

# A LQR Scheme for SCR Process in Combined-Cycle Thermal Power Plants

Santo Wijaya<sup>1</sup> Keiko Shimizu<sup>1</sup> and Masashi Nakamoto<sup>2</sup>

**Abstract**—The paper presents a feedback control of Linear Quadratic Regulator (LQR) scheme for Selective Catalytic Reduction (SCR) process in combined-cycle (CC) power plants. The SCR process is to reduce NO<sub>x</sub> emission at the CC power plants, prior to venting into the atmosphere by decomposing process using Ammonia (NH<sub>3</sub>) at the SCR catalysts. This process can be considered as a nonlinear with time delay system. To improve the process efficiency, we are proposing cascade structure of feedback control system which consists of NO<sub>x</sub> control and NH<sub>3</sub> control that are designed with LQR scheme. A multi-rate sampling system is adopted in the cascade design to cope with a slow SCR process, and a fast NH<sub>3</sub> injection system characteristic. Furthermore, the structures of the controllers are designed to be implemented in the power plants control system. Simulation results on actual power plants conditions shows a satisfied control performance and design practicability.

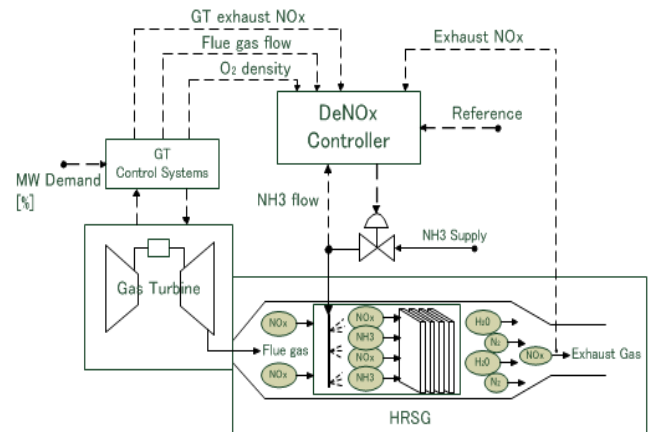


Fig. 1. General SCR process and controller configuration.

## I. INTRODUCTION

The paper addresses a control of SCR process in the CC thermal power plants. We propose a feedback control using LQR scheme with cascade structure in order to control NO<sub>x</sub> emission flow rate and improve the SCR process efficiency. The control configuration is developed to meet the stringent environmental law of NO<sub>x</sub> emission, and as a mean to further improve the current control system performance which is now using Generalized Predictive Control (GPC) scheme.

A general SCR process and NO<sub>x</sub> decomposition (DeNO<sub>x</sub>) control configuration are shown in Fig.1. The SCR process reduces the NO<sub>x</sub> exhaust emission rate by injecting NH<sub>3</sub> into a SCR catalysts inside the Heat Recovery Steam Generator (HRSG) system. In the SCR catalysts, NO<sub>x</sub> is decomposed into Nitrogen (N<sub>2</sub>) and Water (H<sub>2</sub>O). Due to the process inefficiency, there is an incomplete reaction which causes unreacted NO<sub>x</sub> emission releases into the exhaust stack and to the environment at the end. This NO<sub>x</sub> emission must comply with the environmental regulation on the region where the power plants are located. In Japan, especially in Tokyo region, the regulation stated that the 1-Hour Moving Averages (MA1H) of NO<sub>x</sub> emission flow rate of the power plants is limited below 14 Nm<sup>3</sup>/h, and the time-instant of NO<sub>x</sub> emission concentration is limited below 8.5 ppm. Therefore, they become the DeNO<sub>x</sub> control system primary and secondary objectives.

<sup>1</sup>Santo Wijaya and Keiko Shimizu are with the Information and Control Systems Eng. Dept, Toshiba Corporation, Japan. email: santo.wijaya@toshiba.co.jp, keiko.shimizu@toshiba.co.jp

<sup>2</sup>Masashi Nakamoto is with the Central Research Institute of Electric Power Industry, Japan. email: nakamoto@criepi.denken.or.jp

High performance of DeNO<sub>x</sub> control is desirable. However, the objectives are very challenging control task to achieve, due to the nonlinearity of the NO<sub>x</sub>-NH<sub>3</sub> process reaction and large dead-time of the SCR process (NH<sub>3</sub> transport time to catalyst, and NO<sub>x</sub> concentration measurement and analysis time), and also large disturbance from the gas turbine (GT) exhaust NO<sub>x</sub> emission flow rate due to the operation of the power plants. On the other hand, a feasible controller size is preferable from the practicability and economical point of view to reduce engineering time.

