Time and Frequency Performance Assessment of IMC PI Control Loops

Henrique C. Barroso^{*} George Acioli Júnior^{*} Péricles R. Barros^{*}

* Departamento de Engenharia Elétrica, Universidade Federal de Campina Grande Campina Grande, Paraíba, Brasil (e-mail: henriquecunha@ee.ufcg.edu.br, georgeacioli@dee.ufcg.edu.br, prbarros@dee.ufcg.edu.br).

Abstract:

In this paper a methodology is presented to assess the performance of IMC PI controllers from closed-loop response data for a chosen setpoint signal. Time and frequency domains tools are used to assess the closed-loop performance. Simulation examples illustrate the methodology.

Keywords: Performance assessment, PID control, IAE, IMC, Gain margin, Phase margin.

1. INTRODUCTION

Many different approaches for control loop performance assessment have been proposed in the literature. The minimum variance control (MVC) benchmark introduced by Harris (1989) is such a tool to assess the control-loop performance. Several methodologies for the performance assessment have been also reported in a variety of control applications. In Desborough and Harris (1992) the assessment of control loop performance are considered for both feedback and feedforward control using minimum variance as the benchmark cost measure. Reviews of related work can be found in Qin (1998), Huang and Shah (1999), Harris et al. (1999), and Jelali (2006).

Recently, people have been focusing on the integrated absolute error (IAE) based indices. Huang and Jeng (2002) estimated the lower IAE bound for PI/PID control loops from step response by simulations. In Veronesi and Visioli (2009) a procedure that uses the setpoint response for IAEbased performance assessment of PID Control Loop and, if necessary, redesign the PID controller based on the SIMC tuning rules is proposed. More recently, the authors proposed a procedure that uses the setpoint pulse response for process identification and then makes use of the estimated model for IAE-based performance assessment Barroso and Barros (2012).

Another approach for control loop performance assessment is based on the loops dynamic characteristics. In this context, gain and phase margins have been used as important measures of performance and robustness in singleinput single-output (SISO) processes. It is known from classical control theory that the phase margin is related to the damping of the system and that the error-based performance indices are related to these stability margins Jeng et al. (2006). Moreover, the redesigned controller specifications may be expressed by the gain and phase margins that are classical measures of system robustness in frequency-domain.

In this paper a methodology for performance assessment of IMC PI control loops is proposed. A combined excitation signal for reference changes is proposed to experimentally evaluate time and frequency domains aspects of the PI control closed-loop performance.

This paper is organized as follows. In section 2, the problem statement is presented. Then in sections 3 and 4, the time and frequency domains tools, respectively, for PI control loop performance assessment are explained. In section 5, the proposed excitation signal for performance assessment is described. Simulations results are presented in section 6. Conclusions are drawn in section 7.

2. PROBLEM STATEMENT

2.1 The Closed-Loop

Consider the closed-loop system shown in Figure 1. The process transfer function G(s) is represented by a simple first-order plus dead time (FOPDT) model:

$$G(s) = \frac{\mu}{\tau s + 1} e^{-\theta s},\tag{1}$$

while the PI controller is: $C(s) = K_c(1 + \frac{1}{T_{is}}).$

$$y_r \rightarrow e \rightarrow C(s) \rightarrow G(s) \rightarrow G(s)$$

Fig. 1. The Closed-Loop

Assume that the PI controller (C(s)) was defined using IMC-PI design settings for setpoints presented in Rivera et al. (1986). The closed-loop transfer function from the reference signal $y_r(t)$ to the process output y(t) is

^{*} This work was supported in part by the CAPES (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior) and CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico).

$$T(s) = \frac{Y(s)}{Y_r(s)} = \frac{L(s)}{1 + L(s)} = \frac{1}{\tau_c s + 1} e^{-\theta s},$$
 (2)

where L(s) = G(s)C(s) is the Loop Gain Transfer Function.

The IMC-PI tuning parameter τ_c is related to the closedloop time constant. For more details see Rivera et al. (1986) and Skogestad (2003).

By definition, the gain margin (A_m) and phase margin (ϕ_m) of a closed-loop is

$$A_m = \frac{1}{|L(j\omega_c)|},$$

and

$$\phi_m = \pi + \angle L(j\omega_g),$$

where ω_c and ω_g , critical and crossover frequencies respectively, are obtained from $\angle L(j\omega_c) = -\pi$ and $|L(j\omega_g)| = 1$.

