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On-line maximization of biogas production in an anaerobic reactor using a pseudo-super-twisting controller ⋆

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Abstract: We consider an apparently oversimplified first order model of an anaerobic digester operated as a CSTR, where the dilution rate is the controlled input and the biogas production rate is the measured output. The parameters of this model are considered slowly time-varying. The output function depends on the only state (the substrate), and at any instant has a unique maximum. We propose a simple output-feedback controller based on the super-twisting algorithm combined with a state machine, which converges in a practical sense to this maximum. The controller was tested by simulations of an anarobic digester, maximizing the biogas production rate, showing very good results.

Keywords: second order sliding mode; anaerobic digestion; extremum-seeking control.

1. INTRODUCTION

Anaerobic digestion of biomass is a very complex process involving several types of microorganisms in various linked reactions, from the hydrolysis of complex carbohydrates to the methanization of volatile fatty acids (VFA); additionanlly, there are also physico-chemical reactions involved (Antonelli et al., 2003). Probably the most used model for describing this complexity in a tractable way is IWA's anaerobic digestion model 1 (ADM1) (Batstone et al., 2002). Although this model is useful for simulation, it is too complicated for designing controllers, since it involves 19 reactions and 24 states, plus some algebraic constraints. However, a simplification that has found applications in the design of controllers is the so-called AM2 model (Bernard et al., 2001). Still, to use it in a model-based controller, calibration of many parameters may be needed (Méndez-Acosta et al., 2010). Furthermore, in these developments the need for on-line measuring some states or estimating them with an observer has sometimes hindered real-life applications.

In this report, we tackle the problem in another way. Instead of designing a controller based on a model and assuming the measurement of critical state variables, we assume a simplified first order model (possibly the simplest one for bioreactors) and consider that an output function is indeed on-line measurable. Very little assumptions are made on the model, but a critical one is that the output function depends on the unique state and has a maximum value defined by a set of parameters that are allowed to be time-varying. This is general enough to capture the main input-output dynamics of many bioreactors, such as a biogas producing anaerobic digester.

The controller proposed is based on the well known super-twisting controller (STC) (Levant, 1998; Moreno, 2011), which has already been used for bioreactor rate estimation (Lara-Cisneros et al., 2014; Vargas et al., 2014; De Battista et al., 2012). Instead of assuming the measurement of the state and thus building a reference error signal to be used by the controller, we use an approach that uses only the available output measurements. With them we are able to compute an approximation of the absolute value of the error and to use a state-machine that estimates its sign; furthermore, it provides an estimate of a needed parameter, in a similar manner as has previously been done by Moreno et al. (2006). The main result is showing that although we implement a pseudo-super-twisting controller (PSTC) in this sense, we recover the desired properties of the STC in a practical sense. We show its applicability for the AM2 model of anaerobic digestion, maximizing the biogas production rate.

The paper is organized as follows. The next section introduces the AM2 model equations and a proposal about how to view that system as 1-dimensional with time-varying parameters. Section 3 introduces the first part of the PSTC, giving its proof of convergence, followed by a section that complements the design with the state-machine implementation of the error's sign estimator. Section 5 presents and discusses simulation results and finally conclusions are made.

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2. THE ANAEROBIC DIGESTION MODELS

Anaerobic digestion is a complex process, involving several linked reactions, but it has been simplified to a two-reaction four-parameter model, called the AM2 model (Bernard et al., 2001):

$$\dot{X}_1 = \mu_1 X_1 - a X_1 D_{\text{in}}\,, \tag{1a}$$
$$\dot{S}_1 = -k_{11}\mu_1 X_1 + (S_1^{\text{in}} - S_1) D_{\text{in}}\,, \tag{1b}$$
$$\dot{X}_2 = \mu_2 X_2 - a X_2 D_{\text{in}}\,, \tag{1c}$$
$$\dot{S}_2 = k_{12}\mu_1 X_1 - k_{22}\mu_2 X_2 + (S_2^{\text{in}} - S_2) D_{\text{in}}, \tag{1d}$$
$$G = k_G \mu_2 X_2\,, \tag{1e}$$

with

$$\mu_1(S_1) = \frac{\mu_1^{\max} S_1}{K_{S1} + S_1}\,, \tag{1f}$$
$$\mu_2(S_2) = \frac{\mu_2^* S_2^* S_2}{S_2^* S_2 + \beta_2 \left(S_2^* - S_2\right)^2}\,. \tag{1g}$$

Two reactions take place: organic substrate S_1 is consumed by acidogenic bacteria X_1 under Monod kinetics to grow and to produce volatile fatty acids (VFA) S_2, which in turn are consumed by methanogens X_2 to grow and to produce biogas, mainly methane and carbon dioxide; the reaction is of Haldane type (this is an alternative, but equivalent representation [1]). The biogas production rate G is proportional to the growth rate of methanogens. The two types of biomass may be partially fixed and $a \in [0, 1]$ ($a = 1$ represents a perfectly mixed CSTR and $a = 0$ a perfect fixed-mass bioreactor).

