# Optimization of $H_2$ Performance Criterion for Disturbance Attenuation in the Direct Data-based Control Design*

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***Abstract*— The direct data-based control design solves the optimal control parameters which minimize the performance criterion. The input and output data collected from the controlled process derives the performance criterion instead of using process model. The approach has difficulty when the initial input and output data for the control design is generated from a disturbance signal, and the disturbance signal is assumed to be neither known, nor deterministic signals. The paper considers disturbance attenuation problem in the regulatory control using direct data-based control design approach on the assumption that the disturbance is modeled by colored stochastic signal, and is fed at the input signal. The paper formulates $H_2$ performance criterion representing the variance of disturbance response. The performance criterion is derived from the initial input and output data. The paper gives analytical results for the optimization problem. According to the main result in the paper, the problem can be formulated as a quadratic optimization problem with equality constraints. Finally, a simple numerical example is illustrated to show the analytical result.**

## I. INTRODUCTION

The direct data-based control design solves the optimal control parameters which minimize the performance criterion. The input and output data collected from the controlled process derives the performance criterion instead of using process model. These methods are especially effective when closed loop systems are already set in full operation because collecting input and output data for identification of the process model becomes a hard task.

The virtual reference feedback tuning (VRFT)[1] solves the perfect model matching problem directly from data. The method has a distinguished feature which only requires a single experiment. Subsequently, noniterative data-driven controller tuning using the correlation approach (NoCbT)[5], the unfalsified method[11], the fictitious reference iterative tuning (FRIT) [12], [2], [3], and the extended FRIT[6], [4] have been proposed. While the researches have focused on improving the closed loop characteristics from the reference signal to the controlled output, the direct control design methods for disturbance attenuation have been recently proposed[7], [8], [9], [10]. The several disturbance models, which are not only step-type or impulse-type deterministic disturbances, but also colored stochastic model, have been considered. In case of colored stochastic disturbance, model following criterion is not adequate. The prior research[10] has introduced variance evaluation for the performance criterion to solve the disturbance attenuation problem. The performance criterion is formulated as variance ratio of the estimated values to the target values in terms of the disturbance reference model output. The method has advantage that it can avoid an additive experiment where test signals are fed to the closed loop system for collecting the initial input and output data. The feature facilitates applying the proposed method to the industrial applications. However, the research [10] has not given the analytical property for the performance criterion represented as the variance ratio. Hence, the optimized parameters numerically solved by nonlinear optimization tool do not always assure the desired control parameters.

The paper formulates $H_2$ performance criterion representing the variance of disturbance response. The performance criterion is derived from the initial input and output data. The paper gives analytical results for the optimization problem. According to the main result in the paper, the problem can be formulated as a quadratic optimization problem with equality constraints. Finally, a simple numerical example is illustrated to show the analytical result.

## II. PROBLEM STATEMENTS

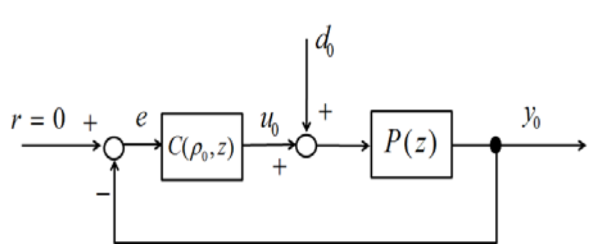


Fig. 1. Closed loop system in the regulator problem

Consider a single-input, single-output, discrete-time, time-invariant, one-degree-of freedom closed-loop system with a disturbance signal at the input signal. shown in Fig. 1. Let the process model be denoted by $P(z)$ in the form of the transfer function. The argument $z$ stands for a shift operator, and the initial values of transfer functions are assumed to be zero.

Attention is restricted to the feedback controller $C(\boldsymbol{\rho}, z)$ linearly parametrized in terms of adjustable control parameters $\boldsymbol{\rho}$. That is, the controller $C(\boldsymbol{\rho}, z)$ can be described as

$$C(\boldsymbol{\rho}, z) = \boldsymbol{\varphi}(z)^{\mathrm{T}}\boldsymbol{\rho} \qquad (1)$$

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where $\boldsymbol{\rho}$ is an $n$-dimensional control parameter vector, and $\boldsymbol{\varphi}(z)$ is also an $n$-dimensional vector whose elements are rational functions of $z$ representing a transfer function.