In recent years, several advanced control scheme for SCR process in the thermal power plants have been proposed to replace the traditional PID controller due to the unsatisfied performance because of the process characteristics. Peng et al. [3] proposed a model based multivariable Generalized Predictive Control (GPC) strategy using ExpARX model. The latter extends the work in [1], proposing a predictive control strategy using ExpARMAX model for nonlinear NO<sub>x</sub> decomposition process. Nakamoto et al. [6] utilizes the cascade structure with a model based controller using GPC for the NO<sub>x</sub> control and LQR for the NH<sub>3</sub> control.

In this paper, we propose a feedback control using a unified LQR scheme with cascade structure for the NO<sub>x</sub> control and the NH<sub>3</sub> control as an improvement from the controller that has been proposed in [6]. The cascade structure comprises of 1-Hour Moving Averages (MA1H) control of NO<sub>x</sub> emission flow rate, time-instant of NO<sub>x</sub> emission concentration control, and a NH<sub>3</sub> flow rate control. The controller is designed to compute the velocity form of the

inputs. The velocity form is used in the digital implementation to avoid wind-up [7]. The unified LQR scheme has been developed due to several reasons: the scheme can be designed with compensator that reduces the effect of high frequency noise, the controller size can be reduced by an order reduction technique, integrated control scheme for NOx control and NH3 control in which reduces design complexity, weighting function can be addressed directly to the desired states and inputs, better disturbance rejection compare to GPC scheme.

The organization of the paper are as follows. In section 2, the process model dynamic modeling is presented. In section 3 problem statements of control design are given. In section 4, control system design and configurations are derived. Next, in section 5, we presents a simulation results to show that the design procedure is sufficient. Finally, concluding remarks are stated in section 6.

## II. PLANT MODELING

### A. SCR Process Model

The SCR process model has been described in detail at [6]. In this section, we briefly describe the nonlinear model as showed in Fig. 2. The model is chosen based on the physical model due to the difficulties and time constraint in doing the identification on the real-plant. Therefore, the transport delay of NH3 flow rate from the injection system to the SCR catalysts is approximated by first-order with time delay model  $G_2(s)$ , and the NOx concentration measurement and analysis time is approximated by first-order with time delay model  $G_1(s)$ . While, the nonlinear function of  $f(x)$  indicates the NOx-NH3 process reaction.

The process model inputs are  $(u_c, v)$ , and the model outputs are  $(y_c, y_a)$ , while the model disturbance is  $(d)$  because the value is largely change within the operation of the power plants, where

$u_c$  : NH3 injection flow rate of feedback controller,

$v$  : NH3 injection flow rate of feedforward controller,

$d$  : GT exhaust NOx flow rate,

$y_c$  : NOx emission concentration in the exhaust stack,

$y_a$  : NOx emission flow rate at exhaust stack.

The  $(w, r)$  are assumed as a fixed value, where

$w$  : Flue gas flow rate,

$r$  : Oxygen concentration at 15% concentration conversion,

The nonlinear model is linearized at the power plants rated condition. While, the SCR process nonlinearity function is linearized around the catalyst inlet mol ratio NH3/NOx = 1 to obtain the function  $y = ax + b$ . The overall SCR process model can be obtained as follows

$$y_c = \frac{K_2 e^{-(L_1+L_2)s}}{(1+T_1s)(1+T_2s)} (u_c + v) + \frac{K_1 e^{-(L_1)s}}{(1+T_1s)} d, \quad (1)$$

where  $(K_1, K_2)$  are the respective system gain,  $(T_1, T_2)$  are the respective system time constant, and  $(L_1, L_2)$  are the respective system time delay. The  $(w, r, a, b)$  are included in the  $(K_1, K_2)$  as a proper system gain.

