Adaptive Trajectory Control of Robot Arm Based on Smooth Projection Adaptive Law and Nonlinear friction Compensation

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***Abstract*— This paper proposes an adaptive trajectory control method for a 2DOF planar rigid link robot arm with arbitrary link length. The transient response can be improved by a dynamic certainty equivalent controller with nonlinear friction compensator and a dead zone adaptation law to be robust to bounded friction compensation error. Also, the adaptation law accompanies with a smooth projection algorithm which not only confines adjustable parameters of adaptive controller into a certain convex set to guarantee a positive definiteness of estimated inertia matrix but also ensures the differentiability of those. The convex set is designed by taking into account the existence region of the parameters of the robot arm with expected loads.**

I. INTRODUCTION

An adaptive trajectory control system for a robot manipulator is well known as a powerful control system to be robust to parameter uncertainties; for example, these are caused by a large fluctuation of tip load, accuracy of identification experiments and so on. Various adaptive trajectory control methods [1], [2] have been proposed. The control law proposed by Middleton *et al.* [2] is superior to them from the view point of transient response of trajectory error. It corresponds to the current adaptive control based on a dynamic certainty equivalent (DyCE) principle. However, the control law requires strictly both the positive definiteness of estimated inertia matrix and the differentiability of adjustable controller parameter. Sawada *et al.* [3], [4] proposed a smooth projection adaptation law in order to satisfy these condition. Tomimura *et al.* [5], [6], [7], [8] proposed a design method of restraining area of the adjustable parameters in order to be able to use the adaptive law of [3], [4] to a robot arm with any link length. Also, Shibayama *et al* [9] proposed a friction direct compensation method based on a nonlinear PI control input in a context of adaptive trajectory control system for 2DOF planar rigid link robot arm.

In this paper, the effect of the combination of these method will be examined. Hence, this paper proposes an adaptive trajectory control method for a 2DOF planar rigid link robot arm with arbitrary link length. The transient response can be improved by a dynamic certainty equivalent controller with nonlinear friction compensator and a dead zone adaptation law to be robust to bounded friction compensation error. Also, the adaptation law accompanies with a smooth projection algorithm which not only confines adjustable parameters of adaptive controller into a certain convex set to guarantee a positive definiteness of estimated inertia matrix but also ensures the differentiability of those. The convex set is designed by taking into account the existence region of the parameters of the robot arm with expected loads.

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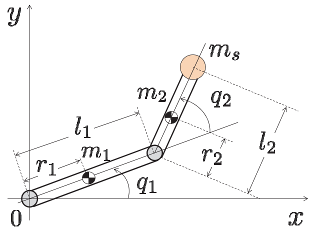

Fig. 1. Definitions of parameters and variables in 2DOF planar rigid link robot arm

II. CONTROLLED OBJECT AND PROBLEM STATEMENT

A. Controlled Object

The controlled object considered here is a 2 degree of freedom (2DOF) planar rigid link robot arm. The inverse dynamics model is presented as

$$\boldsymbol{\tau} = \boldsymbol{M}(\boldsymbol{\rho}, \boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{\rho}, \boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{\phi}(\dot{\boldsymbol{q}}) \tag{1}$$

where $\boldsymbol{\tau} := [\tau_1,\, \tau_2]^{\mathrm{T}}$ is the torque vector, $\boldsymbol{q} := [q_1,\, q_2]^{\mathrm{T}}$, $\dot{\boldsymbol{q}}$ and $\ddot{\boldsymbol{q}}$ mean the link angle vector, the link angular velocity vector and the link angular acceleration vector, respectively. $\boldsymbol{\phi}(\dot{\boldsymbol{q}})$ means the friction term. Also, $\boldsymbol{\rho} := [\rho_1,\, \rho_2,\, \rho_3]^{\mathrm{T}}$ is the parameter vector which depends upon link length (l_1, l_2), link mass (m_1, m_2), load mass m_{s} at the tip of 2nd link, moment of inertia ($I_1\, I_2$) around center of gravity of each link and length ($r_1\, r_2$) from rotation axis to center of gravity of each link (see **Fig. 1**). Then, its elements are defined as

$$\rho_1 := I_1 + I_2 + m_1 r_1^2 + (m_2 + m_{\mathrm{s}})(l_1^2 + l_2^2) \tag{2}$$
$$\rho_2 := I_2 + m_2 r_2^2 + m_{\mathrm{s}} l_2^2 \tag{3}$$
$$\rho_3 := m_2 r_2 l_1 + m_{\mathrm{s}} l_1 l_2 \tag{4}$$

These are clearly positive constant.