The model has three inputs: the dilution rate D_{in} and the two inflow substrate concentrations: S_1^{in} and S_2^{in}, but we consider that only the first one is a manipulated variable, while the latter are perturbations.

The start-up of such a reactor is no easy task, but let us assume that the bioreactor is already at a steady state $\bar{\xi} = [\bar{X}_1, \bar{S}_1, \bar{X}_2, \bar{S}_2]^T$, with input $\bar{u} = \bar{D}_{\text{in}}$ and perturbations $\bar{S}_1^{\text{in}}$ and $\bar{S}_2^{\text{in}}$ constant. It is known that a steady-state for this process that is neither the washout of methanogens nor acidogens is only achievable for certain values of $\bar{u}$, and the region of convergence for the coexistence of the two species with this constant input may be relatively small (Sbarciog et al., 2012).

By determining the possible steady states for given values of constant $\bar{u}$ we get can build an input/output steady-state map $(\bar{u}, \bar{y})$, considering $y = G$, and find that there is a unique point $(\bar{u}_{\text{opt}}, \bar{y}_{\text{opt}})$ where the biogas production rate is maximal.

[1] For the Monod equation its two parameters have relevant meaning: $\mu_{\max}$ is the maximal rate as $S \to \infty$ and K_S is the concentration where $\mu = \mu_{\max}/2$. For the Haldane equation in its usual form as an extension of the Monod equation, i.e.

$$\mu(S) = \frac{\mu_{\max} S}{K_S + S + S^2/K_I}$$

the parameters $\mu_{\max}$, K_S and K_I have no direct meaning. Instead this alternative representation uses three relevant parameters: the maximum μ^*, the concentration where this maximum occurs S^*, and $\beta > 0$, which defines the steepness of the curve:

$$S^* = \sqrt{K_S K_I}, \quad \mu^* = \frac{\mu_{\max}\sqrt{K_I}}{2\sqrt{K_S} + \sqrt{K_I}}, \quad \beta = \frac{\sqrt{K_S}}{2\sqrt{K_S} + \sqrt{K_I}}.$$

If the system has been operating for a sufficiently long time with $u(t) \geq 0$, it approaches a 2-dimensional manifold in the $\mathbb{R}_+^4$ space. To see this, consider writing the system as

$$\dot{X} = M(S)X - aXD_{\text{in}}, \qquad X(0) = X_0, \tag{2a}$$
$$\dot{S} = -KM(S)X + (S_{\text{in}} - S)D_{\text{in}}, \qquad S(0) = S_0\,. \tag{2b}$$

with the following definitions:

$$X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, \qquad K = \begin{bmatrix} k_{11} & 0 \\ -k_{12} & k_{22} \end{bmatrix}, \tag{2c}$$
$$S = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}, \qquad M(S) = \begin{bmatrix} \mu_1(S_1) & 0 \\ 0 & \mu_2(S_2) \end{bmatrix}. \tag{2d}$$

Let us make the coordinate change

$$Z = KX + S$$

so that

$$\dot{Z} = \left(-Z + S_{\text{in}} + KX(1-a)\right) D_{\text{in}}\,.$$

If $D_{\text{in}}(t) > 0$, then after some time (e.g. for $t \geq t_s$, where $\int_0^{t_s} D_{\text{in}}(\tau)\mathrm{d}\tau = 5$), with $\delta(t)$ a small delay or error,

$$Z(t) = S_{\text{in}} + KX(t)(1-a) + \delta(t)\,. \tag{3}$$

If we equate this with the definition $Z = KX + S$, then

$$X(t) = \frac{1}{a} K^{-1} \left(S_{\text{in}} - S(t) + \delta(t)\right)\,, \tag{4}$$

which we substitute in the differential equation for $S(t)$:

$$\dot{S} = -\frac{1}{a} KM(S)K^{-1} \left(S_{\text{in}} - S + \delta\right) + \left(S_{\text{in}} - S\right) D_{\text{in}}$$