$\boldsymbol{\rho}_0$ is an initial control parameter vector, and it is assumed that the controller $C(\boldsymbol{\rho_0})$ stabilizes the closed-loop system, but $\boldsymbol{\rho}_0$ is not well tuned.

Now, let $u_0$ and $y_0$ be the measured one-shot closed-loop experimental input and output signal, respectively when disturbance $d_0$ is added at the input signal of the closed-loop system for a certain period (from time $0$ to time $T$).

The paper considers the disturbance $d$ modeled by the following equation

$$d = F(z)w \tag{2}$$

where $F(z)$ is a stable discrete-time, finite dimensional transfer function, which works as a filter, and $w$ is a white noise with zero mean value, and the variance is $\sigma_w^2$. The filter $F(z)$ and the variance of the white noise $\sigma_w^2$ are assumed to be known. Hence, the disturbance $d_0$, which generates the initial input and output data $u_0$ and $y_0$ is also modeled by (2).

The control objective is to derive the control parameter vector realizing an ideal response by using one-shot closed-loop experimental input and output signal $u_0$, $y_0$ instead of using a process model. The paper defines the ideal response as outputs of reference model transfer function $P_{dr}(z)$ as is shown in $y_{dr} = P_{dr}(z)d$. The paper assumes that there exists the ideal control parameter vector $\boldsymbol{\rho}_d$ such that the following equation is satisfied:

$$P_{dr}(z) = \frac{P(z)}{1 + C(\boldsymbol{\rho}_d, z)P(z)} \tag{3}$$

Since the disturbance $d$ is assumed to be a colored, and stochastic signal defined in (2), the paper evaluates the performance of the reference model output using the variance of the signal. The control problem can be formulated into the optimization problem which minimizes the following performance criterion.

$$J(\boldsymbol{\rho}) = \left| \frac{\mathrm{Var}\left[y_d(\boldsymbol{\rho})\right]}{\mathrm{Var}\left[P_{dr}(z)d\right]} - 1 \right| \tag{4}$$

where Var$[\cdot]$ stands for the variance of the signal, and $y_d(\boldsymbol{\rho})$ is the closed-loop response for the disturbance signal $d$ when the controller $C(\rho, z)$ is employed for the feedback controller.

$$y_d(\boldsymbol{\rho}) = \frac{P(z)}{1 + C(\boldsymbol{\rho}, z)P(z)} d \tag{5}$$

Since $y_d(\boldsymbol{\rho})$ depends on the plant model $P(z)$, the optimization problem to minimize the performance criterion (4) cannot be solved without using the plant model. Hence, the key issue of the paper is to introduce an alternative performance index by only using the one-shot closed-loop experimental input and output signal $u_0$ and $y_0$ instead of using the plant model.

## III. Formulation of the Optimization of $H_2$ Performance Criterion

The section reformulates the optimization of $H_2$ performance criterion. The expectation of the variance of $y_d(\boldsymbol{\rho})$ in (5), and disturbance reference model output $P_{dr}(z)d$ are represented as

$$\mathrm{Var}\left[y_d(\boldsymbol{\rho})\right] = \left\| \frac{P(z)}{1 + C(\boldsymbol{\rho}, z)P(z)} F(z) \right\|_2^2 \sigma_w^2 \tag{6}$$

$$\mathrm{Var}\left[P_{dr}(z)d\right] = \|P_{dr}(z)F(z)\|_2^2 \sigma_w^2 \tag{7}$$

where, $\|\cdot\|_2$ stands for $H_2$ norm of the discrete-time transfer function. Using (6) and (7), the performance criterion (4) becomes

$$J(\boldsymbol{\rho}) = \frac{1}{\|P_{dr}(z)F(z)\|_2^2} \cdot \left| \left\| \frac{P(z)}{1 + C(\boldsymbol{\rho}, z)P(z)} F(z) \right\|_2^2 - \|P_{dr}(z)F(z)\|_2^2 \right| \tag{8}$$

Clearly, the equivalent performance criterion to (8) can be given in the following equation:

$$J_e(\boldsymbol{\rho}) = \left( \left\| \frac{P(z)}{1 + C(\boldsymbol{\rho}, z)P(z)} F(z) \right\|_2^2 - \|P_{dr}(z)F(z)\|_2^2 \right)^2 \tag{9}$$

The performance criterion (9) is continuously differentiable at every $\boldsymbol{\rho}$. Hence, it follows that the performance criterion (9) is well formulated. However, the performance criterion (9) becomes a complex nonlinear function, which is a fourth order function of the parameter $\boldsymbol{\rho}$ when the controller $C(\boldsymbol{\rho})$ is linearly parameterized shown in (1). That is why it becomes a hard task to solve the global minimum.