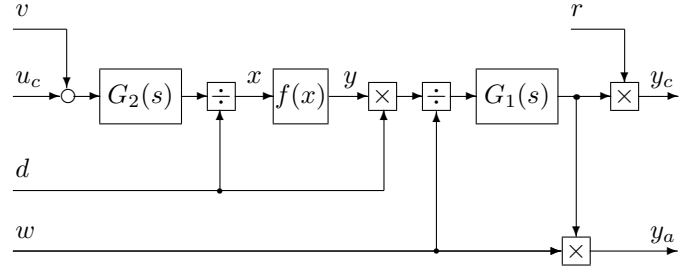


Fig. 2. SCR nonlinear process model

Equation 1 is discretized, and converted into state space system as follows

$$\begin{aligned} x_c(k+1) &= A_c x_c(k) + B_c (u_c(k) + v(k)) + E_c d(k) \\ y_c(k) &= C_c x_c(k), \end{aligned} \quad (2)$$

where  $A_c, B_c, C_c, E_c$  are the state space matrices with respective size and calculated system time delay.

### B. NH3 Process Model

NH3 process model is the dynamic of NH3 system actuator. The dynamic can be modeled with first order system with time delay as follows

$$y_h = \frac{K_3 e^{-L_3 s}}{1 + T_3 s} u_h, \quad (3)$$

where  $K_3$  is the system gain,  $T_3$  is the system time constant, and  $L_3$  is the system time delay. The system input  $u_h$  is the NH3 flow rate SV, and the system output  $y_h$  is the NH3 flow rate PV.

Equation 3 is discretized and converted into state space system as follows

$$\begin{aligned} x_h(k+1) &= A_h x_h(k) + B_h u_h(k) \\ y_h(k) &= C_h x_h(k), \end{aligned} \quad (4)$$

where  $A_h, B_h, C_h$  are the state space matrices with respective size and calculated system time delay.

The model parameters for both SCR process model and NH3 process model are identified using a step response method at the power plants rated condition. The identified parameters are fixed for all operating conditions.

## III. PROBLEM STATEMENT

### A. NOx Control

For ease of control, the primary control of NOx cascade controller directly controls the MA1H of NOx emission flow rate of the power plants to satisfy the primary control objective. The output of this controller is time instant of NOx concentration reference.

The time instant of NOx concentration controller is the secondary control of NOx cascade controller. The controller regulates the time instant of NOx emission concentration based on the reference input from the primary loop to satisfy the secondary control objective. The output of this controller is NH3 flow rate reference.

Due to the time constant differences between the MA1H of NOx emission flow rate and the time instant of NOx emission concentration, a multi-rate sampling is implemented. The sampling time of the MA1H of NOx exhaust emission flow rate controller, the NOx concentration controller are  $T_{s_1} = 30$  sec,  $T_{s_2} = 1$  sec respectively.

### B. NH3 Control

NH3 control is proposed to regulate the NH3 flow rate that will be injected into the SCR process according to NH3 flow rate reference. The NH3 process has a time constant about one-tenth of the NOx process time constant [6]. Hence, a fast sampling rate is needed for NH3 controller. The time sampling of the NH3 controller is  $T_{s_3} = 0.2$  sec.

## IV. CONTROL SYSTEM DESIGN AND CONFIGURATION

### A. Control System Configuration

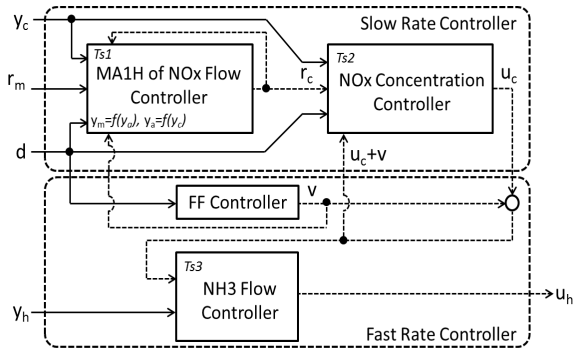


Fig. 3. Controller structure.

The controller structure, as shown in Fig. 3, is a cascade control of NOx controller and NH3 controller that consists of multi-rate sampling time.

The NH3 flow controller state space model refers to the Eq. 4. The objective of this controller is to minimize  $e_h(k) = |r_h(k) - y_h(k)|$  by finding an optimal state feedback control  $u_h(k)$ .

The NOx concentration controller state space model refers to the Eq. 2. The objective of this controller is to minimize  $e_c(k) = |r_c(k) - y_c(k)|$  by finding an optimal state feedback control  $u_c(k)$ .