Since $M(S)$ is diagonal, we notice that

$$KM(S)K^{-1} = \begin{bmatrix} \mu_1(S_1) & 0 \\ \left(\mu_2(S_2) - \mu_1(S_1)\right) k_3 & \mu_2(S_2) \end{bmatrix}, \quad k_3 = \frac{k_{12}}{k_{11}}$$

has as one of its eigenvalues $\mu_2(S_2)$, with right eigenvector $b^T = [k_3, 1]$, so if we define $S_3 = b^T S = k_3 S_1 + S_2$ we get

$$b^T KM(S)K^{-1} = \mu_2(S_2) b^T$$

and thus $\dot{S}_3 = b^T \dot{S}$ is given by

$$\dot{S}_3 = -\rho(S_3, S_1) + \left(S_3^{\text{in}} - S_3\right) D_{\text{in}} \tag{5}$$

where we have defined

$$\rho(S_3, S_1) = \frac{1}{a} \mu_2(S_3 - k_3 S_1) \left(S_3^{\text{in}} - S_3 + \delta_3\right)\,, \tag{6a}$$
$$S_3^{\text{in}} = k_3 S_1^{\text{in}} + S_2^{\text{in}}, \quad k_3 = \frac{k_{12}}{k_{11}}\,, \quad \delta_3 = k_3\delta_1 + \delta_2\,. \tag{6b}$$

In fact, S_3 is the total dissolved substrate in the bioreactor, expressed in VFA units. We can view this as a scalar system driven by two inputs: D_{in}, which we can control, and S_1, which is the state of another scalar system with the same input, namely

$$\dot{S}_1 = -\frac{1}{a} \mu_1(S_1) \left(S_1^{\text{in}} - S_1 + \delta_1\right) + \left(S_1^{\text{in}} - S_1\right) D_{\text{in}} \tag{7}$$

However, we can view ρ also as a function solely of S_3 with S_1 a time-varying parameter.

From (4), we see that $X_2 = \frac{1}{ak_{22}} \left(S_3^{\text{in}} - S_3 + \delta_3\right)$, so the output $G = k_G \mu_2(S_2) X_2$ then can also be written as:

$$G = k_y \rho(S_3)\,, \qquad k_y = \frac{k_G}{k_{22}} \tag{8}$$

It is important to notice that indeed $\rho(S_3)$ has a unique maximum for every time-varying "parameter" S_1; its domain is defined by $S_3 \geq k_3 S_1$ and its range is bounded in $\mathbb{R}_+$. Therefore, we have come up with an alternative representation of the AM2 model, considering only one substrate governed by some reaction rate function and an output function depending on this substrate, which has a unique maximum in its domain. Of course, the parameters are quite time-varying, since they depend on both fairly constant and possibly slowly varying original parameters and the "hidden" state variable S_1.

3. THE PSEUDO-SUPER-TWISTING CONTROLLER

Consider a first order system with relative degree one:

$$\dot{s}(t) = -r_s(s(t), \theta_s(t)) + [s_{\text{in}}(t) - s(t)]\, u \tag{9a}$$
$$y(t) = r_y(s(t), \theta_y(t)) \tag{9b}$$

with u the input (in our case the dilution rate D_{in}), and y the measured output. The functions $r_s(\cdot)$ and $r_y(\cdot)$ can be viewed as scalar functions of s, defined by some time-varying parameters θ_s and θ_y. Their structure may be unknown. In fact, we may even assume that $r_s(\cdot)$ is *completely unknown* at all time, but it is non-negative, i.e. $r_s\colon \mathbb{R}^+ \mapsto \mathbb{R}^+$. However we assume that its time-derivative is bounded: $|\dot{r}_s(t)| < \Delta_s$.

On the other hand, the structure of the function $r_y(\cdot)$ can also be assumed unknown, but at any given moment, it always has a unique maximum defined by two (possibly also time-varying) parameters: s^* and y^*, which are contained in the parameter vector θ_y:

$$r_y(s^*) = y^*, \qquad \frac{\partial r_y}{\partial s}\begin{cases} > 0 & \text{if } s < s^* \\ = 0 & \text{if } s = s^* \\ < 0 & \text{if } s > s^* \end{cases} \tag{10}$$

We will assume that the parameters θ_S and θ_y vary sufficiently slowly with respect to time. By *slow*, we mean that a change in the input $u(t)$ will change $s(t)$ and thus $y(t)$ relatively more than it will change the parameters $\theta_s(t)$ and $\theta_y(t)$ (if they are influenced by it).

