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Robust gain-scheduled controller for linear parameter varying systems

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***Abstract*—Proportional-integral-derivative (PID) controller is the most widely used in industry due to its simplicity and performance characteristics. However, a number of processes with time-variant or nonlinear characteristics are difficult to be handled with a conventional PID controller. To address this problem, an enhanced PID controller exhibiting improved performance than the conventional linear fixed-gain PID controller is proposed. The algorithms proposed use a gain-scheduling technique, and integrates robustness and explicit input-output constraint-handling capabilities in the controller design. Nonlinear processes are modeled as a linear parameter varying (LPV) system. The gain-scheduled P controller is designed by off-line solving robust optimal control problem in order to construct a sequence of state feedback gains. The associated sequence of nested invariant sets is used to define the corresponding operating region of each state feedback gain computed. At each control iteration, the smallest invariant set containing the current state measured is determined, a corresponding feedback gain is then implemented to the process. Further, an interpolation algorithm is proposed to improve control performances. The feedback gain implemented to the process is determined by maximizing its norm subjected to a set of constraints associated with the current invariant set. Stability of a closed loop behaviour can be guaranteed. Simulation example of a spherical level tank is used to illustrate the applicability of the algorithms proposed. Comparison between our algorithms and a conventional PI controller tuned by existing technique is performed. The simulation results showed that the proposed algorithms can stabilize the system while satisfying input-output constraints, and provide a better control performance than the conventional PI controller. Interpolation algorithm can improve control performance while on-line computation is still tractable.**

I. INTRODUCTION

Proportional-integral-derivative (PID) is the most widely used controller in industry for several decades because of its simplicity and performance characteristics. Although a conventional PID controller which uses fixed gain is often sufficient, a number of processes with nonlinear characteristics are often beyond the capabilities of a conventional PID controller. Limitations of the conventional PID control become obvious when applied to more complicated systems such as those with a time-delay, poorly damped, highly nonlinear and time-varying dynamics [1]. To address these issues, the PID controller has evolved to include adaptive features such as self-tuning and gain-scheduling.

Gain-scheduling requires insights on process behaviours in order to define operating region and its optimal PID tuning parameters. The parameters are stored and later recalled for use in the controller, according to the prevailing operational conditions represented by a custom scheduling scheme. A traditional technique is to linearize the process around several operating points and to use linear control tools to design a controller for each of these points. However, a transition of operating regions might lead to instabilities if the PID controller is not designed to make smooth transitions. Thus, success of such an approach depends on establishing a relationship between an original nonlinear system and a linear system associated. Various techniques to design gain-scheduled controllers have been proposed [2].

A mild nonlinear system can be efficiently represented by a linear parameter-dependent description. Thus, linear parameter varying (LPV) systems have recieved increasing attention during last decades [3]. LPV model is a linear system parameterized by scheduling parameters. The dynamics depend on external time varying parameters. The future trajectories are unknown a priori but states can be accurately measured or estimated online. Therefore, information on the current process model is available.

The extension of H_∞ synthesis techniques to allow for controller dependence on time-varying but measured parameters was studied in [4]. Thus, a higher control performance can be achieved by incorporating measurements of these parameters to the control algorithm. In [5], a bounding technique based on parameter-dependent Lyapunov function was used to design PD controllers. The proposed approach represents generalization of the standard sub-optimal H_∞ control problem.

In [6], a design problem of gain-scheduled controllers for LPV systems via parameter-dependent Lyapunov function was addressed. A gain-scheduled controller design for discrete-time systems was proposed in [7]. The design of gain-scheduled PI controller, when the uncertainty of the system is assumed to be a difference between the nonlinear model and the nominal linear model, was studied in [8].