The MA1H NOx flow controller state space model consists of the SCR process model (Eq. 2), and the NOx concentration controller model in the closed-loop configuration. The objective of this controller is to minimize  $e_m(k) = |r_m(k) - y_m(k)|$  by finding an optimal state feedback control  $u_m(k)$  or  $r_c(k)$ . While,  $y_m(k)$  is NOx exhaust emission flow rate in MA1H state space model of the form

$$\begin{aligned} x_m(k+1) &= A_m x_m(k) + B_m y_a(k), \\ y_m(k) &= C_m x_m(k) + D_m y_a(k), \end{aligned} \quad (5)$$

where  $A_m, B_m, C_m, D_m$  are the state space matrices with respective size.

A feedforward controller is proposed to reject the effect of the NOx emission fluctuation by using the NOx emission

flow rate of the gas turbine. The feedforward controller sampling time is the fast rate time sampling  $T_{s_3}$ .

### B. Control System Design

A feedback configuration for the LQR scheme is proposed for NOx controllers and NH3 controller. The controllers state space model are of the form

$$\begin{aligned} x_i(k+1) &= A_i x_i(k) + B_i u_i(k) + B_k v(k) + E_k d(k), \\ y_i(k) &= C_i x_i(k), \end{aligned} \quad (6)$$

where  $A_i \in \mathbb{R}^{n \times n}, B_{i,k} \in \mathbb{R}^{n \times m}, C_i \in \mathbb{R}^{r \times n}, E_k \in \mathbb{R}^{n \times m}$  are the state space matrices, and  $i = \{1, 2, 3\}, k = \{1, 2\}$  are the controllers state-space model index. The controller with index  $\{1, 2, 3\}$  are the NOx exhaust emission flow rate in MA1H controller, the NOx concentration controller, and the NH3 flow controller respectively, where

$$\begin{aligned} x_1(k) &= [x_c(k) \ x_m(k)]^T, \ y_1(k) = [y_a(k) \ y_m(k)]^T, \\ x_2(k) &= x_c(k), \ y_2(k) = y_c(k), \\ x_3(k) &= x_h(k), \ y_3(k) = y_h(k). \end{aligned}$$

In the actual SCR process, all the states cannot be measured directly. Therefore, an observer is used to provide an estimate of the internal states  $x(k)$ . In order to avoid an offset, we adopt servo type controller for each of the controllers. In this configuration, the state-space model for each controllers are of the form

$$\begin{aligned} \hat{x}_i(k+1) &= A_i \hat{x}_i(k) + B_i u_i(k) + B_k v(k) + E_k d(k) + \\ &\quad L_i (y_i(k) - \hat{y}_i(k)), \\ w_i(k+1) &= w_i(k) - y_i(k) + r_i(k). \end{aligned} \quad (7)$$

We now seek to find a state feedback control

$$u_i(k) = [K_{1i} \ K_{2i}] \bar{x}_i(k), \quad (8)$$

that minimizes the performance index

$$J = \sum_{k=0}^{\infty} (\bar{x}_i^T \bar{Q}_i \bar{x}_i(k) + u_i^T R_i u_i(k)), \quad (9)$$

where

$$\bar{Q}_i = \begin{bmatrix} q_{1,i} I + q_{2,i} C_i^T C_i & 0 \\ 0 & q_{3,i} \end{bmatrix}, \quad \bar{x}_i = [\hat{x}_i \ w_i]^T,$$

in which  $q_{1,i}, q_{2,i}, q_{3,i}, R_i$  are a weighting matrix for the plant states, a weighting matrix for the plant outputs, a weighting matrix for the states of servo controller, a weighting matrix for the manipulated variables in the respective  $i$  controller with a respective sizes.

The LQR gains  $\{K_{1i}, K_{2i}\}$ , and the observer gain  $\{L_i\}$  are solved using the Discrete time algebraic Riccati equation (DARE). Furthermore, the controllers size are reduced by an order reduction method.