Given a reference s^* and thus the error $e_1 = s^* - s$, the super-twisting controller (STC) is given by

$$u = k_1|e_1|^{\frac{1}{2}} \operatorname{sign}(e_1) + \eta, \tag{11a}$$
$$\dot{\eta} = k_2 \operatorname{sign}(e_1)\,. \tag{11b}$$

Now define $\varrho(t) = r_s(s^* - e_1(t))$ and consider that $\dot{\varrho}(t) = d(t)$, an unknown, but bounded function. Further defining $e_2 = \varrho - (s_{\text{in}} - s^*)\eta$, using (11) we get the dynamics for the error:

$$\dot{e}_1 = -\kappa_1|e_1|^{\frac{1}{2}} \operatorname{sign}(e_1) + e_2 - e_1 u \tag{12a}$$
$$\dot{e}_2 = -\kappa_2 \frac{1}{2} \operatorname{sign}(e_1) + d(t) \tag{12b}$$

where

$$\kappa_1 = (s_{\text{in}} - s^*)k_1\,, \qquad \kappa_2 = 2(s_{\text{in}} - s^*)k_2\,. \tag{13}$$

It has been shown (Moreno, 2011) that if $u \geq 0$ is (upper) bounded, then it is possible to find (constant) values for κ_1 and κ_2 such that the above system converges in finite time to zero despite the bounded uncertainty on $\varrho(t)$ (the term $e_1 u$ can be considered as a vanishing perturbation). Note that

$$|e_1|^{\frac{1}{2}} \operatorname{sign}(e_1) = \phi_1(e_1) \implies \frac{1}{2}\operatorname{sign}(e_1) = \phi_1'(e_1)\phi_1(e_1).$$

In order to implement this controller, we need to feed back $e_1(t)$, but we can only measure $y(t)$. We will tackle this problem by approximating separately $|e_1|$ and $\operatorname{sign}(e_1)$ from the measurement of y. Let us consider the first case.

Define $f(e_1) = r_y(s^* - e_1)$. According to (10) it will have a maximum at $e_1 = 0$, but it is *not injective* with respect to e_1. For some $y < y^*$, there will exist $e_1^- < 0$ and $e_1^+ > 0$ such that $f(e_1^-) = f(e_1^+) = y$, but usually $|e_1^-| \neq |e_1^+|$.

We propose an *injective* function $g\colon [0, \hat{y}^*] \mapsto \mathcal{D}_g \subset \mathbb{R}_+$, $g(\hat{y}^*) = 0$, with parameter $\hat{y}^* \geq y^*$, such that we can approximate $f(\cdot)$ with its inverse on $|e_1|$:

$$\hat{y} = g^{-1}(|e_1|) \tag{14}$$

This implies that given some value $y \in [0, y^*]$, we can estimate $|e_1|^{\frac{1}{2}}$ with $|g(y)|^{\frac{1}{2}}$ and therefore

$$|g(y, \hat{y}^*)|^{\frac{1}{2}} \operatorname{sign}(e_1) = \varpi(e_1)|e_1|^{\frac{1}{2}} \operatorname{sign}(e_1) \tag{15}$$

where we have made explicit the importance of the chosen parameter $\hat{y}^*$ of $g(\cdot)$ and

$$\varpi(e_1) = \left(\frac{|g \circ f(e_1)|}{|e_1|}\right)^{\frac{1}{2}}, \qquad \varpi(e_1) > 0, \tag{16}$$

where $g \circ f$ represents the composition of the two functions, but respecting the sign:

$$g \circ f(e_1) = g(f(e_1), \hat{y}^*) \operatorname{sign}(e_1) \tag{17}$$

This function is discontinuous at $e_1 = 0$ if $g(y^*) > 0$, leading to unbounded $\varpi(e_1)$ at $e_1 = 0$. However, it is continuous if $g(y^*) = 0$ (e.g. when $\hat{y}^* = y^*$), since then $g \circ f(e_1)$ is continuous and the limit as $e_1 \to 0$ exists; then $\varpi(e_1)$ is upper bounded for $e_1 \in \mathcal{D}_g$ by $\varpi_{\max}$.

The initial version of the proposed pseudo-super-twisting controller (PSTC) is

$$u = k_1|g(y, \hat{y}^*)|^{\frac{1}{2}} \operatorname{sign}(e_1) + \eta, \tag{18a}$$
$$\dot{\eta} = k_2 \operatorname{sign}(e_1)\,, \tag{18b}$$

with which the error system is given as

$$\dot{e}_1 = -\left(\kappa_1 \varpi(e_1) + \alpha_1(t)\right) \phi_1(e_1) + e_2\,, \tag{19a}$$
$$\dot{e}_2 = \left(-\kappa_2 + \alpha_2(t)\right) \phi_1'(e_1)\phi_1(e_1)\,, \tag{19b}$$

where

$$\alpha_1(t) = |e_1(t)|^{\frac{1}{2}} u(t), \qquad \alpha_2(t) = \frac{d(t)}{\frac{1}{2}\operatorname{sign}(e_1(t))}\,.$$

We notice that $|\alpha_2(t)| \leq 2\Delta_s$ and $\alpha_1(t) = 0$ when $e_1 = 0$; this is important in the proof for convergence. Now we state the main result.