Therefore, let the optimization problem be reformulated. Noting that the ideal control parameter vector $\boldsymbol{\rho}_d$ satisfy (3), it follows that the ideal control parameter vector $\boldsymbol{\rho}_d$ satisfy the following constraints with equality:

$$\|P_{dr}(z)F(z)\|_2^2 = \left\| \frac{P(z)}{1 + C(\boldsymbol{\rho}, z)P(z)} F(z) \right\|_2^2 \tag{10}$$

Then, we formulate the following optimization problem:

$$\begin{aligned} \text{Minimize} \quad & \left\| \left(1 - P_{dr}(z)\boldsymbol{\varphi}(z)^{\mathrm{T}}\boldsymbol{\rho}\right) F(z) \right\|_2^2 \\ \text{s.t.} \quad & \text{Eq.}(10) \end{aligned}$$

It should be noted that the objective function becomes the sensitivity function in case of $\boldsymbol{\rho} = \boldsymbol{\rho}_d$. The formulation is reasonable because minimization of sensitivity function achieves the disturbance attenuation.

Since the equality constraint (10) depends on the plant model $P(z)$, the optimization problem cannot be solved without using the process model. Hence, the key issue of the paper is to introduce an alternative optimization problem by only using the one-shot closed-loop experimental input and output signal $u_0$ and $y_0$ instead of using the process model.

## IV. Direct Data-based Control Design for Disturbance Attenuation

### A. Optimization Problem Using Disturbance Response Estimation

The subsection derives disturbance response estimation using the initial input and output data $u_0$ and $y_0$ based on the approach of FRIT(Fictitious Reference Iterative Tuning) [12], [2].

FRIT introduces a fictitious reference signal $r^*(\boldsymbol{\rho})$ that generates closed-loop input and output signals corresponding to the first experimental input-output data $u_0$ and $y_0$ even when the controller $C(\boldsymbol{\rho}, z)$ is employed where the control parameter vector $\boldsymbol{\rho}$ is different from the initial control parameter vector $\boldsymbol{\rho}_0$, but the disturbance $d_0$, which generates the initial input and output data, is invariably added.

In detail, regarding the control input $u_0$ is generated from the fictitious reference signal $r^*(\boldsymbol{\rho})$, we can introduce the following equation.

$$u_0 = C(\boldsymbol{\rho}, z)\left(r^*(\boldsymbol{\rho}) - y_0\right) \tag{11}$$

Noting that the same disturbance $d_0$ is invariably added at the input signal, and using the control input (11), the measured output signal $y_0$ can be represented as

$$\begin{aligned} y_0 &= P(z)(u_0 + d_0) \\ &= P(z)\left\{C(\boldsymbol{\rho}, z)\left(r^*(\boldsymbol{\rho}) - y_0\right) + d_0\right\} \\ &= P(z)C(\boldsymbol{\rho}, z)r^*(\boldsymbol{\rho}) - P(z)C(\boldsymbol{\rho}, z)y_0 + P(z)d_0 \end{aligned}$$

Hence, the output signal $y_0$ becomes

$$y_0 = \frac{P(z)C(\boldsymbol{\rho}, z)}{1 + P(s)C(\boldsymbol{\rho}, z)} r^*(\boldsymbol{\rho}) + \frac{P(z)}{1 + P(z)C(\boldsymbol{\rho}, z)} d_0 \tag{12}$$

On the other hand, from (11), the fictitious reference signal $r^*(\boldsymbol{\rho})$ can be described as

$$r^*(\boldsymbol{\rho}) = C(\boldsymbol{\rho}, z)^{-1} u_0 + y_0 \tag{13}$$

Using (12) and (13), we can get

$$\begin{aligned} y_0 &= \frac{P(z)}{1 + P(z)C(\boldsymbol{\rho}, z)} u_0 + \frac{P(z)C(\boldsymbol{\rho}, z)}{1 + P(z)C(\boldsymbol{\rho}, z)} y_0 \\ &\quad + \frac{P(z)}{1 + P(z)C(\boldsymbol{\rho}, z)} d_0 \end{aligned} \tag{14}$$

