)\overline{x}_j(t-h_{ij})\end{aligned} \tag{25}$$

$$\begin{aligned}\frac{d}{dt}\mathcal{V}(t) \le\ & \sum_{i=1}^{N} e_i^T(t)\left[H_e\left\{\left(A_{K_i}+B_iF_i\right)^T\mathcal{P}_i\right\}\right]e_i(t) + \sum_{i=1}^{N}\overline{x}_i^T(t)\left[H_e\left\{A_{K_i}^T\mathcal{S}_i\right\}\right]\overline{x}_i(t) \\ & + \sum_{i=1}^{N}\left\{\frac{1}{\epsilon_i}\sum_{\substack{j=1\\j\ne i}}^{N} e_j^T(t)\left(\mathcal{D}_{ij}^T\mathcal{D}_{ij}+\mathcal{M}_{ij}^T\mathcal{M}_{ij}\right)e_j(t) + \frac{1}{\epsilon_i}\sum_{\substack{j=1\\j\ne i}}^{N}\overline{x}_j^T(t)\left(\mathcal{D}_{ij}^T\mathcal{D}_{ij}+\mathcal{M}_{ij}^T\mathcal{M}_{ij}\right)\overline{x}_j(t)\right\} \\ & + \sum_{i=1}^{N}\left\{\frac{1}{\epsilon_i}\sum_{\substack{j=1\\j\ne i}}^{N} e_j^T(t-h_{ij})\left(\mathcal{E}_{ij}^T\mathcal{E}_{ij}+\mathcal{N}_{ij}^T\mathcal{N}_{ij}\right)e_j(t-h_{ij}) + \frac{1}{\epsilon_i}\sum_{\substack{j=1\\j\ne i}}^{N}\overline{x}_j^T(t-h_{ij})\left(\mathcal{E}_{ij}^T\mathcal{E}_{ij}+\mathcal{N}_{ij}^T\mathcal{N}_{ij}\right)\overline{x}_j(t-h_{ij})\right\} \\ & + \sum_{i=1}^{N}\sum_{j=1}^{N}\left\{e_j^T(t)\mathcal{P}_{ij}e_j(t) - e_j^T(t-h_{ij})\mathcal{P}_{ij}e_j(t-h_{ij}) + \overline{x}_j^T(t)\mathcal{S}_{ij}\overline{x}_j(t) - \overline{x}_j^T(t-h_{ij})\mathcal{S}_{ij}\overline{x}_j(t-h_{ij})\right\}\end{aligned} \tag{26}$$

$$\begin{aligned}\frac{d}{dt}\mathcal{V}(t) \le\ & \sum_{i=1}^{N} e_i^T(t)\left[H_e\left\{\left(A_{K_i}+B_iF_i\right)^T\mathcal{P}_i\right\} + \sum_{\substack{j=1\\j\ne i}}^{N}\frac{1}{\epsilon_j}\left(\mathcal{D}_{ji}^T\mathcal{D}_{ji}+\mathcal{M}_{ji}^T\mathcal{M}_{ji}\right) + \sum_{j=1}^{N}\mathcal{P}_{ji}\right]e_i(t) \\ & + \sum_{i=1}^{N}\left\{\frac{1}{\delta_i}\sum_{j=1}^{N} e_j^T(t-h_{ij})\left(\mathcal{E}_{ij}^T\mathcal{E}_{ij}+\mathcal{N}_{ij}^T\mathcal{N}_{ij}-\delta_i\mathcal{P}_{ij}\right)e_j(t-h_{ij})\right\} \\ & + \sum_{i=1}^{N}\overline{x}_i^T(t)\left[H_e\left\{A_{K_i}^T\mathcal{S}_i\right\} + \sum_{\substack{j=1\\j\ne i}}^{N}\frac{1}{\epsilon_j}\left(\mathcal{D}_{ji}^T\mathcal{D}_{ji}+\mathcal{M}_{ji}^T\mathcal{M}_{ji}\right) + \sum_{j=1}^{N}\mathcal{S}_{ji}\right]\overline{x}_i(t) \\ & + \sum_{i=1}^{N}\left\{\frac{1}{\delta_i}\sum_{j=1}^{N}\overline{x}_j^T(t-h_{ij})\left(\mathcal{E}_{ij}^T\mathcal{E}_{ij}+\mathcal{N}_{ij}^T\mathcal{N}_{ij}-\delta_i\mathcal{S}_{ij}\right)\overline{x}_j(t-h_{ij})\right\}\end{aligned} \tag{27}$$

