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A Model Reference Adaptive Control Based on On-line FRIT Approaches Using a Normalized Recursive Least Square Method*

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***Abstract*— The model reference adaptive control (MRAC) designed based on the fictitious reference iterative tuning (FRIT) approach in an on-line manner has recently been proposed. The FRIT method is an off-line control parameters tuning method so that the plant output could follow the prescribed reference model output from one-shot experimental input-output data with no need for help from plant model. The MRAC based on on-line FRIT employs a normalized gradient method for the adaptive adjusting law. However, since the gradient algorithm suffers from slow convergence rate. it is desirable to employ an adaptive adjusting law with fast convergence rate. The paper gives a normalized recursive least square (RLS) method for the adaptive adjusting law for the model reference adaptive control based on an on-line FRIT approach. In the traditional MRAC, the RLS method shows faster convergence rate than the gradient algorithm. The paper also proves the boundedness of all signals in the closed loop system as well as asymptotically tracking the reference model output. An effectiveness of the proposed method is shown through a numerical example.**

I. Introduction

The Fictitious Reference Iterative Tuning (FRIT)[1] are one of the direct controller parameters design approaches, such as VRFT (Virtual Reference Feedback Tuning)[2], NCbT(Noniterative Correlation-based Tuning)[4], and unfalsified approach[5]. In the FRIT methods, the fictitious reference signal is parametrized by control parameters using the one-shot experimental input-output data. Then, the control parameters are optimized so that the reference model output from the parametrized fictitious reference signal could follow the closed loop output obtained from the one-shot closed-loop experiment. The obtained control parameters makes the closed loop transfer function exactly match the prescribed reference model output.

The approach can avoid the time-consuming closed loop experiments for control parameters tuning or system identification. However, since the control parameters are designed in an off-line manner, once plant characteristics change, the control performance may be deteriorated. On-line FRIT methods which evaluates the performance index iteratively, and updates the control parameters in an on-line manner, has been proposed [6], [8], [7] in order to avoid the problem of the variation of plant characteristics.

Masuda[6] gave an on-line FRIT method by repeating a modified FRIT approach, where the identification model is linearly parametrized in terms of control parameters. Wakasa et al.[8] introduced a recursive least square method into the on-line FRIT approach in order to save the computation load. However, these researches have not discussed the stability of the closed system. The MRAC designed based on the fictitious reference iterative tuning (FRIT) approach in an on-line manner has recently been proposed[7]. The MRAC based on on-line FRIT employs a normalized gradient method for the adaptive adjusting law. However, since the gradient algorithm suffers from slow convergence rate. it is desirable to employ an adaptive adjusting law with fast convergence rate.

The paper gives a normalized recursive least square (RLS) method[9] for the adaptive adjusting law for the model reference adaptive control based on an on-line FRIT approach. In the traditional MRAC[3], [9], the RLS method shows faster convergence rate than the gradient algorithm. The paper also proves the boundedness of all signals in the closed loop system as well as asymptotically tracking the reference model output. An effectiveness of the proposed method is shown through a numerical example.

II. Problem Statements

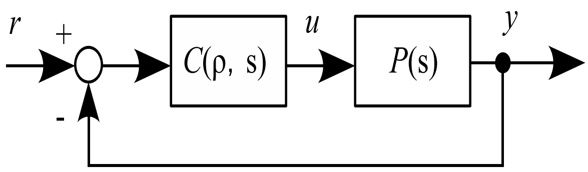

Fig. 1. Closed loop system in the regulator problem

Consider a single-input, single-output, continuous-time, time-invariant, one-degree-of freedom closed-loop system with a disturbance signal at the input signal. shown in Fig. 1. Let the plant model be denoted by $P(s)$ in the form of the transfer function. The argument s stands for a differential operator, and the initial values of transfer functions are assumed to be zero. In addition, $r(t)$, $u(t)$, and $y(t)$ are the reference signal, control input signal, and controlled output signal, which are functions of time t.