Vectors $\boldsymbol{M}(\boldsymbol{\rho}, \boldsymbol{q})\ddot{\boldsymbol{q}}$ and $\boldsymbol{C}(\boldsymbol{\rho}, \boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$ are The term on inertia torque and the term on centripetal or Corioli's torque, respectively. Then, The inertia matrix $\boldsymbol{M}(\boldsymbol{\rho},\boldsymbol{q})$ and the matrix $\boldsymbol{C}(\boldsymbol{\rho},\boldsymbol{q},\dot{\boldsymbol{q}})$ are defined as

$$\boldsymbol{M}(\boldsymbol{\rho},\boldsymbol{q}) := \begin{bmatrix} \rho_1 + 2\rho_3\cos q_2 & \rho_2 + \rho_3\cos q_2 \\ \rho_2 + \rho_3\cos q_2 & \rho_2 \end{bmatrix} \tag{5}$$

$$\boldsymbol{C}(\boldsymbol{\rho},\boldsymbol{q},\dot{\boldsymbol{q}}) := \begin{bmatrix} -2\rho_3\dot{q}_2\sin q_2 & -\rho_3\dot{q}_2\sin q_2 \\ \rho_3\dot{q}_1\sin q_2 & 0 \end{bmatrix} \tag{6}$$

It is well known that $\boldsymbol{M}(\boldsymbol{\rho},\boldsymbol{q})$ is always positive definite matrix for any q_2.

The first term in right-hand side of (1) can be written as linear representation with respect to $\boldsymbol{\rho}$. Hence, the inverse dynamics model can be represented as

$$\boldsymbol{\tau} = \boldsymbol{A}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\boldsymbol{\rho} + \boldsymbol{\phi}(\dot{\boldsymbol{q}}) \tag{7}$$

where

$$\boldsymbol{A}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}) := \begin{bmatrix} \ddot{q}_1 & \ddot{q}_2 & \kappa_1 \\ 0 & \ddot{q}_1+\ddot{q}_2 & \kappa_2 \end{bmatrix} \tag{8}$$

$$\kappa_1 := (2\ddot{q}_1 + \ddot{q}_2)\cos q_2 - (\dot{q}_2 + 2\dot{q}_1)\dot{q}_2\sin q_2 \tag{9}$$

$$\kappa_2 := \ddot{q}_1\cos q_2 + \dot{q}_1^2\sin q_2 \tag{10}$$

Assumptions 1: It is assumed that

(A1) Available signals are $\boldsymbol{\tau}$, $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$.

(A2) $\boldsymbol{\rho}$ is unknown but the upper bound $\bar{m}_s$ of m_s, the following positive constants $\rho_{i0\min}$, $\rho_{i0\max}$ $(i = 1,2,3)$, $l_{j\min}$ and $l_{j\max}$ $(j = 1, 2)$ are known *a priori*;

$$\rho_{i0\min} \le \rho_{i0} \le \rho_{i0\max} \tag{11}$$

$$l_{j\min} \le l_j \le l_{j\max} \tag{12}$$

where ρ_{i0} means the special parameter ρ_i when $m_s \equiv 0$.

(A3) The friction function $\boldsymbol{\phi}(\cdot) := [\phi_1(\cdot),\ \phi_2(\cdot)]^{\mathrm{T}}$ is unknown but the positive constant c_i $(i = 1, 2)$ which satisfy

$$|\phi_i(\dot{q}_i)| \le c_i\cdot(1+|\dot{q}_i|) \tag{13}$$

are known *a priori*.

(A1) is a standard assumption in a trajectory control. Angular velocities $\dot{q}_i$ may be obtained from difference data on rotary encoders. (A2) is a reasonable assumption since $\boldsymbol{\rho}$ is unknown with load change and existence intervals of ρ_i in case of $m_s \equiv 0$ (no load) can be obtained by repeating identification experiments. (A3) is also appropriate assumption without loss of generality when c_i is a conservative upper bound (relatively large positive number) though obtaining a sharper bound (c_i is small) may be difficult.

B. Problem Statement

For a 2DOF planar rigid link robot arm, the control objective considered here is to design a stable trajectory control system which satisfies following properties;

(O1) It is robust to parameter uncertainties including large fluctuation of tip load.

(O2) The influence of friction in the control system is effectively and quickly compensated.

(O3) The transient response of trajectory error is reduced more quckly and the steady-state trajectory error is sufficiently small.

III. ADAPTIVE TRAJECTORY CONTROLLR

Now, we propose here the following adaptive trajectory controller in order to achieve the control objective mentioned above.

A. Synthesis of Control Torque

The control torque is adaptively synthesized by the dynamic certainty equivalent control law with nonlinear friction direct compensator as follows;

$$\begin{aligned}\boldsymbol{\tau} =& \boldsymbol{M}(\hat{\boldsymbol{\rho}},\boldsymbol{q})(\ddot{\boldsymbol{q}}_{\mathrm{r}} - \boldsymbol{K}_{\mathrm{D}}\dot{\tilde{\boldsymbol{q}}} - \boldsymbol{K}_{\mathrm{P}}\tilde{\boldsymbol{q}}) \\ &+ \boldsymbol{C}(\hat{\boldsymbol{\rho}},\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{\tau}_{\mathrm{a}} + \boldsymbol{\tau}_{\mathrm{n}}\end{aligned} \tag{14}$$