For the quadratic functions $\mathcal{V}_{e_i}(t)$ of (20), its time derivative can be computed as (22). Note that for derivation of (22), **Lemma 1** and the well-known inequality

$$2\alpha^T\beta \le \delta\alpha^T\alpha + \frac{1}{\delta}\beta^T\beta. \tag{23}$$

for any vectors α and β with appropriate dimensions and a positive scalar δ have been used. Furthermore for the quadratic functions $\mathcal{V}_{\overline{x}_i}(t)$ of (21), its time derivative can be computed as (24).

Firstly, we consider the case of $B_i^T\mathcal{P}_ie_i(t) \ne 0$. In this case, substituting the variable gain matrix of (13) into (22) and some algebraic manipulations give the inequality of (25). Thus, we can see that the relation of (26) for the quadratic fucntion $\mathcal{V}(t)$ of (19) can be obtained. Since the inequality of (26) can be rewritten as (27), if the matrix inequality conditions of (28) — (30) are holds, then the following inequality is satisfied.

$$\frac{d}{dt}\mathcal{V}(t) < 0 \quad \text{for} \quad \forall\xi(t) \ne 0 \tag{31}$$

where $\xi(t) \overset{\triangle}{=} \left(e_1^T(t), \cdots, e_N^T(t), \overline{x}_1^T(t), \cdots, \overline{x}_N^T(t)\right)^T$.

$$H_e\left\{\left(A_{K_i}+B_iF_i\right)^T\mathcal{P}_i\right\}+\sum_{\substack{j=1\\j\neq i}}^{N}\frac{1}{\epsilon_j}\left(\mathcal{D}_{ji}^T\mathcal{D}_{ji}+\mathcal{M}_{ji}^T\mathcal{M}_{ji}\right)+\sum_{j=1}^{N}\mathcal{P}_{ji}<0 \tag{28}$$

$$H_e\left\{A_{K_i}^T\mathcal{S}_i\right\}+\sum_{\substack{j=1\\j\neq i}}^{N}\frac{1}{\epsilon_j}\left(\mathcal{D}_{ji}^T\mathcal{D}_{ji}+\mathcal{E}_{ji}^T\mathcal{E}_{ji}\right)+\sum_{j=1}^{N}\mathcal{S}_{ji}<0 \tag{29}$$

$$\mathcal{E}_{ij}^T\mathcal{E}_{ij}+\mathcal{N}_{ij}^T\mathcal{N}_{ij}-\delta_i\mathcal{P}_{ij}<0 \text{ and } \mathcal{E}_{ij}^T\mathcal{E}_{ij}+\mathcal{N}_{ij}^T\mathcal{N}_{ij}-\delta_i\mathcal{S}_{ij}<0 \tag{30}$$

$$\begin{aligned}\frac{d}{dt}\mathcal{V}(t)=&\sum_{i=1}^{N}e_i^T(t)\left[H_e\left\{\left(A_{K_i}+B_iF_i\right)^T\mathcal{P}_i\right\}+\sum_{j=1}^{N}\mathcal{P}_{ij}\right]e_i(t)+\sum_{i=1}^{N}\overline{x}_i^T(t)\left[H_e\left\{A_{K_i}^T\mathcal{S}_i\right\}+\sum_{j=1}^{N}\mathcal{S}_{ij}\right]\overline{x}_i(t)\\&-\sum_{i=1}^{N}\sum_{j=1}^{N}e_j^T(t-h_{ij})\mathcal{P}_{ij}e_j(t-h_{ij})-\sum_{i=1}^{N}\sum_{j=1}^{N}\overline{x}_j^T(t-h_{ij})\mathcal{S}_{ij}\overline{x}_j(t-h_{ij})\end{aligned} \tag{32}$$