Theorem 1. Consider that $g(y^*) = 0$ and thus $\varpi(t)$ is upper bounded by $\varpi_{\max}$, and

$$\varpi_M \kappa_1 = \epsilon + \frac{\Delta_2^2}{\epsilon^3 \gamma} + (\nu^2 - 1)\epsilon\gamma \left(1 + \frac{\Delta_2}{\epsilon^2 \gamma}\right)^2, \tag{20a}$$
$$\kappa_2 = \epsilon \left(\varpi_M \kappa_1 + \epsilon\,(\gamma - 1)\right)\,. \tag{20b}$$

Then there exist $\varpi_M \geq \varpi_{\max}$, $\epsilon > 0$, $\gamma > 0$, and $\nu > 1$ such that the error system (19) with the controller (18) using (13) converges globally in finite time to the origin, for all $u(t) \geq 0$.

If $g(y^*) > 0$ then ϖ is unbounded at $e_1 = 0$, but there exist $\varpi_M > 0$, $\epsilon > 0$, $\gamma > 0$, and $\nu > 1$, such that trajectories converge globally in finite time to a neighborhood $\mathcal{N}_e$ of the origin.

The complete proof of this theorem is omitted for lack of space, but it follows very closely the methodology of Moreno (2011) and also explained by Vargas et al. (2014), where a Lyapunov function candidate is defined as $V(e) = \xi^T P \xi$ with $\xi_1 = \phi_1(e_1)$ and $\xi_2 = e_2$. This leads to $\dot{V} = \phi_1'(e_1)\xi^T \left(A^T(t)P + PA(t)\right)\xi = -\phi_1'(e_1)\xi^T Q(t)\xi$ with $A(t)$ depending on the chosen gains and the unknown perturbations α_1 and α_2:

$$A(t) = \begin{bmatrix} -\varpi(t)\kappa_1 - \alpha_1(t) & 1 \\ -\kappa_2 + \alpha_2(t) & 0 \end{bmatrix}.$$

If we can show that for any constant $P = P^T > 0$ we can find a positive definite $Q(t)$ for all considered $\varpi(t)$, $\alpha_1(t)$ and $\alpha_2(t)$, then it is straightforward to prove the theorem. The choice of gains (20a)-(20b) guarantees this.

4. ESTIMATION OF THE SIGN FUNCTION

We have shown so far that under proper tuning of the gains κ_1 and κ_2, the controller (18) would lead to global finite-time convergence to at least a neighborhood of the origin. However, we still need to know sign(e_1), which must be estimated. Even though this is either a value of $+1$ or -1, it is still very important.

A simple qualitative analysis of the system with *wrong sign* shows that it is *globally unstable*. This can be done using as phase-plane analysis. It implies that if a trajectory of the feedback system starts with the correct sign, it will approach the origin, but as soon as $e_1(t)$ changes sign, it will diverge. Nevertheless, this can be used advantageously for detecting that a sign change is needed. Once we change the sign to its correct value, the convergence to the origin follows because of the *global stability* of the closed loop. We repeat this stabilization/destabilization of the system and thus error trajectories will oscillate around the origin.

We now consider a discretized version of the controller. Taking samples every T_s time units, at time $t = kT_s$ we have measurements $y_k, y_{k-1}, \ldots, y_0$ and the controller is implemented as:

$$u_{k+1} = -k_1|g(y_k, \hat{y}_k^*)|^{\frac{1}{2}}\sigma_k + \eta_k, \quad (21a)$$
$$\eta_{k+1} = \eta_k - T_s k_2 \sigma_k\,. \quad (21b)$$

The estimator for sign(e_1) proposed is a state-machine involving several discrete signals:

- $\sigma_k \in \{-1, +1\}$ is the estimate of sign(e_1) at $t = t_k$;
- $\hat{y}^*$ estimates the global maximum y^*;
- $\bar{y}_k$ keeps track of the maximum achieved value of y_k after the last sign change: $\bar{y}_k = \max(\bar{y}_{k-1}, y_k)$.
- ψ_k is used to decide whether to change the sign or not, $\psi \approx \int(\bar{y} - y)\mathrm{d}t$;
- c_k keeps track of the time since the last sign change.

Under a correct sign estimation, trajectories will make $e_1(t) \to 0$ and thus $y(t) \to y^*$. Therefore y_k will be increasing until reaching y^*; then e_1 will change sign. Afterwards, if we do not detect the sign change, trajectories will diverge and thus y_k will start decreasing. We use this to detect that a sign change is needed and also to estimate y^*.