where $\hat{\boldsymbol{\rho}}$ is the adjustable controller parameter which corresponds to $\boldsymbol{\rho}$, $\boldsymbol{q}_{\mathrm{r}}$ is a desired link angle which has second derivative, the triplet $(\boldsymbol{q}_{\mathrm{r}},\ \dot{\boldsymbol{q}}_{\mathrm{r}},\ \ddot{\boldsymbol{q}}_{\mathrm{r}})$ is designed by a trajectory planning and the inverse kinematics solution, $\tilde{\boldsymbol{q}}$ means the trajectory error $\boldsymbol{q} - \boldsymbol{q}_{\mathrm{r}}$, and

$$\boldsymbol{\tau}_{\mathrm{a}} := \boldsymbol{A}_f\dot{\hat{\boldsymbol{\rho}}} - \dot{\boldsymbol{M}}(\hat{\boldsymbol{\rho}},\boldsymbol{q})\boldsymbol{e}_f \tag{15}$$

$$\boldsymbol{\tau}_{\mathrm{n}} := \begin{bmatrix} \tau_{\mathrm{n}1} \\ \tau_{\mathrm{n}2} \end{bmatrix} := \begin{bmatrix} c_1\cdot(1+|\dot{q}_1|)\, z_1\cos z_1 \\ c_2\cdot(1+|\dot{q}_2|)\, z_2\cos z_2 \end{bmatrix} \tag{16}$$

A variable with subscript $_f$ represents the filtered output. Hence, for example, x_f means the output of low pass filter $\dot{x}_f = -f x_f + x$ where the band width $f > 0$ is the design parameter.

Remark 1: It is well known that he matrix signal $\boldsymbol{A}_f$ can be realized with only $\boldsymbol{q}$ and $\dot{\boldsymbol{q}}$. ■

Remark 2: The auxiliary input $\boldsymbol{\tau}_{\mathrm{a}}$ removes the effect of $\dot{\hat{\boldsymbol{\rho}}}$ from the error dynamics of control system. Therefore, a fast adaptation can be achieved. The auxiliary input $\boldsymbol{\tau}_{\mathrm{n}}$ plays central role in order to compensate the effect of friction torque as possible. ■

Gains $\boldsymbol{K}_{\mathrm{P}}$ and $\boldsymbol{K}_{\mathrm{D}}$ are positive definite diagonal matrices which are set so that the ideal control error equation $\boldsymbol{e}_f := (\ddot{\tilde{\boldsymbol{q}}})_f + \boldsymbol{K}_{\mathrm{D}}(\dot{\tilde{\boldsymbol{q}}})_f + \boldsymbol{K}_{\mathrm{P}}(\tilde{\boldsymbol{q}})_f = \boldsymbol{0}$ gives the specified transient response.

Then, the gain $\boldsymbol{z} := [z_1,\ z_2]^{\mathrm{T}}$ is tuned as follows;

$$\boldsymbol{z} := \boldsymbol{\beta}_{\mathrm{P}} + \boldsymbol{\beta}_{\mathrm{I}} \tag{17}$$

$$\boldsymbol{\beta}_{\mathrm{P}} := (\dot{\tilde{\boldsymbol{q}}})_f \tag{18}$$

$$\dot{\boldsymbol{\beta}}_{\mathrm{I}} = \boldsymbol{K}_{\mathrm{P}}\tilde{\boldsymbol{q}}_f + \boldsymbol{K}_{\mathrm{D}}(\dot{\tilde{\boldsymbol{q}}})_f\ ;\ \boldsymbol{\beta}_{\mathrm{I}}(0) = \boldsymbol{\beta}_{\mathrm{I}0} \tag{19}$$

B. Error dynamics

Let $\boldsymbol{x} := [\boldsymbol{x}_1^{\mathrm{T}},\ \boldsymbol{x}_2^{\mathrm{T}}]^{\mathrm{T}}$ be

$$\dot{\boldsymbol{x}}_1 = -f\boldsymbol{x}_1 + \tilde{\boldsymbol{q}}\ ;\ \boldsymbol{x}_1(0) = \boldsymbol{0} \tag{20}$$

$$\dot{\boldsymbol{x}}_2 = -f\boldsymbol{x}_2 + \dot{\tilde{\boldsymbol{q}}}\ ;\ \boldsymbol{x}_2(0) = \boldsymbol{0} \tag{21}$$

Then, according to the proposed controller, the error dynamics of control system becomes;

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \boldsymbol{O} & \boldsymbol{I} \\ -\boldsymbol{K}_{\mathrm{P}} & -\boldsymbol{K}_{\mathrm{D}} \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} \boldsymbol{O} \\ \boldsymbol{I} \end{bmatrix} \boldsymbol{M}^{-1}(\hat{\boldsymbol{\rho}}, \boldsymbol{q}) \{-\boldsymbol{A}_f \tilde{\boldsymbol{\rho}} + \boldsymbol{\xi}_f\} \quad (22)$$

$$\dot{\boldsymbol{z}} = \boldsymbol{M}^{-1}(\hat{\boldsymbol{\rho}}, \boldsymbol{q}) \{-\boldsymbol{A}_f \tilde{\boldsymbol{\rho}} + \boldsymbol{\xi}_f\} \quad (23)$$

where

$$\dot{\boldsymbol{\xi}}_f = -f\boldsymbol{\xi}_f + \{\boldsymbol{\tau}_{\mathrm{n}} - \boldsymbol{\phi}\} \quad (24)$$

Therefore, it can be seen from (22), (23) that $\boldsymbol{x}$ converges to zero if $\dot{\boldsymbol{z}}$ tends to zero and also $\boldsymbol{x}$ becomes small signal if $\dot{\boldsymbol{z}}$ goes to sufficiently small limit cycle.