Attention is restricted to the feedback controller $C(\boldsymbol{\rho}(t), s)$ linearly parametrized in terms of adjustable control parameters $\boldsymbol{\rho}(t)$. That is, the controller $C(\boldsymbol{\rho}(t), s)$ can be described as

$$C(\boldsymbol{\rho}(t), s) = \boldsymbol{\rho}(t)^{\mathrm{T}} \boldsymbol{\varphi}(s) \qquad (1)$$

where $\boldsymbol{\rho}(t)$ is an n-dimensional control parameter vector which are functions of time t, and $\boldsymbol{\varphi}(s)$ is also an n-

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dimensional vector whose elements are rational functions of s representing a transfer function.

The proposed adaptive controller also assumes that all elements of $\boldsymbol{\varphi}(s)$ belong to proper and stable transfer functions. The reference model is given by $P_r(s)$ and the output of the reference model by $y_r(t) = P_r(s)r(t)$.

Now, we assume that the control parameter vector $\boldsymbol{\rho}_d$ exists so that the closed loop transfer function corresponds to the reference model. That is, the following equation is assumed to be satisfied:

$$P_r(s) = \frac{P(s)C(\boldsymbol{\rho}_d, s)}{1 + P(s)C(\boldsymbol{\rho}_d, s)} \tag{2}$$

The control objective is to tune the control parameter vector $\boldsymbol{\rho}(t)$ in an on-line manner so that the plant output $y(t)$ follows the reference model output $y_r(t) = P_r(s)r(t)$ with no need for help from a plant model while the boundedness of all signals in the closed loop system is assured.

III. Control Parameters Tuning Using FRIT Approach

This section briefly reviews the conventional FRIT approach in the continuous-time formulation.

In the FRIT method, the control parameter $\boldsymbol{\rho}_d$ satisfying (2) is identified from the input and output data u_0 and y_0 on the finite interval $[0,\ T]$ measured from the closed loop system when an initial control parameter $\boldsymbol{\rho}_0$ is employed.

To this end, the FRIT method calculates the fictitious reference signal $r^*(\boldsymbol{\rho})$ which generates the initial collected input and output signal u_0 and y_0 even when the control parameter $\boldsymbol{\rho} \neq \boldsymbol{\rho}_0$ is employed. The following equations are fictitious reference signal $r^*(\boldsymbol{\rho})$.

$$r^*(\boldsymbol{\rho}) \quad = \quad C(\boldsymbol{\rho}, s)^{-1} u_0 + y_0 \tag{3}$$

The plant output y_0 can be regarded as the output of the closed loop system where the ideal controller $C(\boldsymbol{\rho}_d)$ and the fictitious reference signal $r^*(\boldsymbol{\rho}_d)$ were employed. Hence, it follows from (3) and (2) that:

$$\begin{aligned} y_0 &= \frac{P(s)C(\boldsymbol{\rho}_d, s)}{1 + P(s)C(\boldsymbol{\rho}_d, s)} r^*(\boldsymbol{\rho}_d) \\ &= P_r(s) r^*(\boldsymbol{\rho}_d) \\ &= P_r(s) C(\boldsymbol{\rho}_d, s)^{-1} u_0 + P_r(s) y_0 \end{aligned} \tag{4}$$

FRIT identifies the ideal control parameter vector $\boldsymbol{\rho}_d$ based on the identification model (4) using one-shot experimental input-output data u_0 and y_0. In detail, let $\hat{y}(\boldsymbol{\rho})$ be defined as:

$$\hat{y}(\boldsymbol{\rho}) = P_r(s) C(\boldsymbol{\rho}, s)^{-1} u_0 + P_r(s) y_0 \tag{5}$$

and determine the optimal control parameter vector which optimizes the following integrated square error between $\hat{y}(\boldsymbol{\rho})$ and y_0:

$$J_F(\boldsymbol{\rho}) = \int_0^T (\hat{y}(\boldsymbol{\rho}) - y_0)^2 dt \tag{6}$$

The optimal parameter vector:

$$\boldsymbol{\rho}^* = \arg\left(\min_{\boldsymbol{\rho}} J_F(\boldsymbol{\rho})\right) \tag{7}$$

is the control parameter vector determined using the FRIT approach.