The paper assumes that there exists the ideal control parameter $\boldsymbol{\rho}_d$ such that (3) is satisfied. Hence, substituting $\boldsymbol{\rho} = \boldsymbol{\rho}_d$ into (14), it follows from (3) that

$$y_0 = P_{dr}(z)u_0 + C(\boldsymbol{\rho}_d, z)P_{dr}(z)y_0 + P_{dr}(z)d_0 \tag{15}$$

Using (15), an estimated disturbance reference model output can be derived as

$$\hat{y}_{dr}(\rho) = y_0 - P_{dr}(z)u_0 - C(\boldsymbol{\rho}, z)P_{dr}(z)y_0 \tag{16}$$

When the control parameters correspond to the desired control parameters $\boldsymbol{\rho}_d$, the left-hand side of (16) becomes the disturbance reference model output $P_{dr}(z)d_0$. Although the signal $P_{dr}(z)d_0$ is a stochastic signal, the variance of $P_{dr}(z)d_0$ becomes a specific variable because the disturbance $d_0$ is modeled by (2), and the filter $F(z)$ and the variance of the white noise $\sigma_w^2$ are assumed to be known. Indeed, the variance of $P_{dr}(z)d_0$ is calculated in the following way.

$$\begin{aligned} \text{Var}\left[P_{dr}(z)d_0\right] &= \text{Var}\left[P_{dr}(z)F(z)w\right] \\ &= \|P_{dr}(z)F(z)\|_2^2 \sigma_w^2 \end{aligned} \tag{17}$$

Hence, the paper considers the optimization problem that the following performance criterion is minimized in order to derive the desired control parameters.

$$J_F(\boldsymbol{\rho}) = \left| \frac{\text{Var}\left[\hat{y}_{dr}(\rho)\right]}{\|P_{dr}(z)F(z)\|_2^2 \sigma_w^2} - 1 \right| \tag{18}$$

where $\hat{y}_{dr}(\rho)$ is defined as (16).

From the discussion analogous to Section III, the equivalent performance criterion can be derived as

$$J_{Fe}(\boldsymbol{\rho}) = \left(\text{Var}\left[\hat{y}_{dr}(\rho)\right] - \|P_{dr}(z)F(z)\|_2^2\right)^2 \tag{19}$$

By solving the optimization problem which minimizes the performance criterion using disturbance response estimation (19), the ideal control parameters can be obtained because the performance criterion becomes zero when the parameter vector $\boldsymbol{\rho}$ is equivalent to the ideal parameter vector $\boldsymbol{\rho}_d$.

Since the performance criterion (19) is also a complex nonlinear function, it becomes a hard task to solve the global minimum. The next subsection examines the property of the performance criterion (19).

### B. Reformulation of optimization problem

Consider the property of the performance criterion (19) in detail using the process model. Using (3), the estimated disturbance reference model output $\hat{y}_{dr}(\rho)$ can be rewritten into the following equation.

$$\begin{aligned} \hat{y}_{dr}(\rho) &= y_0 - \frac{P(z)}{1 + P(z)C(\boldsymbol{\rho}_d, z)} u_0 \\ &\quad - \frac{P(z)C(\boldsymbol{\rho}, z)}{1 + P(z)C(\boldsymbol{\rho}_d, z)} y_0 \end{aligned} \tag{20}$$

Noting that $y_0 = P(z)(u_0 + d_0)$, (20) becomes

$$\begin{aligned} \hat{y}_{dr}(\rho) &= \frac{P(z)(C(\boldsymbol{\rho}_d, z) - C(\boldsymbol{\rho}, z))}{1 + P(z)C(\boldsymbol{\rho}_d, z)} y_0 \\ &\quad + \frac{P(z)}{1 + P(z)C(\boldsymbol{\rho}_d, z)} d_0 \end{aligned} \tag{21}$$

Again, using (3), (21) becomes

$$\hat{y}_{dr}(\rho) = P_{dr}(z)(C(\boldsymbol{\rho}_d, z) - C(\boldsymbol{\rho}, z))y_0 + P_{dr}(z)d_0 \tag{22}$$