Hence, the auxiliary input $\boldsymbol{\tau}_{\mathrm{n}}$ and the adaptation law proposed in the next subsection play central role.

C. Tuning of adjustable parameter

Adjustable parameter $\hat{\boldsymbol{\rho}}$ and $\dot{\hat{\boldsymbol{\rho}}}$ are updated by the robust adaptation law with smooth projection algorithm and dead zone as follows;

$$\dot{\hat{\boldsymbol{\rho}}} = \gamma_0(J^{1/2}) \cdot \boldsymbol{\Gamma}(\hat{\boldsymbol{\rho}}) \cdot \left(-\frac{\partial J}{\partial \hat{\boldsymbol{\rho}}}\right)^{\mathrm{T}} ; \ \hat{\boldsymbol{\rho}}(0) \in C \quad (25)$$

where

$$J := \frac{1}{2}\int_0^t e^{-\lambda(t-\sigma)} \left\| \frac{\tilde{\boldsymbol{\tau}}_f(\sigma) - \boldsymbol{A}_f(\sigma)\hat{\boldsymbol{\rho}}(t)}{N} \right\|^2 \mathrm{d}\sigma \quad (26)$$

$$\gamma_0(J^{1/2}) := \begin{cases} 0 & (J^{1/2} < D) \\ -1 + J^{1/2}/D & (D \le J^{1/2} < 2D) \\ 1 & (J^{1/2} \ge 2D) \end{cases} \quad (27)$$

$$\boldsymbol{\Gamma}(\hat{\boldsymbol{\rho}}) := \boldsymbol{T}\boldsymbol{\Phi}(\hat{\boldsymbol{\rho}})\boldsymbol{T}^{\mathrm{T}}/N_0 \quad (28)$$

$$\boldsymbol{T} := \begin{bmatrix} \boldsymbol{m}_1^{\mathrm{T}} \\ \boldsymbol{m}_2^{\mathrm{T}} \\ \boldsymbol{m}_3^{\mathrm{T}} \end{bmatrix}^{-1} \quad (29)$$

$$\boldsymbol{\Phi}(\hat{\boldsymbol{\rho}}) := \mathrm{diag}\{\gamma_1\boldsymbol{m}_1^{\mathrm{T}}\hat{\boldsymbol{\rho}},\ \gamma_2\boldsymbol{m}_2^{\mathrm{T}}\hat{\boldsymbol{\rho}},\ \gamma_3\boldsymbol{m}_3^{\mathrm{T}}\hat{\boldsymbol{\rho}}\} \quad (30)$$

$$N_0 := [\eta_0 + \mathrm{trace}\{\boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{\Phi}\}]^{1/2} ; \ \eta_0 > 0 \quad (31)$$

$$\tilde{\boldsymbol{\tau}} := \boldsymbol{\tau} - \boldsymbol{\tau}_{\mathrm{n}} \quad (32)$$

$$N := \sqrt{\eta + m} ; \ \eta > 0 \quad (33)$$

$$\dot{m} = -\mu m + \mu \cdot (\|\dot{\boldsymbol{q}}\|^2 + \|\boldsymbol{q}\|^2 + \|\boldsymbol{\tau}\|^2 + \|\boldsymbol{\tau}_{\mathrm{n}}\|^2) \quad (34)$$

where $\gamma_i > 0$ $(i = 1 \sim 3)$, μ is a design parameter satisfying $0 < \mu < 2f$ [10].

The continuous switching function $\gamma_0(J^{1/2})$ (see **Fig. 2**) functionalizes parameter update laws as a robust adaptation law with dead zone. $\lambda > 0$ is a design constant which means forgetting factor; it is known that the estimation become sensitive when λ is relatively large and a relatively small λ contributes to improve the convergence property of $\hat{\boldsymbol{\rho}}$. The positive constant D is the dead zone width and it is chosen by trial and error in advance so as to satisfy the following inequality;

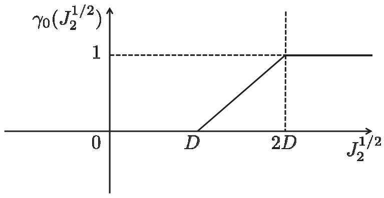

Fig. 2. The definition of switching function $\gamma_0(J^{1/2})$

$$D > \sup_t \zeta \quad (35)$$

where

$$\zeta := \left(\int_0^t e^{-\lambda(t-\sigma)} \|\boldsymbol{\xi}_{\mathrm{f}}/N\|^2 d\sigma\right)^{1/2} \quad (36)$$

The matrix $\boldsymbol{\Gamma}(\hat{\boldsymbol{\rho}})$ achieves a *smooth projection* function which means that adjustable parameters are constrained to the specified convex set C and their differential values always exist. The set C is defined as

$$C := \{\hat{\boldsymbol{\rho}} \mid \boldsymbol{\Phi}(\hat{\boldsymbol{\rho}}) \succ 0\} \quad (37)$$

Parameters $\boldsymbol{m}_1$, $\boldsymbol{m}_2$ and $\boldsymbol{m}_3$ defining C are designed beforehand with the procedure shown in the next section.