When the assumption that the parameter vector $\boldsymbol{\rho}_d$ exists, namely (2) is satisfied, it follows from simple calculations that the performance index (6) become:

$$J_F(\boldsymbol{\rho}) = \int_0^T \left(\frac{C(\boldsymbol{\rho}_d, s) - C(\boldsymbol{\rho}, s)}{1 + P(s)C(\boldsymbol{\rho}_d, s)} \frac{1}{C(\boldsymbol{\rho}, s)} y_0 \right)^2 dt \tag{8}$$

Hence, the determined control parameter vector (7) in the FRIT approach turned out to be $\boldsymbol{\rho}_d$, which makes the performance index $J_F(\boldsymbol{\rho})$ be identical to zero.

IV. Adaptive Parameter Tuning Using a Normalized Least Square Method

The first, the identification model linearly parameterized in terms of control parameter is introduced from (4). Multiply both side of the equation (4) by $C(\boldsymbol{\rho}_d, s)$ and arrange the equation, we get

$$C(\boldsymbol{\rho}_d, s)\,(1 - P_r(s))\, y_0 = P_r(s) u_0 \tag{9}$$

Using the linearly parametrized controller defined in (1), (9) becomes.

$$\boldsymbol{\rho}_d^{\mathrm{T}} \boldsymbol{\varphi}(s)\,(1 - P_r(s))\, y_0 = P_r(s) u_0 \tag{10}$$

The identification model (10) is obviously satisfied when the input and output signal u_0 and y_0 on finite interval $[0, T]$ are replaced by $u(t)$ and $y(t)$ which are measured signal at the time t in an on-line manner. Hence, (10) becomes

$$\boldsymbol{\rho}_d^{\mathrm{T}} \boldsymbol{\varphi}(s)\,(1 - P_r(s))\, y(t) = P_r(s) u(t) \tag{11}$$

Using the identification model (11) and replacing the desired control parameters $\boldsymbol{\rho}_d$ by adjustable control parameters $\boldsymbol{\rho}(t)$, the following identification error $\varepsilon(t)$ is defined

$$\varepsilon(t) = \boldsymbol{\rho}(t)^{\mathrm{T}} \boldsymbol{\xi}(t) - \eta(t) \tag{12}$$

where $\boldsymbol{\xi}(t)$ and $\eta(t)$ are defined as

$$\boldsymbol{\xi}(t) \quad = \quad \boldsymbol{\varphi}(s)(1 - P_r(s)) y(t) \tag{13}$$

$$\eta(t) \quad = \quad P_r(s) u(t) \tag{14}$$

The next theorem gives the adaptive adjusting law using a recursive least square method assuring the boundedness of the control parameters[9].

Theorem 4.1: The following adaptive adjusting law assures the boundedness of the control parameters $\boldsymbol{\rho}(t)$

$$\frac{d}{dt}\boldsymbol{\rho}(t) \quad = \quad -g \frac{\boldsymbol{P}(t)\varepsilon(t)\boldsymbol{\xi}(t)}{1 + \gamma \boldsymbol{\xi}(t)^{\mathrm{T}} \boldsymbol{P}(t) \boldsymbol{\xi}(t)} \tag{15}$$

$$\frac{d}{dt}\boldsymbol{P}(t) \quad = \quad -g \frac{\boldsymbol{P}(t)\boldsymbol{\xi}(t)\boldsymbol{\xi}(t)^{\mathrm{T}}\boldsymbol{P}(t)}{1 + \gamma \boldsymbol{\xi}(t)^{\mathrm{T}} \boldsymbol{P}(t) \boldsymbol{\xi}(t)} \tag{16}$$

where g, γ is a positive real number, and $\boldsymbol{\xi}(t)$ and $\varepsilon(t)$ are defined in (13) and (14). In addition, the following equations are satisfied.