The signal ψ_k approximates the integral of $(\bar{y}_k - y_k)$ since the last sign change. We change $\sigma_k = -\sigma_{k-1}$ (i.e. a sign change) when $\psi_k > \psi^*$, with $\psi^* > 0$ a threshold value to be proposed. At a sign change, we reset $\psi_k = 0$, $c_k = 0$, $\bar{y}_k = y_k$ and $\hat{y}_k^* = \bar{y}_{k-1}$. This last reset allows having a better estimate for y^* to be used in the estimation of $|e_1|$ according to $g(y_k, \hat{y}_k^*)$. We use a counter $c_k = c_{k-1} + T_s$ to force a sign change when $c_k > T_\sigma^{\max}$, where $T_\sigma^{\max} > 0$ is a design parameter, in order to make the controller signal oscillate at least at some minimal frequency. The signal c_k is also used to fix $\psi_k = 0$ while $c_k < T_\sigma^{min}$, with $0 < T_\sigma^{\min} < T_\sigma^{\max}$, thus establishing a maximum frequency for sign changes; $\psi_k = 0$ is also reset whenever a maximum is detected ($y_k > \bar{y}_{k-1}$) to deal with possible noise in $y(t)$. We also use $\hat{y}_k^* = \max(\hat{y}_{k-1}^*, p\bar{y}_{k-1})$ with $p > 1$ (slightly greater) to ensure that $\hat{y}_k^*$ is always larger than y_k.

Table 1 defines the boolean variables for events that might occur in the sign estimation. Table 2 summarizes how each signal should change; for example,

$$\psi_k = \begin{cases} 0 & \text{if } C \text{ or } L \\ \psi_{k-1} + T_s(\bar{y}_{k-1} - y_k) & \text{otherwise} \end{cases}$$

Event	Boolean	Condition for TRUE
Sign change	C	$c_k \geq T_\sigma^{\max}$ or $\psi_k \geq \psi^*$
In a lag phase	L	$c_k < T_\sigma^{\min}$
New local maximum	N	$y_k > \bar{y}_{k-1}$
New global maximum	M	$y_k > \hat{y}_{k-1}^*$

Table 1. Events defining a sign change

Signal	New value	@Event	Default
$\psi_k =$	0	C or L	$\psi_{k-1} + T_s(\bar{y}_{k-1} - y_k)$
$\bar{y}_k =$	y_k	C or N	$\bar{y}_{k-1}$
$\hat{y}^* =$	$p\bar{y}_{k-1}$	C or M	$\hat{y}_{k-1}^*$
$c_k =$	0	C	$c_{k-1} + T_s$
$\sigma_k =$	$-\sigma_{k-1}$	C	σ_{k-1}

Table 2. Actions upon combinations of events

The controller implementation (21) will therefore make trajectories of $y(t)$ oscillate around the optimal value y^* that defines the current maximum of function r_y, even if this function has time-varying parameters.

5. SIMULATION RESULTS

For the model (1) there exists an optimal steady state $(\bar{u}_{\text{opt}}, \bar{y}_{\text{opt}}) = (\bar{D}_{\text{in}}^{\text{opt}}, \bar{G}_{\text{opt}})$ where the biogas production rate is maximum, assuming that S_1^{in} and S_2^{in} are kept constant. The controller aims at reaching at each instant the value y^* of the function $r_y(s)$ of model (9). In the case of the simplified anaerobic digester model, for (9) we use $s = S_3$ and $r_y(s) = k_y\rho(S_3, S_1)$; see (5)-(6). The output function's optimal value is time-varying, since it depends on S_1, which evolves according to (7) and is thus affected by the same input. If the system in closed loop were to reach a steady-state $(\bar{u}^*, \bar{y}^*)$, for example when assuming the implementation of the STC (11), nothing guarantees that $\bar{y}^* = \bar{y}_{\text{opt}}$. In fact, a further analysis of the model equations shows that $\bar{u}_{\text{opt}} < \bar{u}^*$; however, for the parameters tested they are quite close and therefore y^* results only slightly smaller than $\bar{y}_{\text{opt}}$.

We simulated the closed loop system using a modification of the nominal parameters presented by Bernard et al. (2001), which were originally fitted from data of a laboratory bioreactor. Table 3 presents these parameters, while Table 4 shows the parameters used for the PSTC. For the simulations we considered constant values for S_1^{in} and S_2^{in}

in the range used by Bernard et al. (2001). For estimating $|e_1| = |S_3 - S_3^*|$ we used

$$g(y, y^*) = 12.5 \left| \log \left(\frac{y^*}{y} \right) \right|^{\frac{1}{2}}$$

That is, the output function $f(e_1)$ is estimated by $\hat{f}(e_1) = y^* \exp\left(-0.08 e_1^2\right)$.