Remark 3: The adaptation law (25) can be represented as follows. Hence, $\dot{\hat{\boldsymbol{\rho}}}$ is defined as the equation below and $\hat{\boldsymbol{\rho}}$ is updated by solving numerically the differential equation;

$$\dot{\hat{\boldsymbol{\rho}}} = \gamma_0(J^{1/2})\boldsymbol{\Gamma}(\hat{\boldsymbol{\rho}})[\boldsymbol{p} - \boldsymbol{R}\hat{\boldsymbol{\rho}}] ; \ \hat{\boldsymbol{\rho}}(0) \in C \quad (38)$$

where

$$\dot{\boldsymbol{p}} = -\lambda_1\boldsymbol{p} + \boldsymbol{A}_f^{\mathrm{T}}\boldsymbol{\tau}_f/N ; \ \boldsymbol{p}(0) = \boldsymbol{0} \quad (39)$$

$$\dot{\boldsymbol{R}} = -\lambda_1\boldsymbol{R} + \boldsymbol{A}_f^{\mathrm{T}}\boldsymbol{A}_f/N ; \ \boldsymbol{R}(0) = \boldsymbol{O} \quad (40)$$

■

IV. DESIGN OF CONVEX SET C

The estimated inertia matrix $\boldsymbol{M}(\hat{\boldsymbol{\rho}}, \boldsymbol{q})$ has to be non-singular for guaranteeing stability of control system based on (22) and (23). Also, $\boldsymbol{M}(\boldsymbol{\rho}, \boldsymbol{q})$ is originally a positive definite matrix and $\rho_i > 0$ $(i = 1 \sim 3)$. Then, the set C_0 on $\hat{\boldsymbol{\rho}}$-space which satisfies $\boldsymbol{M}(\hat{\boldsymbol{\rho}}, \boldsymbol{q}) \succ 0$ and $\hat{\rho}_i > 0$ $(i = 1 \sim 3)$ is the following convex set;

$$C_0 := \left\{ \hat{\boldsymbol{\rho}} \ \middle|\ \left(\frac{\hat{\rho}_2}{\hat{\rho}_1} - \frac{1}{2}\right)^2 + \left(\frac{\hat{\rho}_3}{\hat{\rho}_1}\right)^2 < \left(\frac{1}{2}\right)^2, \ \hat{\rho}_1 > 0,\ \hat{\rho}_3 > 0 \right\} 0 \quad (41)$$

The set is shown in **Fig. 3**. Unfortunately, a smooth projection algorithm which constrains $\hat{\boldsymbol{\rho}}$ to the convex set having a curved surface like C_0 and guarantees the existence of $\dot{\hat{\boldsymbol{\rho}}}$ has not been proposed so far.

The positive gain matrix $\boldsymbol{\Gamma}(\hat{\boldsymbol{\rho}})$ in (28) has the ability which constrains $\hat{\boldsymbol{\rho}}$ to the convex set C and guarantees the existence of $\dot{\hat{\boldsymbol{\rho}}}$. C is a subset of C_0 and it has a triangular pyramid form if $\boldsymbol{m}_i$ $(i = 1 \sim 3)$ are designed properly.

Therefore, we propose a design method to determine the optimal $\boldsymbol{m}_i$ in advance by the following procedure;

Step 1: Find the existence region C_{t} from prior knowledge; $\rho_{i0\min}$, $\rho_{i0\max}$ $(i = 1 \sim 3)$, $l_{j\min}$, $l_{j\max}$ $(j = 1, 2)$ and $\bar{m}_{\mathrm{s}}$.

Step 2: Search the special set C having largest Lebesgue measure which satisfies two conditions;
- (C1) $C_{\mathrm{t}} \subset C \subset C_0$
- (C2) A separation distance between C_{t} and C is greater than some given value

Step 3: Adopt $(\boldsymbol{m}_1, \boldsymbol{m}_2, \boldsymbol{m}_3)$ corresponding to the C selected in Step 2 as the optimal value.

Then, C_{t} on the normalized $(\hat{\rho}_2/\hat{\rho}_1, \hat{\rho}_3/\hat{\rho}_1)$ plane is estimated as shown in **Fig. 4**. The small hexagon of the lower part in Fig. 4 means the existence region of $\boldsymbol{\rho}$ when $m_{\mathrm{s}} \equiv 0$.