Robust model predictive control (RMPC) is another promising approach capable of determining optimal state feedback gain for LPV systems. RMPC for linear time varying (LTV) systems has been developed in [9]. However, RMPC requires solving convex optimization involving linear matrix inequalities and a computational complexity of the optimization problem associated grows exponentially with the number of vertice of polytopic uncertain set, therefore, the problem on computational complexity limits wide application of RMPC. An off-line RMPC for LPV systems was introduced in [10]. Sequences of state feedback gains corresponding to sequences of nested ellipsoidal invariant sets are pre-computed off-line. At each control iteration, the smallest ellipsoid containing the current state measured is determined. The state feedback gain implemented to the process is obtained by linear interpolation between the pre-computed state feedback gains. An off-line RMPC algorithm based on polyhedral invariant set has been developed in

[11]. A sequence of nested polyhedral invariant sets corresponding to a sequence of pre-computed state feedback gains is constructed off-line. At each control iteration, the smallest polyhedral invariant set containing the current state measured is determined. The corresponding state feedback gain is then implemented to the process

In this paper, a nonlinear single input single output (SISO) system is considered. We present a framework for a gain-scheduled controller design targeting at nonlinear processes formulated as an LPV system. The paper is organized as follows. In section 2, formulation of an LPV system is discussed. In section 3, gain-scheduled controller based on robust model predictive is presented. The proposed algorithm is described in section 4. In section 5, we illustrate an implementation of the algorithm proposed in a case study of the spherical level tank. Finally, we conclude the paper in the last section.

II. LINEAR PARAMETER VARYING SYSTEMS

A nonlinear process can be represented as a linear parameter varying system. The most common approach is linearization scheduling, based on Jacobian linearization of the process at a number of equilibrium points. Typically, the parameterization corresponds to a fixed value of scheduling parameter. The other approach is quasi-LPV scheduling. Quasi-LPV scheduling does not involve any Jacobian linearization. Plant dynamics are reformulated to conceal nonlinearities as time-varying parameters used as scheduling parameters. A discrete-time SISO LPV system is described as in Eq. 1.

$$x(k+1) = a(p(k))x(k) + b(p(k))u(k). \tag{1}$$

where $x(k) \in R$ is a state of the plant and $u(k) \in R$ is a control input. A scheduling parameter $p(k)$ is assumed to be on-line measurable at each control iteration k. In addition, $a(p(k))$ and $b(p(k))$ are assumed to be within a polytope Ω,

$$\Omega = Co\{[a_1,b_1],[a_2,b_2],...,[a_L,b_L]\}. \tag{2}$$

Co denotes a convex hull of $[a_j,b_j]$ vertex. Any $[a(p(k)),b(p(k))]$ being inside the polytope Ω is a convex combination of all vertices such that

$$[a(p(k)),b(p(k))] = \sum_{j=1}^{L} p_j(k)[a_j,b_j], \tag{3}$$

$$\sum_{j=1}^{L} p_j(k) = 1, 0 \le p_j(k) \le 1. \tag{4}$$

III. ROBUST MODEL PREDICTIVE GAIN-SCHEDULING

In this work, a nonlinear system formulated as a discrete-time LPV system is taken into account. The objective is to find a state feedback gain for a control law

$$u(k+i/k) = Kx(k+i/k), \tag{5}$$

that stabilises the system and achieves the minimum worst case performance cost while satisfying input and output constraints.

An on-line robust model predictive control for linear time varying (LTV) systems was introduced [9]. A control objective is to minimize an upper bound on infinite horizon worst-case performance cost subjected to input and state constraints. The optimization problem associated with this controller for the problem considered is shown in Eqs. 6-10.