$$\boldsymbol{\rho}(t) \quad \in \quad \mathcal{L}^\infty \tag{17}$$

$$\frac{d}{dt}\boldsymbol{\rho}(t) \quad \in \quad \mathcal{L}^2 \cap \mathcal{L}^\infty \tag{18}$$

$$\frac{\varepsilon(t)}{\sqrt{1 + \boldsymbol{\xi}(t)^{\mathrm{T}} \boldsymbol{P}(t) \boldsymbol{\xi}(t)}} \quad \in \quad \mathcal{L}^2 \cap \mathcal{L}^\infty \tag{19}$$

From Theorem 4.1, we can see that a new adaptive adjusting law (15) and (16) has been proposed, which assures the boundedness of the adjustable parameters. However, the theorem 4.1 does not mean that the plant output asymptotically track the reference model output, and the all the signals in the closed loop system are bounded. The next section discusses on the stability analysis.

V. Stability Analysis

To begin with, let's focus on the relation between the tracking error

$$e(t) = y(t) - y_r(t) \tag{20}$$

and the identification error $\varepsilon(t)$ calculated from (12).

By simple calculation, the tracking error (20) and the identification error $\varepsilon(t)$ can represented in the following way.

$$e(t) = \frac{P(s)}{1+P(s)C(\boldsymbol{\rho}_d, s)} v(t) \tag{21}$$

$$\varepsilon(t) = \frac{P(s)C(\boldsymbol{\rho}(t), s)}{1+P(s)C(\boldsymbol{\rho}_d, s)} v(t) \tag{22}$$

where

$$v(t) = \frac{C(\boldsymbol{\rho}_d, s) - C(\boldsymbol{\rho}(t), s)}{1+P(s)C(\boldsymbol{\rho}(t), s)} r(t) \tag{23}$$

From (21) and (22), it follows that

$$e(t) = \frac{1}{C(\boldsymbol{\rho}(t), s)} \varepsilon(t) \tag{24}$$

Then, let $\boldsymbol{\rho}^*$ be denoted as the control parameters into which the adjustable control parameters $\boldsymbol{\rho}(t)$ converge. Namely, the following equation is satisfied.

$$\lim_{t\to\infty} \|\tilde{\boldsymbol{\rho}}(t)\| = 0, \ \ \tilde{\boldsymbol{\rho}}(t) = \boldsymbol{\rho}(t) - \boldsymbol{\rho}^* \tag{25}$$

From Theorem 4.1, it is assured that there exists $\boldsymbol{\rho}^*$ as a limit of the the adjustable control parameters $\boldsymbol{\rho}(t)$.

Using $\boldsymbol{\rho}^*$ and (1) and (24) can be rewritten into

$$e(t) = \frac{1}{C(\boldsymbol{\rho}^*, s)} \left(\varepsilon(t) - \tilde{\boldsymbol{\rho}}(t)^{\mathrm{T}} \boldsymbol{\varphi}(s) e(t)\right) \tag{26}$$

The following theorem gives the proof of the stability of the closed loop system incorporated with the adaptive adjusting law (15) and (16). The proof can be done by using the similar procedure which is conducted for proving stability of MRACS(Model Reference Adaptive Control) shown in [3].

Theorem 5.1: Assume the following conditions.

1) $\dfrac{1}{C(\boldsymbol{\rho}^*, s)}$ is asymptotically stable.
2) All elements of $\boldsymbol{\varphi}(s)$ are asymptotically stable.

Then, all the signals of the closed loop system

$$y(t) = P(s)u(t) \tag{27}$$

$$u(t) = C(\boldsymbol{\rho}(t), s)(r(t) - y(t)) \tag{28}$$

and the adaptive adjusting law (15) and (16) are assured to be boundedness, and the control error (20) asymptotically goes to zero.

For the preparation, the following definitions[3] are given.

Definition 5.1: $\mathcal{PC}_{[0,\infty]}$ is defined as the set of all real piecewise continuous functions defined on the interval $[0, \infty)$ which have bounded discontinuities.