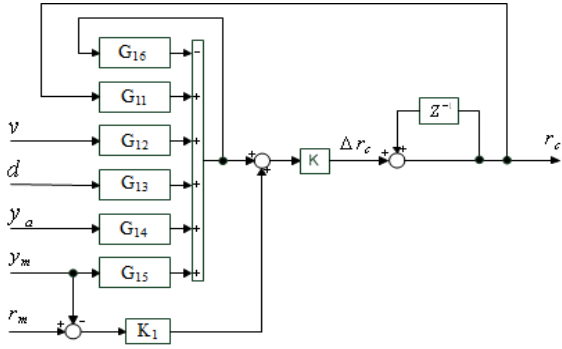


Fig. 4. The MAIH of NOx flow controller structure.

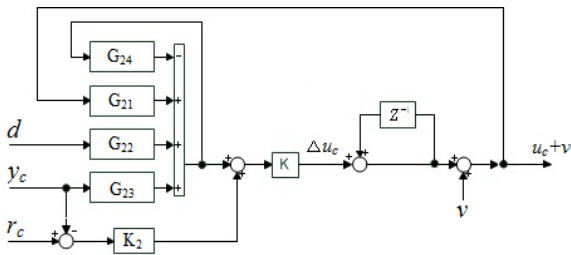


Fig. 5. The time instant of NOx concentration controller structure.

### C. Controller Structure

The observer of NOx controllers and NH3 controller are reduced to 9<sup>th</sup> order controller, and 1<sup>st</sup> order controller. NOx controllers order are based on balanced realization method and the controllers size is a factor between the realization result and control logic block limitation in the control device.

A schematic diagram of the NOx controllers are shown in Fig. 4, and Fig. 5, where  $g_1, g_2, g_3, g_4, g_5, g_6$  are observer in discrete filter form of  $z^{-1}$  with the respective gains.  $K$  is the multi-rate sampling time gain. The controller is designed to compute the input change of  $\Delta u_i(k)$ , at each time  $k$ .

The comparison of the controller size between GPC scheme [6] and the proposed LQR scheme is shown in Table I. The comparison is particularly to show that the controller designed with LQR scheme have a smaller size especially in the observer structures, while it can have similar of even better controller performance.

TABLE I  
GPC [6] AND LQR CONTROLLER SIZE

Observer Gain and Discrete Filter Qty.	GPC	LQR
NOx exhaust emission flow rate in MAIH controller	160	60
NOx concentration controller	110	50

The schematic diagram of the NH3 controller is not shown here due to the same structure as in [4]. The interested reader is suggested to refer to the cited reference.

## V. SIMULATION RESULTS

A simulation is presented to demonstrate the effectiveness of the proposed method. Simulation model comprises of

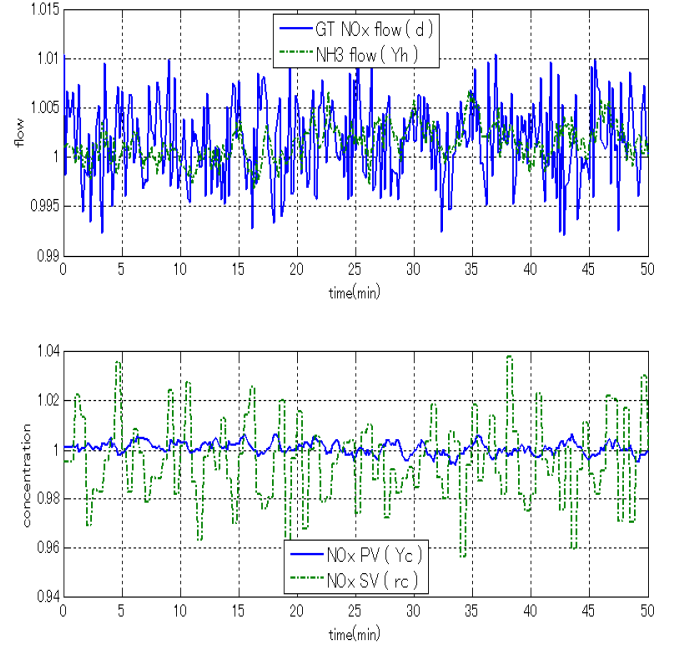


Fig. 6. GPC Controller performance against disturbance changes.

SCR nonlinear model and DeNOx controller in closed-loop configuration. There are three simulation conditions considered in this paper, the controller performance against disturbance changes, start-up condition, and MW-demand operating condition.

The design parameters for the LQR scheme is shown in Table II. The parameters are time-invariant and constant to all simulation conditions.