$$\min_{\gamma,y,q} \gamma \tag{6}$$

$$\text{s.t.} \begin{bmatrix} 1 & x(k) \\ x(k) & q \end{bmatrix} \ge 0, \tag{7}$$

$$\begin{bmatrix} q & a_l q + b_l y & \theta^{\frac{1}{2}} q & \zeta^{\frac{1}{2}} y \\ a_l q + b_l y & q & 0 & 0 \\ \theta^{\frac{1}{2}} q & 0 & \gamma & 0 \\ \zeta^{\frac{1}{2}} y & 0 & 0 & \gamma \end{bmatrix} \ge 0,$$

$$\forall l = 1,2,...,L, \tag{8}$$

$$\begin{bmatrix} u_{\max}^2 & y \\ y & q \end{bmatrix} \ge 0, \forall i = 1,2,...,N, \tag{9}$$

$$\begin{bmatrix} x_{\max}^2 & a_l q + b_l y \\ a_l q + b_l y & q \end{bmatrix} \ge 0, \forall l = 1,2,...,L, \tag{10}$$

where $\theta > 0$ is a weighting factor of state, $\zeta > 0$ is a weighting factor of control input. The optimization problem is solved and the control input $u(k) = yq^{-1}x(k)$ is implemented to the process.

Theorem 1: Given an initial measured state $x(k)$, the control law obtained by solving the associated optimization problem shown in Eqs. 6-10 assures robust stability to the closed-loop system while satisfying input and state constraints.

Proof: The satisfaction of Eq. 8 for the feedback gain K ensures that $[a_l + b_l Kx(k)]\ \gamma\ q^{-1}[[a_l + b_l Kx(k)] - x(k)\ \gamma q^{-1}\ x(k) \le -\ [x(k)\theta x(k) + u(k)\zeta u(k)]$, $\forall l = 1, 2, ..., L$.

Thus, $V(k) = x(k)\gamma q^{-1}x(k)$ is a strictly decreasing Lyapunov function and the closed-loop system is robustly stabilized by the state feedback gain K. Eq. 7 also defines the corresponding invariant set. Any states $x \in S$, $S = \{x| - 1/\sqrt{q} \le x \le 1/\sqrt{q}\}$ can be stabilized by this state feedback gain. In addition, the satisfaction of Eq. 10 guarantees that $(a_l + b_l y)q^{-1}(a_l + b_l y) \le x_{\max}^2$, $\forall l = 1, 2, ..., L$. Thus, the future states move closer to the origin, and their norms are always lower than $|x_{\max}|$. Moreover, the satisfaction of Eq. 9 ensures that $yq^{-1}y \le u_{\max}^2$, and a magnitude of future control inputs are always lower than $|u_{\max}|$. Thus, the corresponding state feedback control law $u(k) = Kx(k)$, $K = yq^{-1}$, assures robust stability to the closed-loop system. ∎

IV. THE PROPOSED ALGORITHM

A. *Offline computation*

- Choose a sequence of states $x_m, m = 1,2,...,m_{\max}$. For each x_m, solve the optimization problem in Eqs. 6-10 by replacing $x(k)$ with x_m in order to obtain the corresponding state feedback gain

$K_m = y_m q_m^{-1}$. x_m is chosen such that $S_{m+1} \subset S_m$. Where, $S_m = \{x| -1/\sqrt{q_m} \leq x \leq 1/\sqrt{q_m}\}$, and $S_{m+1} = \{x| -1/\sqrt{q_{m+1}} \leq x \leq 1/\sqrt{q_{m+1}}\}$. Moreover, for each $m \neq m_{\max}$, the following inequalities must be satisfied $q_m^{-1} - (a_l + b_l K_{m+1}) q_m^{-1} (a_l + b_l K_{m+1}) > 0, \forall l = 1,2,...,L$ to assure robust stability satisfaction of a convex combination between K_m and K_{m+1}. This condition is required in Algorithm 2 of online computation. The state feedback gains are derived based on the minimization of upper bound of infinite horizon worst-case performance with input output constraints satisfaction. The corresponding invariant set $S_m = \{x| -1/\sqrt{q_m} \leq x \leq 1/\sqrt{q_m}\}$ defines the operating region of each feedback gain K_m.

B. Online computation

At each control iteration, $x(k)$ is measured, the smallest invariant set $S_m = \{x| -1/\sqrt{q_m} \leq x \leq 1/\sqrt{q_m}\}$ containing the current state measured is determined. The state feedback gain $K(k)$ for the control law $u(k) = K(k)x(k)$ is determined by using either Algorithm 1 or Algorithm 2.