Definition 5.2: Let $x, y \in \mathcal{PC}_{[0,\infty]}$. We denote $y(t) = O[x(t)]$ if there exist positive constants M_1, M_2, and $t_0 \in \boldsymbol{R}^+$ such that $|y(t) \leq M_1|x(t)| + M_2, \forall t \geq t_0$.

Definition 5.3: Let $x, y \in \mathcal{PC}_{[0,\infty]}$. We denote $y(t) = o[x(t)]$ if there exist a function $\beta(t) \in \mathcal{PC}_{[0,\infty]}$, and $t_0 \in \boldsymbol{R}^+$ such that $y(t) = \beta(t)x(t), \forall t \geq t_0$, and $\lim_{t\to\infty} \beta(t) = 0$.

(Proof of Theorem 5.1) From Theorem 4.1, it follows that the adjustable control parameters $\boldsymbol{\rho}(t)$ is bounded. Hence, all the signals of the closed loop system belong to $\mathcal{PC}_{[0,\infty]}$.

Then, we assume that the signal $y(t)$ grows in an unbounded manner. The proof will lead the assumption into contradiction, and show that the $y(t)$ is a bounded signal. From (19), it follows that

$$\varepsilon(t) = \beta(t)\sqrt{1 + \boldsymbol{\xi}(t)^{\mathrm{T}} \boldsymbol{P}(t) \boldsymbol{\xi}(t)}, \quad \beta(t) \in \mathcal{L}^2 \cap \mathcal{L}^\infty \tag{29}$$

Since $\boldsymbol{\varphi}(s)$ are asymptotically stable from the assumption of the Theorem 4.1, and $P_r(s)$ is also asymptotically stable, the following equation is satisfied.

$$\|\boldsymbol{\xi}(t)\| = O\left[\sup_{\tau\leq t} |y(\tau)|\right] \tag{30}$$

From (29) and (30) and Lemma 2.9 in [3], it follows that

$$\varepsilon(t) = o\left[\sup_{\tau\leq t} |y(\tau)|\right] \tag{31}$$

From Lemma 2.11 and $\frac{d}{dt}\tilde{\boldsymbol{\rho}}(t) \in \mathcal{L}^2$, it follows that

$$\tilde{\boldsymbol{\rho}}(t)^{\mathrm{T}} \frac{1}{C(\boldsymbol{\rho}^*, s)} \boldsymbol{\varphi}(s) e(t) - \frac{1}{C(\boldsymbol{\rho}^*, s)} \tilde{\boldsymbol{\rho}}(t)^{\mathrm{T}} \boldsymbol{\varphi}(s) e(t) = o\left[\sup_{\tau\leq t} |e(\tau)|\right] \tag{32}$$

From (25) it follows that

$$\tilde{\boldsymbol{\rho}}(t)^{\mathrm{T}} \frac{1}{C(\boldsymbol{\rho}^*, s)} \boldsymbol{\varphi}(s) e(t) = o\left[\sup_{\tau\leq t} |e(\tau)|\right] \tag{33}$$

From (20), (26), (29), (30), (31), (32), and (33) it follows that

$$e(t) = o\left[\sup_{\tau\leq t} |e(\tau)|\right] \tag{34}$$

Clearly, (34) contradicts $e(t) = O\left[\sup_{\tau\leq t} |e(\tau)|\right]$. Hence, $y(t)$ is a bounded signal. From (28) and $\boldsymbol{\varphi}(s)$ are asymptotically stable, $u(t)$ is also a bounded signal. Hence, all the signal of the closed loop system is assured to be bounded. In addition, noting that (29) and the boundedness of $\boldsymbol{\xi}(t)$, it can leads to $\lim_{t\to\infty} \varepsilon(t) = 0$. Therefore, from (26), it can be concluded that $\lim_{t\to\infty} e(t) = 0$. ∎

The following assumptions of Theorem 5.1 may be rather restrictive.

$$\frac{1}{C(\boldsymbol{\rho}^*, s)} \text{ is asymptotically stable}$$

However, it is not so restrictive in the case where the number of the control parameters is less than or equal to three because the assumption is satisfied if all the sign of the control parameters are positive. If the projection algorithm is employed to the adaptive adjusting law (15) and (16) in order to keep all the sign of the control parameters to be positive, the assumption can be satisfied straightforwardly.