TABLE II  
DESIGN PARAMETERS

Parameter	$i = 1$	$i = 2$	$i = 3$
$K_i$	-0.2819	0.3131	1.0
$T_i$	22.0	80.0	2.0
$L_i$	48.0	10.0	2.0
$q_{1,i}$	0.0	0.0	0.1
$q_{2,i}$	1.0	1.0	0.0
$q_{3,i}$	1.0	5.0	1.0
$R_i$	100.0	2500.0	10.0

All the simulation results value that are shown in the figures in this paper has been normalized to the process value respectively, due to the undisclosed restriction.

### A. Controller Performance

The controller performance is tested against disturbance or GT NOx flow ( $d$ ) changes. The disturbance is modeled with uniform distributed random number. In order to track the  $Y_c$  back to the set-point  $r_c$ , the proper NH3 flow ( $Y_h$ ) is injected to the system. The GPC controller performance is as shown in Fig. 6, and the LQR controller performance is as shown in Fig. 7.

An error between NOx concentration PV ( $Y_c$ ) and the set-point ( $r_c$ ) is presented as a mean of absolute values and a

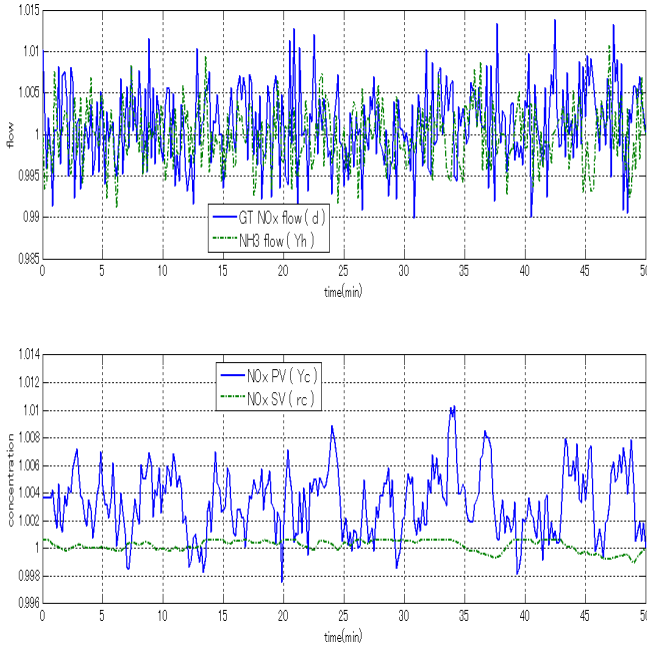


Fig. 7. LQR Controller performance against disturbance changes.

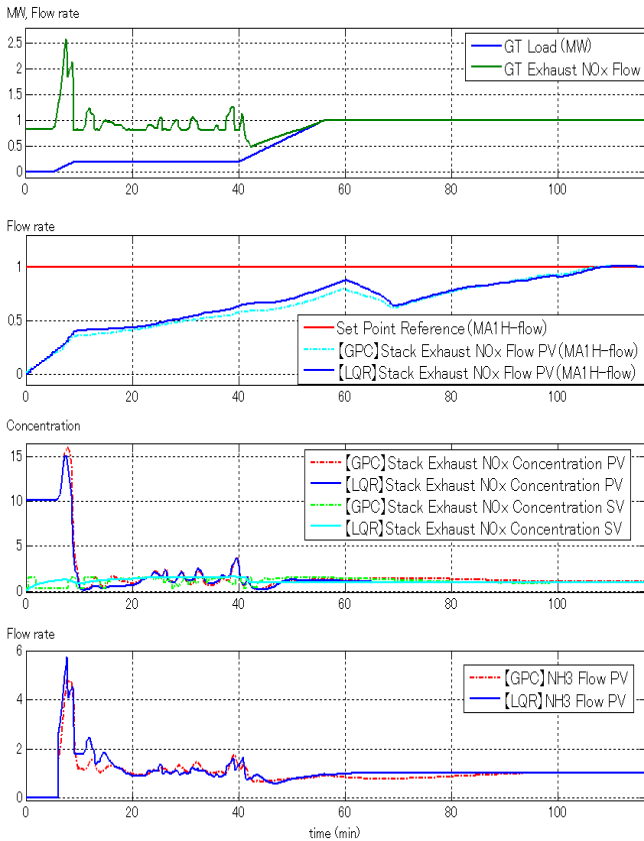


Fig. 8. Start-up simulation.

mean of squared values of the errors that is shown in Table III.