$$H_e\left\{A_{K_i}\mathcal{Y}_i+B_i\mathcal{W}_i\right\}+\sum_{\substack{j=1\\j\neq i}}^{N}\frac{1}{\epsilon_j}\mathcal{Y}_i\left(\mathcal{D}_{ji}^T\mathcal{D}_{ji}+\mathcal{M}_{ji}^T\mathcal{M}_{ji}\right)\mathcal{Y}_i+\mathcal{Y}_i\left(\sum_{j=1}^{N}\mathcal{P}_{ji}\right)\mathcal{Y}_i<0 \tag{33}$$

$$\mathcal{Y}_{ij}\left(\mathcal{E}_{ij}^T\mathcal{E}_{ij}+\mathcal{N}_{ij}^T\mathcal{N}_{ij}\right)\mathcal{Y}_{ij}-\delta_i\mathcal{Y}_{ij}<0 \tag{34}$$

$$\begin{aligned}&A_{11}=\begin{pmatrix}-1.0 & 1.0\\ 0.0 & 1.0\end{pmatrix},\ A_{22}=\begin{pmatrix}0.0 & 1.0\\ -1.0 & -1.0\end{pmatrix},\ A_{33}=\begin{pmatrix}1.0 & 0.0\\ -1.0 & -3.0\end{pmatrix},\\
&B_1=\begin{pmatrix}0.0\\1.0\end{pmatrix},\ B_2=\begin{pmatrix}1.0\\1.0\end{pmatrix},\ B_3=\begin{pmatrix}1.0\\0.0\end{pmatrix},\mathcal{L}_{11}^T=\begin{pmatrix}1.0\\0.0\end{pmatrix},\ \mathcal{L}_{22}^T=\begin{pmatrix}1.0\\2.0\end{pmatrix},\ \mathcal{L}_{33}^T=\begin{pmatrix}1.0\\1.0\end{pmatrix},\\
&\mathcal{D}_{12}^T=\begin{pmatrix}1.0\\2.0\end{pmatrix},\ \mathcal{D}_{13}^T=\begin{pmatrix}2.0\\2.0\end{pmatrix},\ \mathcal{D}_{21}^T=\begin{pmatrix}1.0\\0.0\end{pmatrix},\ \mathcal{D}_{23}^T=\begin{pmatrix}1.0\\1.0\end{pmatrix},\ \mathcal{D}_{31}^T=\begin{pmatrix}2.0\\1.0\end{pmatrix},\ \mathcal{D}_{32}^T=\begin{pmatrix}0.0\\3.0\end{pmatrix},\\
&\mathcal{M}_{12}^T=\begin{pmatrix}2.0\\1.0\end{pmatrix},\ \mathcal{M}_{13}^T=\begin{pmatrix}2.0\\2.0\end{pmatrix},\ \mathcal{M}_{21}^T=\begin{pmatrix}1.0\\0.0\end{pmatrix},\ \mathcal{M}_{23}^T=\begin{pmatrix}0.0\\3.0\end{pmatrix},\ \mathcal{M}_{31}^T=\begin{pmatrix}1.0\\1.0\end{pmatrix},\ \mathcal{M}_{32}^T=\begin{pmatrix}3.0\\1.0\end{pmatrix},\\
&\mathcal{E}_{11}^T=\begin{pmatrix}1.0\\2.0\end{pmatrix},\ \mathcal{E}_{12}^T=\begin{pmatrix}1.0\\1.0\end{pmatrix},\ \mathcal{E}_{13}^T=\begin{pmatrix}2.0\\1.0\end{pmatrix},\mathcal{E}_{21}^T=\begin{pmatrix}1.0\\1.0\end{pmatrix},\ \mathcal{E}_{22}^T=\begin{pmatrix}2.0\\1.0\end{pmatrix},\ \mathcal{E}_{23}^T=\begin{pmatrix}1.0\\0.0\end{pmatrix},\\
&\mathcal{E}_{31}^T=\begin{pmatrix}2.0\\2.0\end{pmatrix},\ \mathcal{E}_{32}^T=\begin{pmatrix}2.0\\2.0\end{pmatrix},\ \mathcal{E}_{33}^T=\begin{pmatrix}1.0\\0.0\end{pmatrix},\ \mathcal{N}_{11}^T=\begin{pmatrix}1.0\\0.0\end{pmatrix},\ \mathcal{N}_{12}^T=\begin{pmatrix}1.0\\1.0\end{pmatrix},\ \mathcal{N}_{13}^T=\begin{pmatrix}2.0\\3.0\end{pmatrix},\\
&\mathcal{N}_{21}^T=\begin{pmatrix}1.0\\1.0\end{pmatrix},\ \mathcal{N}_{22}^T=\begin{pmatrix}3.0\\1.0\end{pmatrix},\ \mathcal{N}_{23}^T=\begin{pmatrix}0.0\\3.0\end{pmatrix},\mathcal{N}_{31}^T=\begin{pmatrix}1.0\\0.0\end{pmatrix},\ \mathcal{N}_{32}^T=\begin{pmatrix}3.0\\1.0\end{pmatrix},\ \mathcal{N}_{33}^T=\begin{pmatrix}2.0\\1.0\end{pmatrix}\end{aligned} \tag{35}$$