Algorithm 1: If $x(k) \in S_m, \forall m \leq m_{\max}$, the state feedback gain $K(k) = K_m$ for the control law $u(k)$=$K(k)x(k)$ is implemented to the process.

Theorem 2: Given an initial measured state $x(k) \in S_m$, the control law provided by Algorithm 1 assures robust stability to the closed-loop system with input and state constraints satisfaction.

Proof: Each K_m is derived based on Theorem 1, thus we prove theorem 2 the same way as we prove theorem 1. The satisfaction of Eq. 8 for the feedback gain K_m ensures that $[a_l + b_l K_m x(k)]\ \gamma_m\ q_m^{-1}[[a_l + b_l K_m x(k)] - x(k)\ \gamma_m q_m^{-1}\ x(k) \leq -\ [x(k)\theta x(k) + u(k)\zeta u(k)]$, $\forall l = 1,2, ...,L$. Thus, $V(k) = x(k)\gamma_m q_m^{-1} x(k)$ is a strictly decreasing Lyapunov function and the closed-loop system is robustly stabilized by the state feedback gain K_m. Moreover, the satisfaction of Eq. 10 guarantees that $(a_l + b_l y_m) q_m^{-1} (a_l + b_l y_m) \leq x_{\max}^2$, $\forall l = 1,2, ...,L$. Thus, the future states bound between $-x_{\max}$ and $x_{\max}$. Moreover, the satisfaction of Eq. 9 ensures that $y_m q_m^{-1} y_m \leq u_{\max}^2$, and a magnitude of future control inputs are always lower than $|u_{\max}|$. Thus, the corresponding state feedback control law $u(k) = K_m x(k)$ drive the initial state to S_{m+1} toward the origin without input and state constraints violation.

Any initial states $x(k) \in S_m$ are guaranteed that all future states remain in S_m without input and state constraints violation. Any initial states $x(k) \notin S_m$ lead to the future states that violate input and output constraints for at least one realization of the uncertainty. Thus, the corresponding state feedback control law $u(k) = K_m x(k)$ assures robust stability to the closed-loop system. ■

Algorithm 2: The state feedback gain $K(k)$ is calculated by linear interpolation between the pre-computed state feedback gains K_m and K_{m+1} to obtain the largest norm of state feedback gain.

If $x(k) \in S_m$ and $x(k) \notin S_{m+1}, \forall m \leq m_{\max} - 1$, the state feedback gain $K(k) = \lambda(k)K_m + (1 - \lambda(k))K_{m+1}$ is obtained by solving the optimization problem in Eqs. 11-15. The control law $u(k)$=$K(k)x(k)$ is implemented to the process.

$$\min_{\lambda(k)} \lambda(k) \tag{11}$$

s.t.

$$-1/\sqrt{q_m} \leq \sum_{i=1}^{L} p_i(k)(a_l + b_l K(k))x(k) \leq 1/\sqrt{q_m}, \tag{12}$$

$$-u_{\max} \leq K(k)x(k) \leq u_{\max}, \tag{13}$$

$$K(k) = \lambda(k)K_m + (1 - \lambda(k))K_{m+1}, \tag{14}$$

$$0 \leq \lambda(k) \leq 1, i = 1,2,...,N. \tag{15}$$

If $x(k) \in S_{m_{\max}}$, the state feedback gain $K(k) = K_{m_{\max}}$ for the control law $u(k)$=$K(k)x(k)$ is implemented to the process.

Theorem 3: Given an initial measured state $x(k) \in S_m$, the control law provided by Algorithm 2 assures robust stability to the closed-loop system.