$\mu_1^{\max}$=1.2 d^{-1}	μ_2^*=0.536 d^{-1}	k_{11}=42.14 g g^{-1}
K_{S1}=7.1 g L^{-1}	S_2^*=30 mmol L^{-1}	k_{12}=116.5 mmol g^{-1}
a=0.8	β=0.3	k_{22}=268 mmol g^{-1}
S_1^{in}=19 g L^{-1}	S_2^{in}=50 mmol L^{-1}	k_y=453 mmol g^{-1}

Table 3. Parameters of the model

k_1=0.02	ψ^*=0.9	$T_\sigma^{\max}$=5 d	T_s=10 min
k_2=0.03	p=1.001	$T_\sigma^{\min}$=0.1 d	$u_{\max}$=1 d^{-1}

Table 4. Parameters of the PSTC

Figure 1 presents the steady state map for the parameters used (green solid curve), where the maximum ocurs at $(u_{\text{opt}}, y_{\text{opt}})$. We also build a locus of maxima (orange dashed curve) for r_y as follows: assuming that a constant $u = D_{\text{in}}$ would make S_1 reach a steady state $\bar{S}_1$ according to (7), we calculate the maximum ρ^* of $\rho(S_3, \bar{S}_1)$ and plot (D_{in}, ρ^*). Their intersection defines the steady state $(u_{\text{ss}}, y_{\text{ss}})$ that would be reached when using the STC in closed loop. The values obtained are

$$u_{\text{opt}} = 0.607 \text{ d}^{-1}, \qquad y_{\text{opt}} = 74.26 \text{mmol} CH_4 \text{Ld}^{-1}$$

$$u_{\text{ss}} = 0.628 \text{ d}^{-1}, \qquad u_{\text{ss}} = 73.86 \text{ mgmmol} CH_4 \text{d}^{-1}$$

Notice that they are very close. This is what would happen if we could indeed implement the STC (11), but we cannot. Instead, we implemented the PSTC (21), so we expect not to reach a steady state, but trajectories will be oscillating around it. Figure 2 shows the results for a simulation of 25 d. In the figure the optimal steady state reached when operating at u_{opt} is shown as a dotted line, while the calculated STC steady state is shown as a grey horizontal line.

Figure 3 shows the input and output signals, as well as those used for the estimation of the sign. It is noticeable how y_k reaches local maxima, and that $\bar{y}_k$ remains at these maxima, being reset upon a sign change. A dashed (orange) line shows the instantaneous value of y^*, which is unknown to the controller and quite time-varying, since it depends on the current value of $S_1(t)$, which is changing. Despite this, y_k follows it. The middle subfigure shows the input; the discontinuities at a sign change are noticeable. However, the input oscillates around the optimal value in steady state, where u_{ss} and u_{opt} are shown as dotted and dashed lines, respectively.

Figure 4 shows the results of a simulation where band-limited white noise was added to the output measurements, while the inflow concentrations S_1^{in} and S_2^{in} were allowed to be slowly time-varying. The PSTC is able to follow the trend of the maximal theoretically achievable biogas production rate (the orange dotted curve), despite the measurement noise and the time-varying perturbations. However, note that this $y^*(t)$ to be followed is calculated with the simplified model equations and at any given moment we may not really know which biogas rate is the true maximally achievable, since it could only be calculated for a steady state. Nevertheless the PSTC achieves its

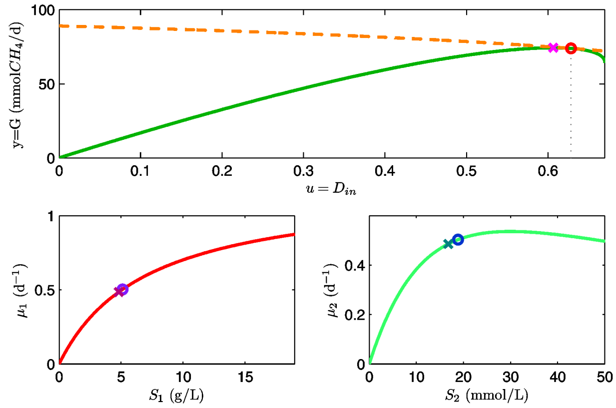

Fig. 1. Top: input/output steady state (green solid) and maximum value of r_y for fixed steady state S_1 (orange dashed), showing optimal (cross) and STC feedback (circle) steady states. Bottom: specific COD and VFA consumption rates, i.e. Monod and Haldane curves.