In addition, recalling the initial output $y_0$ is represented as

$$y_0 = \frac{P(z)}{1 + P(z)C(\boldsymbol{\rho}_0, z)} d_0$$

$\hat{y}_{dr}(\rho)$ becomes

$$\begin{aligned} \hat{y}_{dr}(\rho) &= P_{dr}(z)\left(\frac{P(z)(1 + C(\boldsymbol{\rho}_d, z) - C(\boldsymbol{\rho}, z))}{1 + P(z)C(z, \boldsymbol{\rho}_0)}\right) d_0 \\ &= P_{dr}(z)F(z)\left(1 + \boldsymbol{\xi}(z)^{\mathrm{T}}(\boldsymbol{\rho}_d - \boldsymbol{\rho})\right) w \end{aligned} \tag{23}$$

where $\boldsymbol{\xi}(z)$ is defined as

$$\boldsymbol{\xi}(z) = \frac{P(z)\boldsymbol{\varphi}(z)}{1 + P(z)C(z, \boldsymbol{\rho}_0)} \tag{24}$$

From (23), it follows that

$$\begin{aligned} &\mathrm{Var}\left[\hat{y}_{dr}(\rho)\right] \\ &= \left\| P_{dr}(z)F(z)\left(1 + \boldsymbol{\xi}(z)^{\mathrm{T}}(\boldsymbol{\rho}_d - \boldsymbol{\rho})\right)\right\|_2^2 \sigma_w^2 \\ &= (\boldsymbol{\rho}_d - \boldsymbol{\rho})^{\mathrm{T}} \boldsymbol{\Phi} (\boldsymbol{\rho}_d - \boldsymbol{\rho}) + \gamma^{\mathrm{T}}(\boldsymbol{\rho}_d - \boldsymbol{\rho}) + v \end{aligned} \tag{25}$$

where

$$\boldsymbol{\Phi} = \frac{\sigma_w^2}{2\pi} \int_{-\pi}^{\pi} \left\{ |P_{dr}(e^{j\theta})F(e^{j\theta})|^2 \boldsymbol{\xi}(e^{j\theta})\boldsymbol{\xi}(e^{-j\theta})^{\mathrm{T}} \right\} d\theta \tag{26}$$

$$\boldsymbol{\gamma} = \frac{\sigma_w^2}{\pi} \int_{-\pi}^{\pi} \mathrm{Re}\left(P_{dr}(e^{j\theta})F(e^{j\theta})\boldsymbol{\xi}(e^{-j\theta})^{\mathrm{T}}\right) d\theta \tag{27}$$

$$v = \|P_{dr}(z)F(z)\|_2^2 \sigma_w^2 \tag{28}$$

$\mathrm{Re}(\,\cdot\,)$ stands for the real part of the complex number, and $j$ stands for the imaginary unit.

Using (25), the performance index (19) becomes

$$J_{Fe}(\boldsymbol{\rho}) = \left((\boldsymbol{\rho}_d - \boldsymbol{\rho})^{\mathrm{T}} \boldsymbol{\Phi} (\boldsymbol{\rho}_d - \boldsymbol{\rho}) + \gamma^{\mathrm{T}}(\boldsymbol{\rho}_d - \boldsymbol{\rho})\right)^2 \tag{29}$$

the performance criterion (29) becomes a complex nonlinear function, which is a fourth order function of the parameter $\boldsymbol{\rho}$.

Therefore, through the analogous discussion in the section III, it follows that the ideal control parameter vector $\boldsymbol{\rho}_d$ satisfy the following constraints with equality:

$$(\boldsymbol{\rho}_d - \boldsymbol{\rho})^{\mathrm{T}} \boldsymbol{\Phi} (\boldsymbol{\rho}_d - \boldsymbol{\rho}) + \gamma^{\mathrm{T}}(\boldsymbol{\rho}_d - \boldsymbol{\rho}) = 0 \tag{30}$$

Hence, we can formulate the following optimization problem:

$$\begin{aligned} \text{Minimize} \quad & \left\| \left(1 - P_{dr}(z)\boldsymbol{\varphi}(z)^{\mathrm{T}}\boldsymbol{\rho}\right) F(z) \right\|_2^2 \\ \text{s.t.} \quad & \text{Eq.}(30) \end{aligned}$$

Similarly to the discussion in the section III, the objective function becomes the sensitivity function in case of $\boldsymbol{\rho} = \boldsymbol{\rho}_d$. The formulation is reasonable because minimization of sensitivity function achieves the disturbance attenuation.