VI. A Numerical Example

Consider the following 3rd order stable minimum phase plant model.

$$P(s) = \frac{2}{s^3 + 3s^2 + 3s + 1} \tag{35}$$

The reference signal is given by

$$r(t) = \begin{cases} 1 & t \in [40k,\ 40k+20] \\ -1 & t \in [40k+20,\ 40(k+1)] \end{cases}$$
$$k = 0, 1, 2, \cdots \tag{36}$$

In the simulation, the following controller is considered.

$$C(\boldsymbol{\rho}(t), s) = \boldsymbol{\rho}^{\mathrm{T}} \boldsymbol{\varphi}(s) \tag{37}$$
$$\boldsymbol{\rho}(t)^{\mathrm{T}} = \begin{bmatrix} k_P(t), & k_I(t), & k_D(t) \end{bmatrix} \tag{38}$$
$$\boldsymbol{\varphi}^{\mathrm{T}}(s) = \begin{bmatrix} 1, & \frac{1}{s+\alpha}, & \frac{s}{\tau s+1} \end{bmatrix}, \quad \tau > 0 \tag{39}$$

$k_P(t)$, $k_I(t)$, and $k_D(t)$ correspond to adjustable proportional, integral, and differential gain, respectively. τ is a small positive real number: $\tau = 0.01$, which stands for a time constant of the approximate differentiation. α is also a small positive real number: $\alpha = 0.01$, which stands for an inverse of time constant of the approximate integrator.

The reference model is given by

$$P_r(s) = \frac{219.9s^2 + 264.1s + 104}{s^5 + 103s^4 + 304s^3 + 523.9s^2 + 367.1s + 105} \tag{40}$$

The reference model corresponds to the closed loop transfer function when the desired control parameters are $k_{Pd} = 1.3046$, $k_{Id} = 0.5070$, and $k_{Dd} = 1.0865$ are employed. Namely, the simulation considers the case where the assumption (2) is satisfied.

The Fig. 2 is a simulation result when the proposed RLS adaptive controller is applied, where the initial control parameters are

$$k_P(0) = 0.3, \quad k_I(0) = 0.2, \quad k_D(0) = 0 \tag{41}$$

and the adaptive gain in (15) and (16) is set to be $\boldsymbol{P}(0) = 100\boldsymbol{I}$ and $\gamma = 10$. From the figure, the relatively large control error between the plant output and the reference model output in the beginning of the simulation, but the error quickly goes to zero. Hence, we can see that the proposed adaptive control works well shows a good convergence property. The Fig. 3 shows that the plant input signal in the simulation. From the figure, we can see that the input signal remains bounded, which shows that the stability of the closed loop system is assured.

The Fig. 4 shows the tuned control parameters in the simulation. From the figure, the tuned parameters quickly converge to the constant values, respectively:

$$k_P(300) = 1.3040, \quad k_I(300) = 0.5070,$$
$$k_D(300) = 1.0830$$

These parameters almost correspond to the true values which make the closed loop transfer functions is the given reference model transfer function $P_r(s)$. Hence, we can see that the adaptive adjusting law properly works in the propose method.

The Fig. 5 shows the identification error $\varepsilon(t)$ shown in (12) and the control error $e(t)$. It should be noted that the identification error $\varepsilon(t)$ rapidly goes to zero.

Fig. 6, Fig. 7, Fig. 8 and Fig. 9 are the simulation results[7] using the gradient algorithm for the adaptive adjusting law. Obviously, it follows that the RLS case shows even better convergence rate.