TABLE III  
CONTROLLER PERFORMANCE AGAINST DISTURBANCE REJECTION

Control type	$\frac{1}{N} \sum  e $	$\frac{1}{N} \sum e^2$
GPC	0.01501	0.00034
LQR	0.00333	0.00002

### B. Start-up Simulation

The simulation intends to show the performance of the proposed LQR scheme compare to the performance of the GPC control scheme [6]. The simulation condition is the CC power plants start-up sequence where the gas turbine load or GT Load (MW) changes from 0% to 100% within  $t = 60$  minutes. Following the changes in GT Load (MW), GT exhaust NOx flow ( $d$ ) also changes due to the combustion process in the gas turbine. These start-up conditions are shown in the first graph of Fig. 8.

The simulation results of the proposed control and the GPC control in the case of start-up condition are shown in the second to the fourth graphs of Fig. 8. The control objective is to track unity set-point of NOx exhaust emission flow rate in MA1H regardless of the power plants operating condition. It is shown at the early stage of the power plants start-up (GT Load  $< 0.3$ ) that GT exhaust NOx flow ( $d$ ) is high and it follows NOx concentration also high. Furthermore, it decreases at time 10 – 15 minute when the NH3 injection is started.

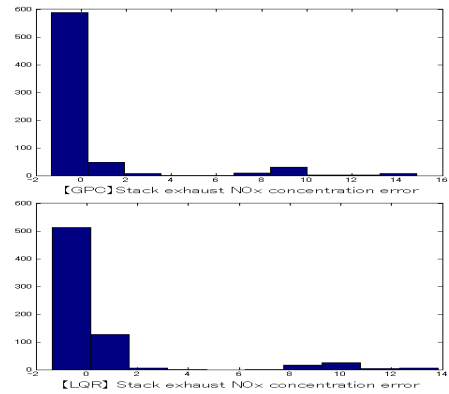


Fig. 9. Histograms of the stack exhaust NOx concentration output error distribution at start-up condition.

A histogram for an error between NOx concentration PV ( $Y_c$ ) and the set-point ( $r_c$ ) is shown in Fig. 10, and a mean of absolute values and a mean of squared values of the errors is shown in Table IV.

TABLE IV  
OUTPUT ERROR AT START-UP

Control type	$\frac{1}{N} \sum  e $	$\frac{1}{N} \sum e^2$
GPC	1.0525	8.2607
LQR	1.0053	7.9441

### C. MW Demand Simulation

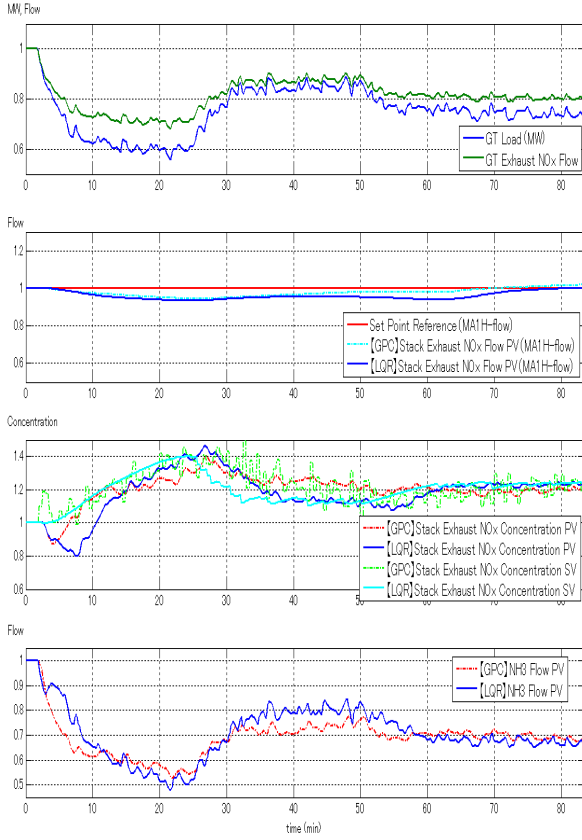


Fig. 10. MW Demand simulation.