The equality constraint (30) is analytically described clearly. However, since the equality constraint requires the process model. The following equation is equality constraint using the initial input and output data $u_0$ and $y_0$:

$$\begin{aligned} &\mathrm{Var}\left[y_0 - P_{dr}(z)u_0 - \boldsymbol{\rho}^{\mathrm{T}} P_{dr}(z)\boldsymbol{\varphi}(z)y_0\right] \\ &\quad - \|P_{dr}(z)F(z)\|_2^2 \sigma_w^2 = 0 \end{aligned} \tag{31}$$

The following optimization problem is an alternative one by only using the one-shot closed-loop experimental input and output signal $u_0$ and $y_0$ instead of using the process model.

$$\begin{aligned} \text{Minimize} \quad & \left\| \left(1 - P_{dr}(z)\boldsymbol{\varphi}(z)^{\mathrm{T}}\boldsymbol{\rho}\right) F(z) \right\|_2^2 \\ \text{s.t.} \quad & \text{Eq.}(31) \end{aligned}$$

The optimization problem is the the proposed one in the paper. The equality constraint is so strictly that it may possibly be difficult to solve the optimization. The relaxation of the equality condition is considered to be a method to derive the solution.

## V. A NUMERICAL EXAMPLE

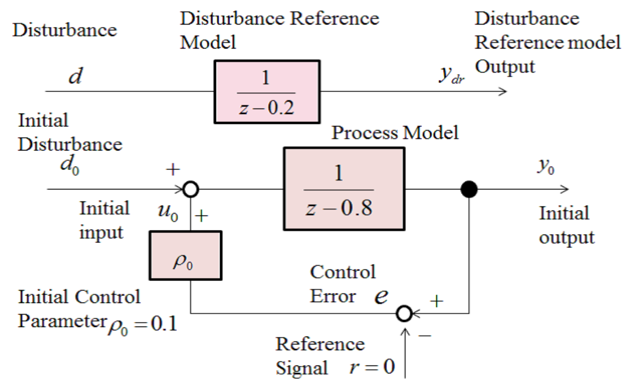


Fig. 2. Closed loop system for a simple numerical example

Consider the following discrete-time 1st order, stable process model.

$$P(z) = \frac{1}{z - 0.8} \tag{32}$$

The reference signal is set to zero, namely a regulatory control is considered. The disturbance $d$ is assumed to be a white noise with zero mean, the variance $\sigma_w^2 = 1$. The simple control law $u = \rho y$ is employed shown in Fig. 3.

Let the initial input and output data be collected when the initial control parameter $\rho_0 = 0.1$ from $t = 0$ to $t = 1000$. The variance of the initial output $y_0$ is $1.9435$. The control

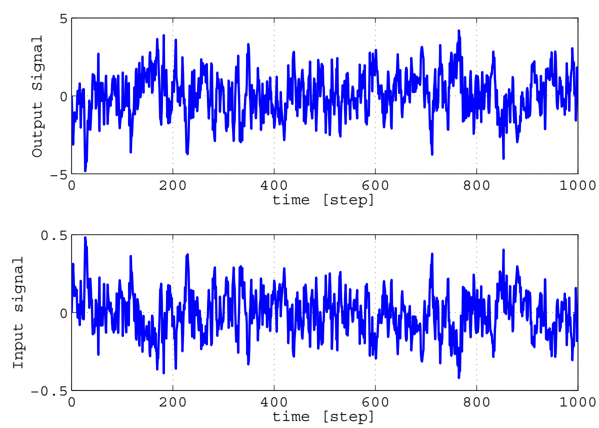


Fig. 3. The initial input and output signal

objective is to reduce the variance of output signal due to the disturbance fed at the input signal.

The disturbance reference model is set in the following way.

$$P_{dr}(z) = \frac{1}{z - 0.2} \tag{33}$$

The square of the $H_2$ norm of $P_{dr}(z)$ is $\|P_{dr}(z)\|_2^2 = 1.0417$. It follows that the output variance reduces when the control parameter tuning could succeed.