Also, an example of containment relationship among C_{t}, C and C_0 is illustrated in **Fig. 5**.

V. ANALYSIS OF CONTROL SYSTEM

A. Stability

The following lemma and theorem are hold with respect to the adaptation loop and the main feedback loop respectively.

Lemma 1: (Stability of adaptation loop) The adaptation loop constructed by the adaptation law satisfies the following properties;
- (P1) $\hat{\boldsymbol{\rho}}(\cdot) \in L_\infty$
- (P2) $\gamma_0(J^{1/2}(\cdot))J^{1/2}(\cdot)/N_0(\cdot) \in L_1$
- (P3) $\dot{\hat{\boldsymbol{\rho}}} \in L_2 \cap L_\infty$
- (P4) $\hat{\boldsymbol{\rho}}$ converges some constant and $J^{1/2}$ enters within the dead zone D as time increases.

Proof: These properties can be derived from the fact that the following inequality holds;

$$\dot{V}(\tilde{\boldsymbol{\rho}}) \leq -c \cdot \gamma_0(J^{1/2}) \cdot J^{1/2}/N_0 \leq 0 \tag{42}$$

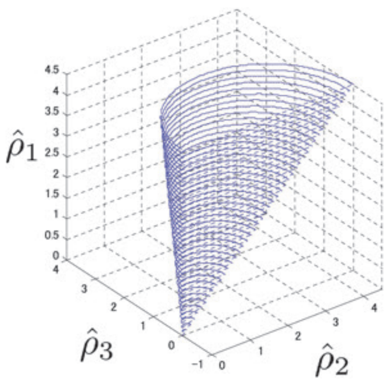

Fig. 3. The convex set C_0

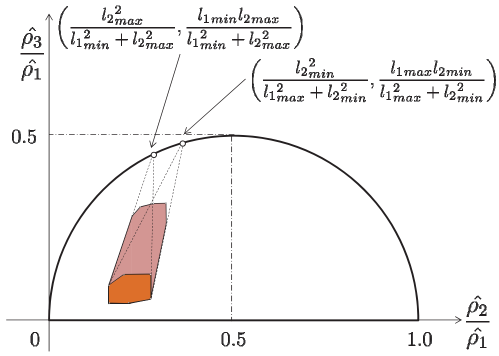

Fig. 4. Existence region of the true value $\boldsymbol{\rho}$ based on prior knowledge on the normalized plane

where the positive definite function $V(\tilde{\boldsymbol{\rho}})$ and the parameter estimation error $\tilde{\boldsymbol{\rho}}$ are defined as

$$V(\tilde{\boldsymbol{\rho}}(t)) = \sum_{i=1}^{3} \frac{1}{\gamma_i} \left\{ \hat{\alpha}_i - \alpha_i \ln \left(\frac{\hat{\alpha}_i}{\alpha_i} \right) \right\} \tag{43}$$

$$\tilde{\boldsymbol{\rho}} := \boldsymbol{\rho} - \hat{\boldsymbol{\rho}} \tag{44}$$

$$[\alpha_1, \alpha_2, \alpha_3]^{\mathrm{T}} := \boldsymbol{T}^{-1}\boldsymbol{\rho} \tag{45}$$

$$[\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3]^{\mathrm{T}} := \boldsymbol{T}^{-1}\hat{\boldsymbol{\rho}} \tag{46}$$

and c is a positive constant which satisfies

$$J^{1/2} - \zeta > c \tag{47}$$

The rest can be proven in the same way as [3], [4], [10]. Q. E. D.

Theorem 1: (Stability of main loop) All variable in the control system which consists of (1), (14) and (25) are bounded under properties of the lemma 1 if the dead zone can be chosen sufficiently small such that $D < D^*$ where D^* is some constant depending on the property of controlled object and it can be derived according to the same procedure proposed in [10].

Proof: The proof is abbreviated for simplicity since it can be done in the same manner as [3], [4], [10] . Q. E. D.

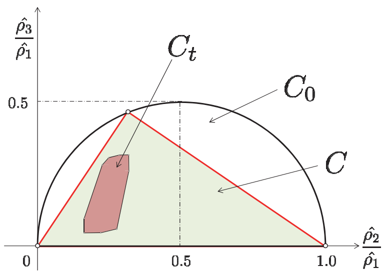

Fig. 5. C_{t}, C and C_0

B. Performance

In this subsection, the performance improvement mechanism of the proposed control system is outlined.

From (22), $\tilde{\boldsymbol{q}}$ and $\dot{\tilde{\boldsymbol{q}}}$ converges to zero while following transient response that is specified by $\boldsymbol{K}_{\rm P}$ and $\boldsymbol{K}_{\rm D}$ if $-\boldsymbol{A}_f\tilde{\boldsymbol{\rho}}+\boldsymbol{\xi}_f$ is L_2 signal. Unfortunately, $\tilde{\boldsymbol{q}}$ and $\dot{\tilde{\boldsymbol{q}}}$ can never converge to zero since $-\boldsymbol{A}_f\tilde{\boldsymbol{\rho}}+\boldsymbol{\xi}_f$ does not belong to L_2. However, a better control performance may be expected if $\dot{\boldsymbol{z}} = -\boldsymbol{A}_f\tilde{\boldsymbol{\rho}}+\boldsymbol{\xi}_f$ becomes small signal as time increases since $\boldsymbol{x}$ can be regarded as the output of asymptotically stable system having the bounded input $-\boldsymbol{A}_f\tilde{\boldsymbol{\rho}}+\boldsymbol{\xi}_f$.