Proof: As $q_m^{-1} - (a_l + b_l K_{m+1}) q_m^{-1} (a_l + b_l K_{m+1}) > 0$, $\forall l = 1,2,...,L$ are satisfied, a convex combination between K_m and K_{m+1}, $K(k) = \lambda(k)K_m + (1 - \lambda(k))K_{m+1}$, $0 \leq \lambda(k) \leq 1$, can robustly stabilize the closed-loop system with $V(k) = x(k)\gamma_m q_m^{-1} x(k)$ which is a strictly decreasing Lyapunov function. The satisfaction of Eq. 13 guarantees that the control input at current time step bounds between $-u_{\max}$ and $u_{\max}$. Eq. 12 defines the current invariant set associated with K_m. The satisfaction of Eq. 12 guarantees that under given uncertainty a one step prediction $x(k+1)$ remains inside the current invariant set. Thus, the state feedback control law $u(k) = K(k)x(k)$ obtained from solving the optimization problem in Eqs. 11-15 assures robust stability to the closed-loop system. ■

V. Simulation of Spherical Level Tank

In this section, we present an example that illustrates an implementation of the proposed algorithms. The numerical simulations have been performed in 2.3 GHz Intel Core i-5 with 16 GB RAM, using SDPT3[12], Gurobi[13] and YALMIP [14] within Matlab R2011b environment.

We consider the application of our approach to the spherical level tank as shown in Fig. 1. A radius of the tank is 0.5 m and the outflow from the tank depends on a liquid level as $F = 1.6971\sqrt{h}$. The system is described by Eq. 16.

$$\frac{dh}{dt} = -\frac{1.6971 h^{0.5}}{\pi h - \pi h^2} + \frac{F_i}{\pi h - \pi h^2}. \tag{16}$$

Where h is a water level in the spherical tank. F_i is a water flowrate fed into the spherical tank. Let $\bar{h} = h - h_{eq}$ and $\bar{F_i} = F_i - F_{i,eq}$. Where subscript eq denotes the corresponding variable at equilibrium condition, $h_{eq} = 0.5$ m and $F_{i,eq} = 1.2$ m^3/hr. The objective is to regulate $\bar{h}$ to the origin by manipulating F_i. The input constraint is symmetric $0.7 \leq F_i \leq 1.7$ m^3/hr. In addition, symmetric output constraint $-0.45 \leq \bar{h} \leq 0.45$ is considered. We further assume that the maximum difference between $-1.6971/(\pi h^{1.5} - \pi h^{2.5})$ and

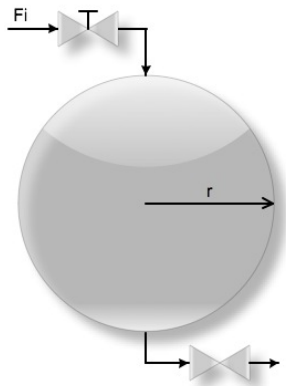

Fig. 1. Schematic diagram of the spherical tank.

$-1.6971/(\pi h_{eq}^{1.5} - \pi h_{eq}^{2.5})$ is small enough to be neglected. Also the maximum difference between $1/(\pi h - \pi h^2)$ and $1/(\pi h_{eq} - \pi h_{eq}^2)$ is neglected. Thus, we describe our system in terms of deviation variables as in Eq. 17.

$$\frac{d\bar{h}}{dt} = -\frac{1.6971}{\pi h^{1.5} - \pi h^{2.5}}\bar{h} + \frac{1}{\pi h - \pi h^2}\bar{F}_i. \tag{17}$$

By rearranging Eq. 17 along all four vertices of an uncertainty set, the solution of Eq. 16 is also the solution of the following differential inclusion

$$\frac{d\bar{h}}{dt} \in \sum_{l=1}^{4} p_l(k)[\tilde{A}_l\bar{h} + \tilde{B}_l\bar{F}_i], \tag{18}$$
$$p_1 = \lambda\gamma, \tag{19}$$
$$p_2 = (1-\lambda)\gamma, \tag{20}$$
$$p_3 = \lambda(1-\gamma), \tag{21}$$
$$p_4 = (1-\lambda)(1-\gamma), \tag{22}$$
$$\lambda = \frac{1}{47.9545}(50.8603 - \frac{1.6971}{\pi h^{1.5} - \pi h^{2.5}}), \tag{23}$$
$$\gamma = \frac{1}{4.7981}(6.7013 - \frac{1}{\pi h - \pi h^2}), \tag{24}$$