Next we consider the case of $B_i^T\mathcal{P}_ie_i(t)=0$. In this case, from (22) and (24) the time derivative of the quadratic function $\mathcal{V}(t)$ of (19) can be written as (32). If the matrix inequalities of (28) — (30) are satisfied, then one can see that the first and the second terms of (32) are negative. Besides, the third and fourth terms are also negative. Namely in the case of $B_i^T\mathcal{P}_ie_i(t)=0$, the relation of (31) holds too.

From the above, robust stability of the overall error system is clearly guaranteed, because the nominal subsystem is asymptotically stable.

Finally, we consider the matrix inequalities of (28) – (30). By introducing the matrices $\mathcal{Y}_i\stackrel{\triangle}{=}\mathcal{P}_i^{-1}$, $\mathcal{Y}_{ij}\stackrel{\triangle}{=}\mathcal{P}_{ij}^{-1}$ and $\mathcal{W}_i\stackrel{\triangle}{=}F_i\mathcal{P}_i$ and pre- and post-multiplying both sides of the matrix inequality of (28) and the first inequality of (30) by $\mathcal{Y}_i$ and $\mathcal{Y}_{ij}$ respectively, we have the inequality of (33) and (34). Thus by applying **Lemma 2** (Schur complement) to (29), the second inequality of (30), (33) and (34), we find that these inequalities are equivalent to the LMIs of (10), (12), (9) and (11), respectively. Thus by solving the LMIs of (9) – (12), the fixed compensation gain matrix is determined as $F_i=\mathcal{W}_i\mathcal{Y}_i^{-1}$, and the variable one is given by (13). Thus the proof of **Theorem 1** is accomplished. ■

V. NUMERICAL EXAMPLES

In this example, we consider the uncertain large-scale interconnected system consisting of three two-dimensional subsystems, i.e. $N=3$. The system parameters are given as (35) and the time delay $h_{ij}=1$.