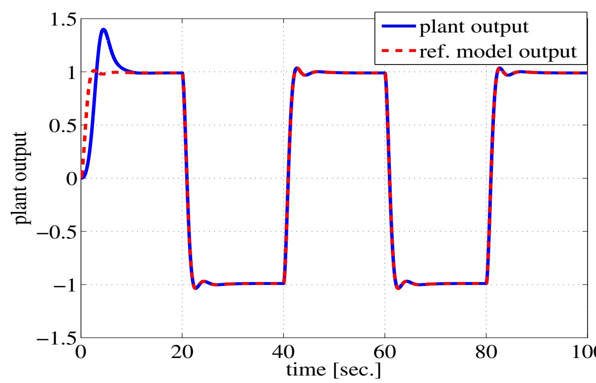

Fig. 2. Plant output in the proposed adaptive control and the reference model output using the RLS algorithm

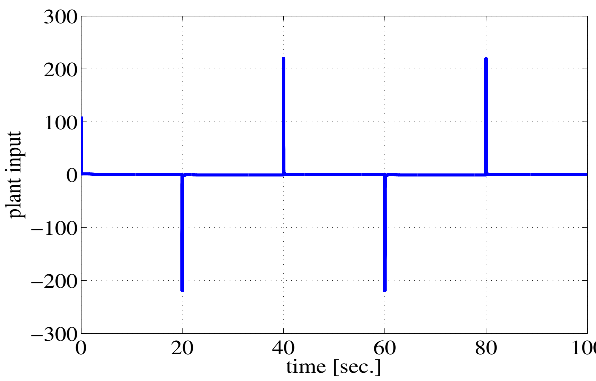

Fig. 3. Plant input in the proposed adaptive control using the RLS algorithm

VII. Conclusion

The paper gives a normalized recursive least square method for the adaptive adjusting law for the model reference adaptive control based on an on-line FRIT approach. The boundedness of all signals in the closed loop system as well

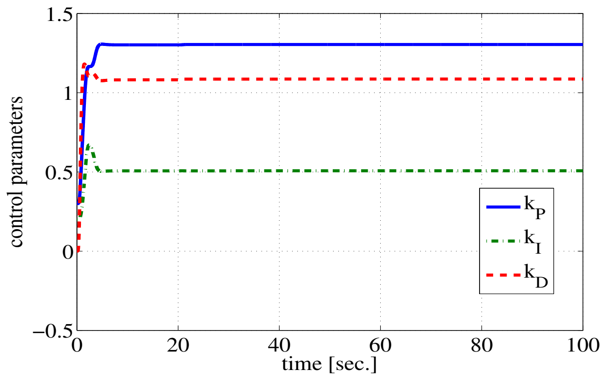

Fig. 4. The tuned control parameters in the proposed adaptive control using the RLS algorithm

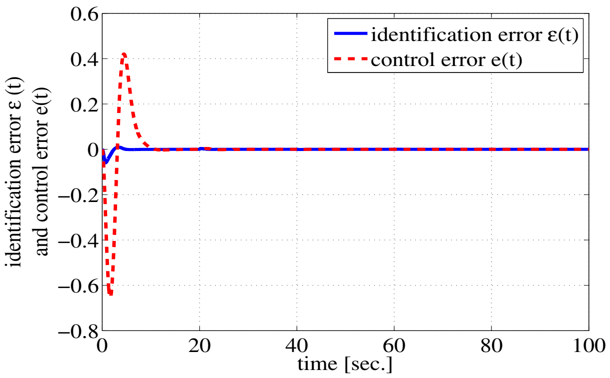

Fig. 5. The identification error $\varepsilon(t)$ and control error $e(t)$ using the RLS algorithm