The simulation condition is the CC power plants operation to meet the demand of the power output. Hence, the GT load (MW) changes when it operates at rated condition as shown in the first graph of Fig. 9. We simulate that the GT load (MW) decrease to 0.75 after swinging in the range of 0.6 to 0.82. The control objective is to track unity set-point of NOx exhaust emission flow rate in MA1H regardless of the power plants operating condition. Thus, the NH3 injection flow is reduced to 0.7.

A histogram for an error between NOx concentration PV ( $Y_c$ ) and the set-point ( $r_c$ ) is shown in Fig. 11, and a mean of absolute values and a mean of squared values of the errors is shown in Table V.

TABLE V  
OUTPUT ERROR AT MW DEMAND

Control type	$\frac{1}{N} \sum  e $	$\frac{1}{N} \sum e^2$
GPC	0.0523	0.0044
LQR	0.0461	0.0047

### VI. CONCLUSION

The LQR scheme in the controller design for NOx decomposition process in a CC thermal power plants is presented in the paper. The feedback controller with cascade structure

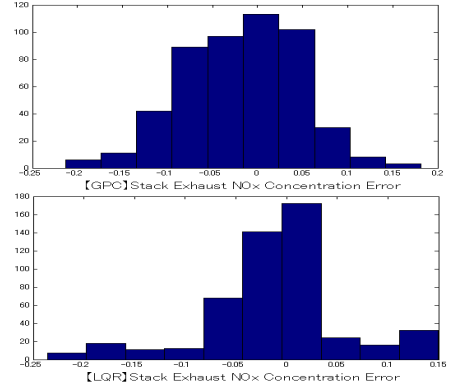


Fig. 11. Histograms of the stack exhaust NOx concentration output error distribution at MW Demand condition.

which consists of NOx controller and NH3 controller with multi-rate sampling time are integrately designed with LQR scheme and compensator. The proposed control scheme is less complex and the controllers size is reduced properly while the control structure is designed for suitable implementation in the power plants. Finally, the simulation results illustrated the effectiveness of the proposed control scheme.

Extension of current work would be, to consider the use of adaptive model parameters due to large ranges of power plants states (start-up, load changes, disturbance changes). Also, change in controller parameters with change in power plants states for improved performance.

### REFERENCES

- [1] H. Peng, W. Gui, H. Shioya, and R. Zou, A predictive control strategy for nonlinear NOx decomposition process in thermal power plants, *IEEE Trans. on Systems, Man, and Cybernetics-Part A:Systems and Humans*, vol. 36, no. 5, pp. 904–921, 2006.
- [2] M. Nakamoto, T. Kokubo, A. Kamito, and K. Shimizu, Cascade control using GPC and LQR for a NOx reduction process of a thermal power plant (in Japanese), *Trans. of the Society of Instrument and Control Engineers*, vol. E-2, no. 1, pp. 98–107, 2002.
- [3] H. Peng, T. Ozaki, V. Haggan-Ozaki, and Y. Toyoda, A nonlinear exponential ARX model-based multivariable generalized predictive control strategy for thermal power plants, *IEEE Trans. Control System Technology*, vol. 10, no. 2, pp. 256–262, 2002.
- [4] M. Nakamoto, T. Kokubo, A. Nakai, and H. Tanabe, Application of linear quadratic regulator to an interactive flow control system for a thermal power plant (in Japanese), *Trans. of the Society of Instrument and Control Engineers*, vol. 33, no. 6, pp. 494–501, 1997.
- [5] K. Shimizu, M. Nakamoto, T. Kokubo, and H. Tanabe, Generalized predictive control for a NOx decomposition process (in Japanese), *Trans. of the Society of Instrument and Control Engineers*, vol. 32, no. 6, pp. 912–920, 1996.
- [6] M. Nakamoto, K. Shimizu, K. Nagat, and T. Kokubo, Generalized Predictive Control for a NOx Decomposition Process of a Combined Cycle Power Plant, *IFAC Control of Power Plants and Power Systems*, Mexico, 1995, pp. 251–256.
- [7] G. Pannocchia, and J.B. Rawlings, *The velocity algorithm LQR: a survey*, Technical Report, no. 2001-01, 2001.
- [8] D. Flynn, *Thermal power plant simulation and control*, United Kingdom, The Institution of Electrical Engineers, 2003.