On the other hand, the adaptive loop guarantees the convergence of $\tilde{\boldsymbol{\rho}}$ and

$$\left(\int_0^t e^{-\lambda(t-\sigma)}\left\|\frac{-\boldsymbol{A}_f(\sigma)\tilde{\boldsymbol{\rho}}+\boldsymbol{\xi}_f(\sigma)}{N(\sigma)}\right\|^2 {\rm d}\sigma\right)^{1/2} < D \quad (48)$$

after a certain large time. Therefore, a better performance may be expected since the dead zone width D can be chosen as a possibly small constant.

D is usually selected through trial-and-error so as to satisfy (P4) of the Lemma 1. Hence, (35) holds if it is satisfied. Therefore, it means that the lower limit of the settable D depends on ζ. From the discussion above, it is understood that small ζ (it means that $\boldsymbol{\xi}_f$ is small) leads to a good control performance in a steady state. So that, in the following, it will be shown that the presence of $\boldsymbol{\tau}_{\rm n}$ included in the control input of the proposed method satisfies this requirement.

Notice that

$$\dot{\boldsymbol{z}} \approx \boldsymbol{M}^{-1}(\hat{\boldsymbol{\rho}},\boldsymbol{q})\begin{bmatrix} \frac{c_1}{f}(1+|\dot{q}_1|)\left\{z_1\cos z_1 - \frac{\phi_1(\dot{q}_1)}{c_1\cdot(1+|\dot{q}_1|)}\right\} \\ \frac{c_2}{f}(1+|\dot{q}_2|)\left\{z_2\cos z_2 - \frac{\phi_2(\dot{q}_1)}{c_2\cdot(1+|\dot{q}_2|)}\right\}\end{bmatrix} \quad (49)$$

if $\boldsymbol{A}_f\tilde{\boldsymbol{\rho}}$ is sufficiently small according to the adaptation law and f is relatively large.

Therefore, further roughly speaking, it may be considered that the solution of $(z_i, \dot{z}_i)$ has similar dynamics to the following;

$$\dot{z}_i = k\cdot\left\{z_i\cos z_i - \frac{\phi_i(\dot{q}_i)}{c_i\cdot(1+|\dot{q}_i|)}\right\} \ ; \ k>0 \quad (50)$$

Also, Note

$$\left|\frac{\phi_i(\dot{q}_i)}{c_i\cdot(1+|\dot{q}_i|)}\right| \le 1$$

and The form of $z_i\cos z_i - \frac{\phi_i(\dot{q}_i)}{c_i\cdot(1+|\dot{q}_i|)}$ with $\phi_i(\dot{q}_i)/c_i\cdot(1+|\dot{q}_i|) = -0.7$ is illustrated by **Fig. 6**. Indeed, under certain conditions, $(\|\boldsymbol{z}\|, \|\dot{\boldsymbol{z}}\|)$ in a numerical example has been obtained as **Fig. 7**. Therefore, a better performance of the proposed control system is expected.

An example is illustrated in **Fig. reffig:trajectory1** and **Fig. 9** when the proposed controller is used and friction exists. On the other hand, the correspoding example is also shown in **Fig. 10** when any friction compensation is not applied.

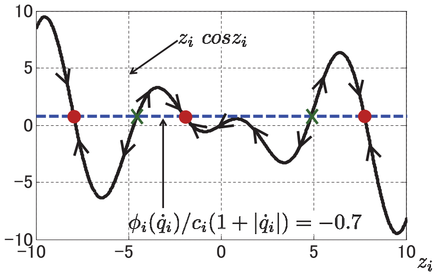

Fig. 6. Characteristics of $z_i\cos z_i$ and $\phi_i(q_i)/c_i(1+|\dot{q}_i|)$

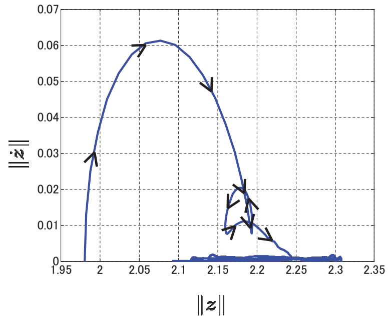

Fig. 7. Phase plane trajectory of $\|\dot{\boldsymbol{z}}\|$ and $\|\boldsymbol{z}\|$

VI. CONCLUSIONS

This paper proposes an adaptive trajectory control method for a 2DOF planar rigid link robot arm with arbitrary link length. The transient response can be improved by a dynamic certainty equivalent controller with nonlinear friction compensator and a dead zone adaptation law to be robust to bounded friction compensation error. Also, the adaptation law accompanies with a smooth projection algorithm which not only confines adjustable parameters of adaptive controller into a certain convex set to guarantee a positive definiteness of estimated inertia matrix but also ensures the differentiability of those. The convex set is designed by taking into account the existence region of the parameters of the robot arm with expected loads.