The direct control parameter tuning method is applied to the example. From the proposed method shown in section IV,

$$\text{Minimize} \left\| 1 - \frac{1}{z - 0.2}\rho \right\|_2^2$$
$$\text{s.t.} \left| \text{Var}\left[ \frac{z - 1.2}{z - 0.2} y_0 - \frac{1}{z - 0.2} u_0 \right] - 1.0417 \right| < \varepsilon$$

where the equality constraint is relaxed to the inequality condition. $\varepsilon = 0.001$ is used in the example.

The optimization solver 'fmincon.m' in the Optimization Toolbox, Matlab/Simulink Ver. 7.12.9.635 (R0211a) was used. On the ground the disturbance signal is a stochastic signal, the 100 trials to solve the optimization problem are repeated. Fig. 4 shows that the histogram of calculated optimum for the 100 trials.

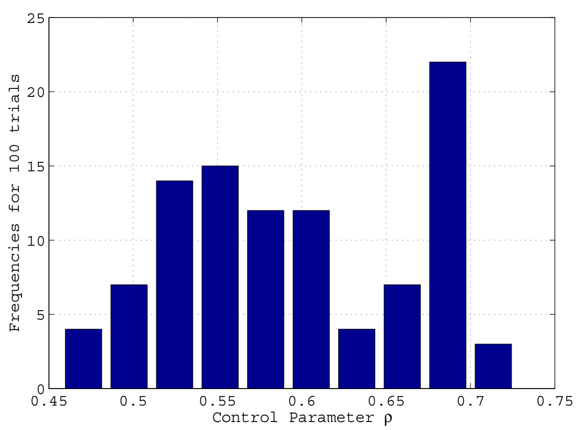


Fig. 4. The histogram of calculated optimum for the 100 trials

Obviously, the ideal control parameter $\rho_d = 0.6$. Indeed, the frequencies are distributed centering $\rho_d = 0.6$ shown in Fig. 4. The mean value of the optimized values for the 100 trials is $\rho^* = 0.5952$.

Fig. 5 shows the comparison result between process output with controller before tuning (blue dashed line) and one with controller after tuning (red solid line). From the figure Fig. 5, it follows that the process output variance can be attenuated by tuning the control parameter from the initial control parameter $\rho_0 = 0.1$ to $\rho^* = 0.5952$. The variance of the process output with controller after tuning is $0.9733$, while one with before tuning is $1.9435$.

## VI. CONCLUSIONS

The paper examines a direct control parameter tuning method for disturbance attenuation in the regulatory control problem. The disturbance is modeled by a stochastic, colored noise signal. The paper analyzed the approach based on the variance evaluation from $H_2$ performance criterion. Then, the optimization problem was reformulated as optimization problem with constraints with equality, which can be relaxed so that with inequality constraints. Finally, a numerical example was illustrated to show the reformulated optimization problem results the ideal control parameter vector.

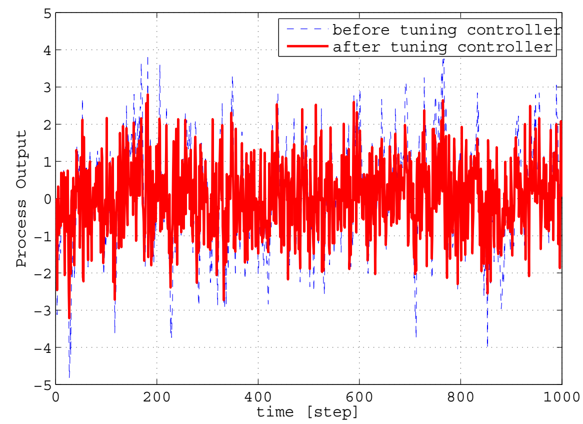


Fig. 5. The comparison result between process output with controller before tuning (blue dashed line) and one with controller after tuning (red solid line)

The initial input and output data generated by the disturbance are rather easily collected from the industrial real process because the method does not have to add a test signal to the industrial process. That is why the proposed approach is especially useful for the industrial applications.

The paper has not discussed on the case where there is no ideal control parameters. In addition, there remain problems on the stability analysis and the way of selecting the disturbance reference model to be solved for the future work. However, the key idea focusing the variance evaluation is a prospective approach and it is expected to open a new development of direct control parameter tuning method from both theoretical and practical point of view.

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