where $[\tilde{A}_1\ \tilde{B}_1]$ = [−2.9058 1.2732], $[\tilde{A}_2\ \tilde{B}_2]$ = [−50.8603 1.2732], $[\tilde{A}_3\ \tilde{B}_3]$ = [−2.9058 6.7013], and $[\tilde{A}_4\ \tilde{B}_4]$ = [−50.8603 6.7013]. The discrete-time model is obtained by discretization of Eq. 18 using Euler first-order approximation with a sampling period of 0.0005 hr.

$$\bar{h}(k+1) \in \sum_{l=1}^{4} p_l[A_l\bar{h}(k) + B_l\bar{F}_i(k)], \tag{25}$$

where $[A_1\ B_1]$ = [0.9985 0.0006], $[A_2\ B_2]$ = [0.9746 0.0006], $[A_3\ B_3]$ = [0.9985 0.0034], and $[A_4\ B_4]$ = [0.9746 0.0034].

The wighting parameters for control input and state are $\zeta = 0.01$ and $\theta = 1$, respectively.

A sequence of four invariant sets with associated state feedback gains were generated by using the following states h = (0.95, 0.85, 0.75, 0.51), equivalently $\bar{h}$ = (0.45, 0.35, 0.25, 0.01). Table I shows operating regions and corresponding state feedback gains constructed off line.

TABLE I. THE OPERATING REGION AND THE STATE FEEDBACK GAIN ASSOCIATED.

Operating region	State feedback gain
$-0.45 \le \bar{h} \le 0.45$	−1.1111
$-0.35 \le \bar{h} \le 0.35$	−1.4286
$-0.25 \le \bar{h} \le 0.25$	−1.6157
$-0.01 \le \bar{h} \le 0.01$	−1.6163

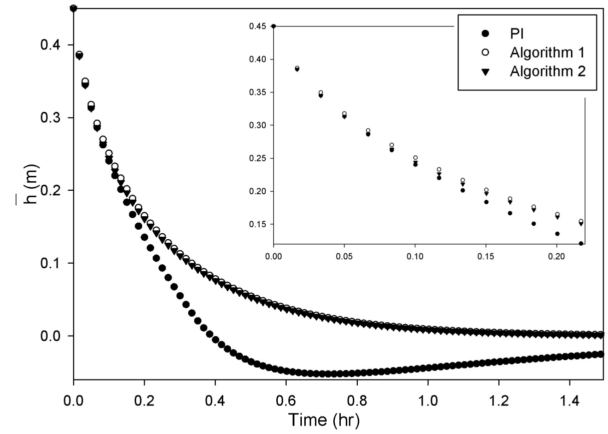

Fig. 2. Regulated state of the spherical tank.

Simulation is performed to stabilize the system from the deviated state of h = 0.95 ($\bar{h} = 0.45$) to the origin. The performance of each algorithm proposed is then compared with a conventional PI controller tuned by SIMC method [15]. Figure 2 depicts the performance of each algorithm in terms of regulated state ($\bar{h}$). A profile of $\bar{h}$ obtained by Algorithm 1 is similar to a profile of $\bar{h}$ resulted from Algorithm 2. A small picture inside Fig. 2 shows responses of each algorithm at time from 0 to 0.2 hr. A slight difference between responses of Algorithm 1 and Algorithm 2 is notice. Algorithm 2 yields a slightly faster response with lower performance cost than Algorithm 1 because the interpolation technique used usually provides higher norm of state feedback gain. Both algorithms achieve less conservative results as compared to PI controller. Moreover, PI controller leads to a response with overshoot resulted from an integral action, and it requires longer time to reach the origin.