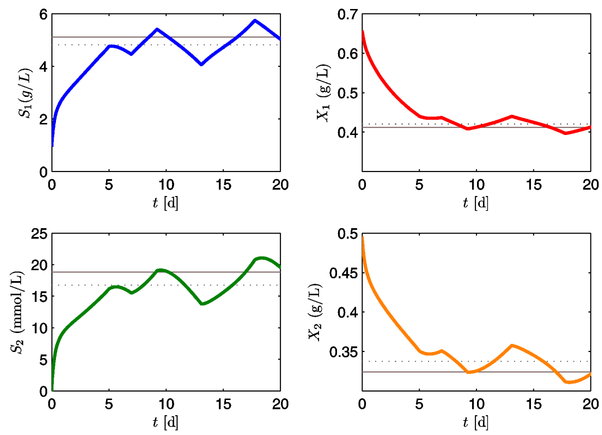

Fig. 2. Trajectories of the states in closed loop without noise.

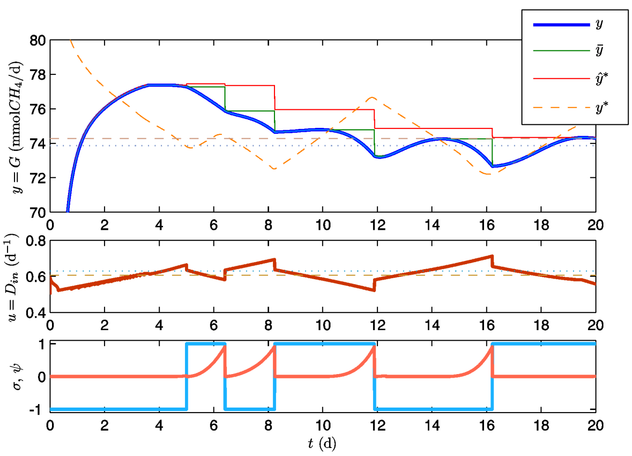

Fig. 3. Trajectories of the output and input signals without noise. Top: evolution of signals true $y(t)$ (dark blue), noisy $y(t)$ (light blue) $\bar{y}(t)$ (green), $\hat{y}^*(t)$ (red) and $y^*(t)$ (orange). Middle: input $u(t)$. Bottom: $\psi(t)$ (blue) and sign estimate $\sigma(t)$ (orange).

objective and brings trajectories close the oprating point where a maximum would occur by adjusting the dilution rate accordingly.

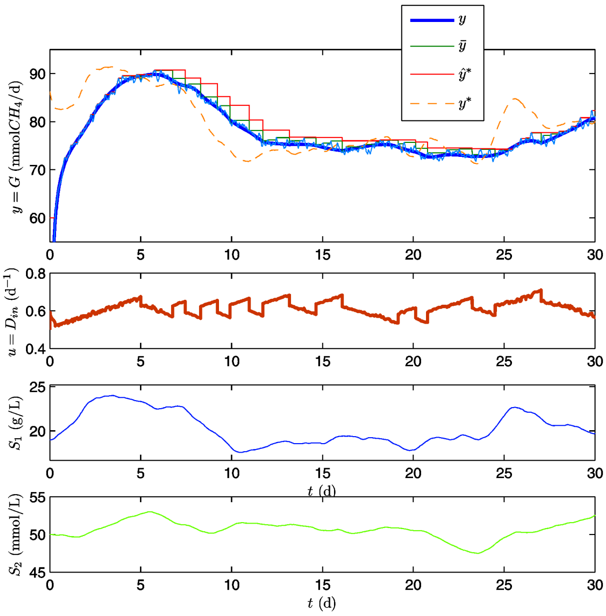

Fig. 4. Trajectories of the output and input signals with noise. Top: evolution of signals $y(t)$ (blue), $\bar{y}(t)$ (green), $\hat{y}^*(t)$ (red) and $y^*(t)$ (orange). Middle: input $u(t)$. Bottom graphs: $S_1^{in}(t)$ (blue) and $S_2^{in}(t)$ (green), respectively.

The controller proposed is thus a type of model-free extremum-seeking controller (ESC) (Wang et al., 1999; Cougnon et al., 2011), but with a different design philosophy. Whereas in the ESC we perturb the system slowly enough to infer the input-output steady state map, here the resulting oscillations in the input are naturally occurring due to the state-machine and the fact that practically we will never know the value of y^*.

6. CONCLUSIONS

The paper has proposed a simple, yet effective, output feedback controller that maximizes the output in a bioprocess model. It is based on a modification of the well-known super-twisting controller, but estimating the absolute value of the tracking error with a proposed function of the output and the sign of the error with a state-machine which additionally provides estimates of some other critical parameters. Simulation results show the applicability of such a control strategy to maximize the biogas production rate in a simplified anaerobic digester model, even under measurement noise and time-varying perturbations and parameters. These results were obtained using the AM2 bioreactor model and not using the more realistic ADM1 model, in order to keep the calculations more tractable. Nevertheless current investigations are under way to test it with simulations using this more complex model and afterwards experimentally in a laboratory bioreactor.

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