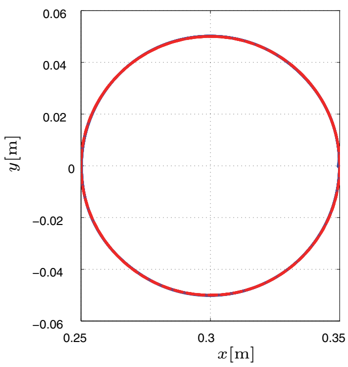

Fig. 8. Tip trajectory (line) and desired trajectory (dashed line) when nonlinear PI control input is used and the angular velocity is π[rad/s]

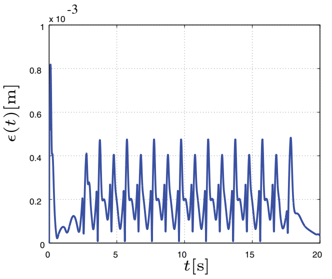

Fig. 9. Error norm of Tip trajectory and desired trajectory when nonlinear PI control input is used and the angular velocity is $\pi[\mathrm{rad/s}]$

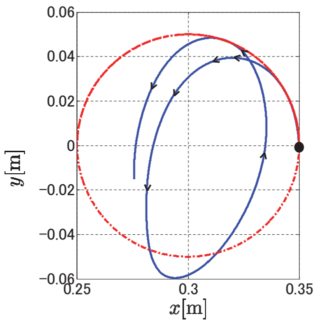

Fig. 10. Tip trajectory (line) and desired trajectory (dashed line) when a friction compensation is not used

References

[1] J. -J. E. Slotine and W. Li , Adaptive manipulator control; A case study, IEEE Trans. on Automatic Control, vol. 33, no. 11, 1988, pp. 995.

[2] R. H. Middleton and G. C. Goodwin, Adaptive Computed Torque Control for Rigid Link Manipulators, Systems & Control Letters, vol. 10, 1988, pp. 9.

[3] M. Sawada and K. Itamiya, An Improvement of Transient Response in Adaptive Trajectory Control System Based on The Computed Torque Law for Robot Arm — Applying Smooth Projection Algorithm —, SICE Trans. on Industrial Automation, vol. 10, no. 7, 2011, pp. 58. (in Japanese)

[4] M. Sawada and K. Itamiya, An Improvement of Transient Response of Dynamic Certainty Equivalent Model Reference Adaptive Control System Based on New Smooth Parameter Projection High Order Tuner, SICE Journal of Control, Measurement, and System Integration, vol. 6, no. 4, 2013, pp. 243.

[5] R. Tomimura, K. Itamiya and M. Sawada, Crass Extension of Robot Manipulator in Adaptive Trajectory Control Based on Smooth Projection Parameter Tuning Rule, Proceedings CD of SICE 13th Annual Conference on Control Systems, 8E3-2, 2013. (in Japanese)

[6] R. Tomimura, K. Itamiya and M. Sawada, An Prior Estimation Method of Sufficient Parameter Convex Set for Guaranteeing Positive Definiteness of Estimated Inertia Matrix Used in Adaptive Trajectory Control of Rigid Link Robot Arm, Proceedings CD of the 57th Annual Conference of the Institute of Systems, Control and Information Engineerings 2013, 113-5 (in Japanese)

[7] R. Tomimura, K. Itamiya and M. Sawada, A Prior Design Scheme of Constraint Space of Adjustable Parameters for Guaranteeing Positive Definiteness of Estimated Inertia Matrix Used in Adaptive Trajectory Control of 2 Link Rigid Robot Arm, Proceedings CD of JJACC 2013, 410, 2013, pp. 327. (in Japanese)

[8] R. Tomimura, K. Itamiya and M. Sawada, A Prior Design Scheme of Constraint Space of Adjustable Parameters for Guaranteeing Positive Definiteness of Estimated Inertia Matrix Used in Adaptive Trajectory Control of 2 Link Rigid Robot Arm, Trans. of ISCIE, vol. 27, no. 3, 2014. (to appear) (in Japanese)

[9] T. Shibayama, K. Itamiya and M. Sawada, A Prior Design Scheme of Constraint Space of Adjustable Parameters for Guaranteeing Positive Definiteness of Estimated Inertia Matrix Used in Adaptive Trajectory Control of 2 Link Rigid Robot Arm, Proceedings CD of JJACC 2013, 509, 2013, pp. 418. (in Japanese)

[10] S.Tanahashi and K. Itamiya, A stability and Control Performance of Robust Model Reference Adaptive Control System: An Approach Based on Its Generalized Structure, SICE JCMSI , vol. 4, no. 2, 2011, pp. 97.