Figure 3 shows profiles of control input $\bar{F}_i$ obtained from each algorithm. A profile of $\bar{F}_i$ of Algorithm 1 is also similar to that of Algorithm 2. A difference between control input profiles of Algorithm 1 and Algorithm 2 can be clearly seen in a small picture inside Fig. 3. A jerking in control input is observed in the profile of Algorithm 1 at the time from 0 to 0.15 hr. The jerking is caused by a switching of feedback gains. We can overcome this issue by using the interpolation technique proposed in Algorithm 2. In PI control, control input is saturated at the time from 0 to 0.35 hr.

Table II shows cumulative performance costs obtained from each Algorithm. The lowest cumulative performance

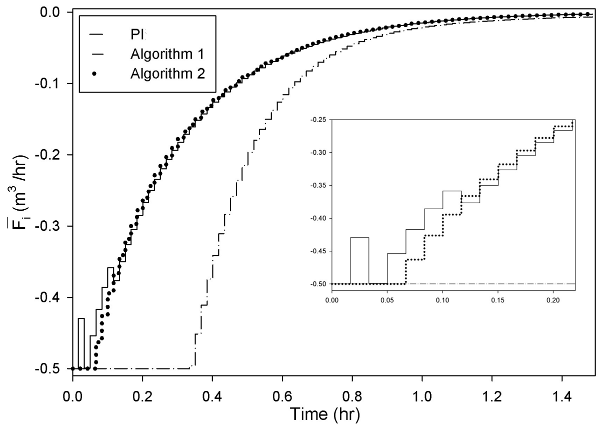

Fig. 3. Control input of the spherical tank.

TABLE II. CUMMULATIVE PERFORMANCE COSTS $\sum_{i=1}^{N}(x(i)\theta x(i)+u(i)\zeta u(i))$

Algorithm	The cummulative performance cost
PID	6.87
Algorithm 1	5.56
Algorithm 2	5.48

cost is obtained by using Algorithm 2. In addition, both Algorithm 1 and Algorithm 2 produce lower cumulative performance cost than PI controller.

For both Algorithms 1 and 2, most of computational burdens are moved off-line so an on-line computation is still tractable. The optimization problem involved in Algorithm 2 requires solving a linear programming. In contrast, Algorithm 1 does not require solving any optimization problems.

We further investigate control performance of each controller in a case where setpoint changes as step from 0.5 to 0.4, 0.6, 0.4 and 0.5 m. Figures reffig:output2 and reffig:input2 show profiles of regulated state (h) and control input (F_i), respectively. The algorithms proposed perform

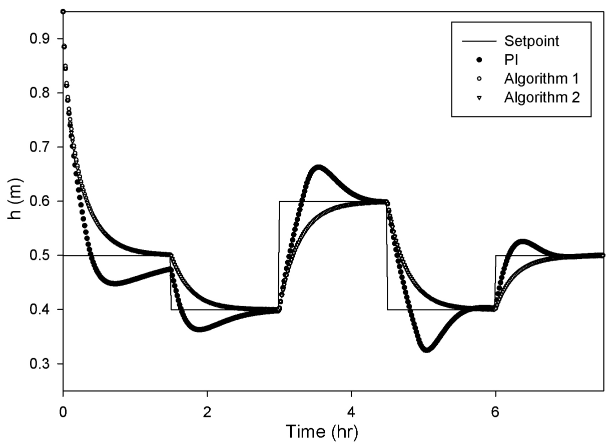

Fig. 4. Regulated state of the spherical tank.

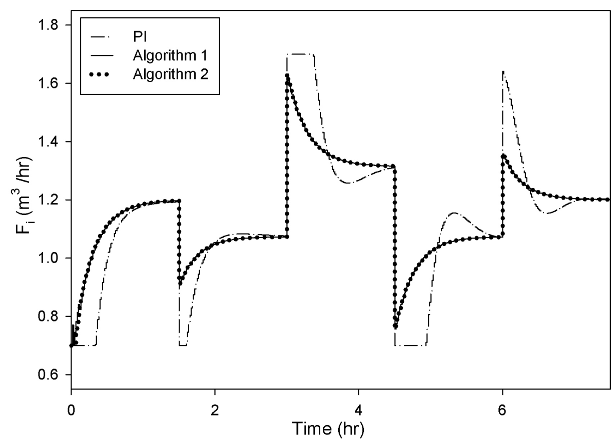

Fig. 5. Control input of the spherical tank.

better than PI controller.