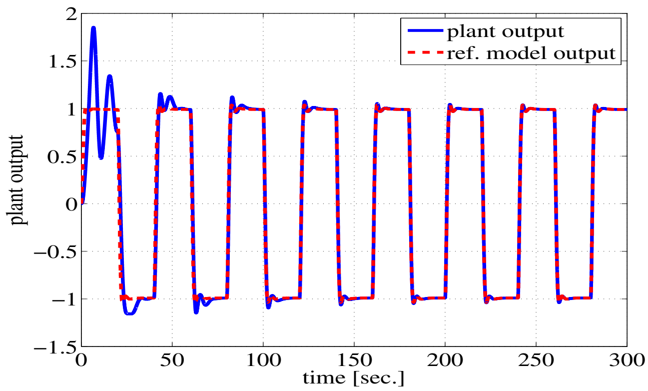

Fig. 6. Plant output in the proposed adaptive control and the reference model output using the gradient algorithm

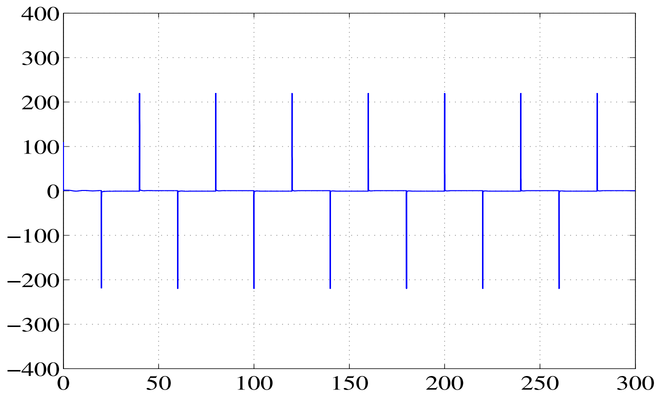

Fig. 7. Plant input in the proposed adaptive control using the gradient algorithm

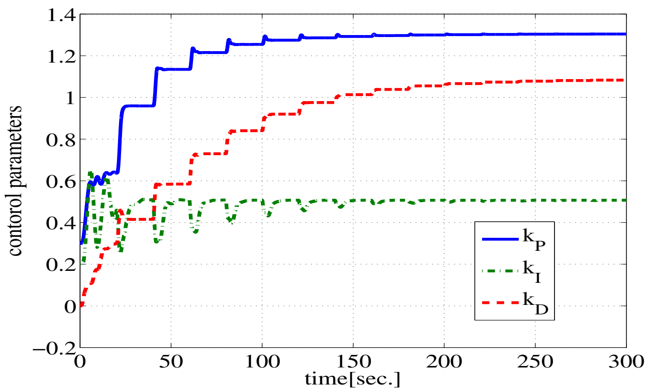

Fig. 8. The tuned control parameters in the proposed adaptive control using the gradient algorithm

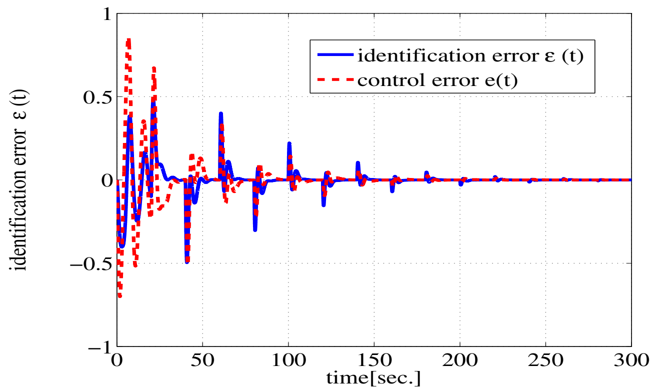

Fig. 9. The identification error $\varepsilon(t)$ and control error $e(t)$ using the gradient algorithm

as asymptotically tracking the reference model output were proved. An effectiveness of the proposed method was shown through a numerical example comparing with the simulation results[7] using the gradient algorithm for adaptive adjusting law.

In the proposed method, the controller has to be represented as linear combination of proper and stable transfer function. However, the integrator does not belong to the class of the controller which the paper has discussed. Hence, strictly to say, the adaptive PID gains tuning cannot be treated by the proposed method. The point as well as the case of the presence of disturbances remain future works.

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