Firstly, we choose the weighting matrices $\mathcal{Q}_i\in\mathbb{R}^{2\times2}$ and $\mathcal{R}_i\in\mathbb{R}^{1\times1}$ $(i=1,\cdots,3)$ for the nominal subsystems such as $\mathcal{Q}_1=\mathrm{diag}\,(1.0,2.0)$, $\mathcal{Q}_2=\mathrm{diag}\left(1.0,1.0\times10^1\right)$, $\mathcal{Q}_3=I_2$, $\mathcal{R}_1=1.0$, $\mathcal{R}_2=1.0\times10^1$ and $\mathcal{R}_3=1.0\times10^1$, respectively. Thus by using the solution of the algebraic Riccati equation of (5), the optimal gain matrices $K_i\in\mathbb{R}^{1\times2}$ of (36) are derived.

Next, by using **Theorem 2** we design the proposed decentralized variable gain robust controller. By solving LMIs of (9) – (12), we have positive definite matrices $\mathcal{Y}_i\in\mathbb{R}^{2\times2}$,

$$K_1 = \begin{pmatrix} -1.71572 \times 10^{-1} & -2.82843 \end{pmatrix}, \; K_2 = \begin{pmatrix} -4.45866 & -3.42538 \end{pmatrix} \times 10^{-1},$$
$$K_3 = \begin{pmatrix} -2.05272 & 4.11182 \times 10^{-3} \end{pmatrix} \tag{36}$$

$$\mathcal{Y}_1 = \begin{pmatrix} 3.8762 & -2.4544 \\ -2.4544 & 6.1636 \end{pmatrix}, \; \mathcal{Y}_2 = \begin{pmatrix} 1.7105 & 2.3201 \times 10^{-1} \\ 2.3201 \times 10^{-1} & 3.4236 \end{pmatrix}, \; \mathcal{Y}_3 = \begin{pmatrix} 4.7580 & -1.6903 \\ -1.6903 & 2.4082 \end{pmatrix},$$
$$\mathcal{W}_1^T = \begin{pmatrix} -6.8344 \\ -1.8326 \times 10^1 \end{pmatrix}, \; \mathcal{W}_2^T = \begin{pmatrix} -1.2970 \times 10^1 \\ -1.2772 \times 10^1 \end{pmatrix}, \; \mathcal{W}_3^T = \begin{pmatrix} -2.4493 \times 10^1 \\ -6.5471 \end{pmatrix}$$
$$\mathcal{Y}_{11} = \begin{pmatrix} 9.8827 & -4.9281 \\ -4.9281 & 5.0870 \end{pmatrix}, \; \mathcal{Y}_{12} = \begin{pmatrix} 2.0325 & -1.8081 \\ -1.8081 & 2.1079 \end{pmatrix} \times 10^1, \; \mathcal{Y}_{13} = \begin{pmatrix} 6.2162 & -4.8969 \\ -4.8969 & 4.9965 \end{pmatrix},$$
$$\mathcal{Y}_{21} = \begin{pmatrix} 2.1510 & -1.7946 \\ -1.7946 & 2.1244 \end{pmatrix} \times 10^1, \; \mathcal{Y}_{22} = \begin{pmatrix} 5.7023 & -1.2191 \times 10^1 \\ -1.2191 & 3.2416 \times 10^1 \end{pmatrix},$$
$$\mathcal{Y}_{23} = \begin{pmatrix} 1.2642 \times 10^1 & -7.5138 \times 10^{-2} \\ -7.5138 \times 10^{-2} & 1.6080 \end{pmatrix}, \; \mathcal{Y}_{31} = \begin{pmatrix} 2.1671 \times 10^1 & -2.9690 \times 10^{-1} \\ -2.9690 \times 10^{-1} & 6.7533 \end{pmatrix},$$
$$\mathcal{Y}_{32} = \begin{pmatrix} 6.8967 & -9.1273 \\ -9.1273 & 1.7469 \end{pmatrix}, \; \mathcal{Y}_{33} = \begin{pmatrix} 9.2870 & -1.1141 \times 10^1 \\ -1.1141 \times 10^1 & 3.1129 \times 10^1 \end{pmatrix},$$
$$\mathcal{S}_1 = \begin{pmatrix} 6.3037 & 1.7142 \\ 1.7142 & 4.9951 \end{pmatrix} \times 10^1, \; \mathcal{S}_2 = \begin{pmatrix} 1.1304 \times 10^1 & 7.3226 \\ 7.3226 & 5.1528 \times 10^1 \end{pmatrix}, \; \mathcal{S}_3 = \begin{pmatrix} 6.9010 \times 10^1 & 5.1262 \\ 5.1262 & 2.6386 \times 10^1 \end{pmatrix},$$
$$\mathcal{S}_{11} = \begin{pmatrix} 3.3758 \times 10^1 & -1.2279 \\ -1.2279 & 3.7294 \times 10^1 \end{pmatrix}, \; \mathcal{S}_{12} = \begin{pmatrix} 3.1356 \times 10^1 & 2.9011 \\ 2.9011 & 3.3014 \times 10^1 \end{pmatrix},$$
$$\mathcal{S}_{13} = \begin{pmatrix} 3.4532 \times 10^1 & -1.0951 \\ -1.0951 & 3.9176 \times 10^1 \end{pmatrix}, \; \mathcal{S}_{21} = \begin{pmatrix} 3.3740 \times 10^1 & -1.2462 \\ -1.2462 & 3.7207 \times 10^1 \end{pmatrix},$$
$$\mathcal{S}_{22} = \begin{pmatrix} 3.1634 \times 10^1 & 2.9308 \\ 2.9308 & 3.2895 \times 10^1 \end{pmatrix}, \; \mathcal{S}_{23} = \begin{pmatrix} 3.4273 \times 10^1 & -1.3811 \\ -1.3811 & 3.9056 \times 10^1 \end{pmatrix},$$
$$\mathcal{S}_{31} = \begin{pmatrix} 3.3702 \times 10^1 & -1.2973 \\ -1.2973 & 3.7209 \times 10^1 \end{pmatrix}, \; \mathcal{S}_{32} = \begin{pmatrix} 3.1456 \times 10^1 & 2.8880 \\ 2.8880 & 3.2906 \times 10^1 \end{pmatrix},$$
$$\mathcal{S}_{33} = \begin{pmatrix} 3.4314 \times 10^1 & -1.3550 \\ -1.3550 & 3.8834 \times 10^1 \end{pmatrix},$$
$$\epsilon_1 = 1.7117 \times 10^1, \; \epsilon_2 = 1.3128 \times 10^1, \; \epsilon_3 = 3.1313 \times 10^1, \; \delta_1 = 1.4271 \times 10^1, \; \delta_2 = 1.8887 \times 10^1, \; \delta_3 = 3.7029 \times 10^1 \tag{37}$$