VI. CONCLUSIONS

We have proposed a framework for robust gain-scheduled controller targeting at nonlinear processes formulated as an LPV system. The proposed algorithms integrate robustness and explicit input state constraint-handling capabilities in the controller design. Feasibility and stability can be guaranteed. Simulation example of a spherical level tank is used to illustrate an applicability of the algorithms proposed. Comparison between our algorithms and a conventional PI controller tuned by SIMC is performed. The proposed algorithms can stabilize the system while satisfying input and state constraints, and provide a better control performance than a conventional PI controller. Interpolation algorithm can improve control performance while on-line computation is still tractable.

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REFERENCES

[1] W. Chang, R. Hwang, and J. Hsieh, "A self-tuning pid control for a class of nonlinear systems based on the lyapunov approach," *J. Process Contr.*, vol. 12, pp. 233–242, 2002.

[2] D. Leith and W. Leithead, "Survey of gain-scheduling analysis and design," *Int. J. Control*, vol. 73, pp. 1001–1025, 2000.

[3] W. Rugh and J. Shamma, "Research on gain-scheduling," *Automatica*, vol. 36, pp. 1401–1425, 2000.

[4] P. Apkarian, P. Gahinet, and G. Becker, "Self-scheduled h_∞ control of linear parameter-varying systems: a design example," *Automatica*, vol. 31, pp. 1251–1261, 1995.

[5] F. Wu, X. Yang, A. Packard, and G. Becker, "Induced l_2-norm control for lpv systems with bounded parameter variation rates," *Int. J. Robust Nonlin.*, vol. 6, pp. 2379–2383, 1996.

[6] M. Sato and D. Peaucelle, "Gain-scheduled output-feedback controllers depending solely on scheduling parameters via parameter-dependent lyapunov functions," *Automatica*, vol. 47, pp. 2786–2790, 2011.

[7] F. Adegas and J. Stoustrup, “Structured control of affine linear parameter varying systems,” in *Proceedings of the 2011 American Control Conference (ACC)*, 2011, pp. 739–744.

[8] J. Gao and H. Budman, “Design of robust gain-scheduled pi controllers for nonlinear processes,” *J. Process Contr.*, vol. 15, pp. 807–817, 2005.

[9] M. Kothare, V. Balakrishnan, and M. Morari, “Robust constrained model predictive control using linear matrix inequalities,” *Automatica*, vol. 32, pp. 1361–1379, 1996.

[10] P. Bumroongsri and S. Kheawhom, “An ellipsoidal off-line model predictive control strategy for linear parameter varying systems with applications in chemical processes,” *Syst. Control Lett.*, vol. 61, pp. 435–442, 2012.

[11] P. Bumroongsri and S. Kheawhom, “An off-line robust mpc algorithm for uncertain polytopic discrete-time systems using polyhedral invariant sets,” *J. Process Contr.*, vol. 22, pp. 975–983, 2012.

[12] R. Tütüncü, K. Toh, and M. Todd, “Solving semidefinite-quadratic-linear programs using sdpt3,” *Math. Prog. Series B*, vol. 95, pp. 189–217, 2003.

[13] Gurobi Optimization Inc., “Gurobi optimizer reference manual,” 2013. [Online]. Available: http://www.gurobi.com

[14] J. Löfberg, “Yalmip: A toolbox for modeling and optimization in matlab,” in *Proc. IEEE international symposium on computer aided control systems design*, 2004, pp. 284–289.

[15] S. Skogestad, “Simple analytic rules for model reduction and pid controller tuning,” *Journal of Process Control*, vol. 13, pp. 291–309, 2003.