$$F_1 = \begin{pmatrix} -5.2064 & -1.2109 \end{pmatrix}, \; F_2 = \begin{pmatrix} -6.7449 & -2.8876 \end{pmatrix} \times 10^{-1}, \; F_3 = \begin{pmatrix} -2.6803 & -2.1929 \end{pmatrix} \tag{38}$$

$\mathcal{Y}_{ij} \in \mathbb{R}^{2\times 2}$, $\mathcal{S}_i \in \mathbb{R}^{2\times 2}$, $\mathcal{S}_{ij} \in \mathbb{R}^{2\times 2}$, matrices $\mathcal{W}_i \in \mathbb{R}^{1\times 2}$ and positive scalars ϵ_i and δ_i of (37). Thus the fixed gain matrices $F_i \in \mathbb{R}^{1\times 2}$ can be computed as (38).

VI. Conclusions

In this paper, for the uncertain large-scale interconnected system with state delays, a decentralized variable gain robust controller which achieves not only robust stability but also satisfactory transient performance has been proposed. Since the derived LMI conditions in this paper are feasible, the proposed decentralized controller synthesis is very useful.

In the future, we will extend the proposed controller to the design problem for such a broad class of systems as large-scale systems with general uncertainties, large-scale systems with Lipschitz nonlinearities and so on.

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