2.2 The Performance Assessment Problem

In order to quantify how far a PI control loop is from the IMC-PI achievable performance, it is therefore necessary to determine a suitable performance index. One possible way is to use the IAE as a benchmark which is a time-domain tool. However, it is possible to observe that, in some situations, the isolated valuation of IAE does not lead to reliable estimates to assess the performance of PI control loops.

A closed-loop step response for two different tunings of a PI controller applied to the same process is shown in Figure 2. It is possible to observe that, despite having similar values for the IAE, the two tunings have quite different dynamics characteristics. This fact indicates that the IAE alone can not be considered as a good performance index in the case of evaluation a system with an oscillatory response.



Fig. 2. Closed-Loop Step Responses

The problem statement is: Given a closed-loop system, assess experimentally the PI control loop regarding the achievable performance with IMC-PI design through time and frequency domains tools. That is, the IMC-PI design is the performance benchmark.

3. TIME-DOMAIN PERFORMANCE ASSESSMENT

The IAE for a step response for the IMC PI control loop in (2) is (see Veronesi and Visioli (2009))

Copyright © 2015 IFAC

$$IAE_{Step} = \int_{0}^{\infty} |e(t)| dt = A \left(\theta + \tau_c\right), \qquad (3)$$

where e(t) := r(t) - y(t) and A is the step amplitude.

If the excitation signal is composed as a sequence of steps occurring at time T_k then the IAE can be computed as given by the following propositions:

Proposition 1. The control error e(t) for a sequence of steps with transitions at time T_k is given by:

$$e(t) = \alpha_0 R_0 + \alpha_0 R_0 e^{-(t-T_0-\theta)/\tau_c}$$
(4)
+ $\sum_{k=1}^{N-1} \left[\alpha_k R_k + \sum_{i=0}^{k-1} \alpha_i R_i e^{-(t-T_i-\theta)/\tau_c} \right]$ + $\sum_{k=1}^{N-1} \left[\sum_{j=0}^k \alpha_j R_j e^{-(t-T_j-\theta)/\tau_c} \right],$

where N is the number of transitions from closed-loop reference and $R_0, R_1, ..., R_{n-1}$ is the value corresponding to the reference amplitude variation between two transitions. The index α is the sign of the control error in the corresponding time interval, and is given by:

$$\alpha_n = \begin{cases} 1, & R_n \ge Y_n \\ & \\ -1, & R_n < Y_n. \end{cases}$$
(5)

Proof.

For simplicity, the proof will show only the case with two transitions. Consider a pulse signal and its decomposition as shown in Figure 3. In that one can observe the existence of two time intervals located between reference signal transitions $T_0 = 0s$ and $T_1 = 4s$ and the final instant of the excitation $T_2 = 8s$ (see Figure 3).

$$e(t) = \alpha_0 R_0 + \alpha_0 R_0 e^{-(t-T_0-\theta)/\tau_c}$$
(6)
+ $\sum_{k=1}^{2-1} \left[\alpha_k R_k + \sum_{i=0}^{k-1} \alpha_i R_i e^{-(t-T_i-\theta)/\tau_c} \right]$ + $\sum_{k=1}^{2-1} \left[\sum_{j=0}^k \alpha_j R_j e^{-(t-T_j-\theta)/\tau_c} \right] .$
 $e(t) = \alpha_0 R_0 + \alpha_0 R_0 e^{-(t-T_0-\theta)/\tau_c}$ + $\alpha_1 R_1 + \alpha_0 R_0 e^{-(t-T_0-\theta)/\tau_c}$ + $\sum_{i=0}^1 \alpha_j R_j e^{-(t-T_j-\theta)/\tau_c} .$ (7)

Analyzing the sign of the error in the intervals between transitions one obtain $\alpha_0 = 1$ and $\alpha_1 = -1$, then

$$e(t) = R_0 + R_0 e^{-(t-T_0-\theta)/\tau_c}$$

$$-R_1 + R_0 e^{-(t-T_0-\theta)/\tau_c} + R_0 e^{-(t-T_0-\theta)/\tau_c}$$

$$-R_1 e^{-(t-T_1-\theta)/\tau_c},$$
(8)

which corresponds exactly to the equation error for a pulse signal shown in Barroso and Barros (2012).



Fig. 3. Pulse Signal Decomposition

Proposition 2. For a number N of transitions of the reference signal, the IAE is given by:

$$IAE = \sum_{i=0}^{N-1} \left[R_i(\theta + \tau_c) + (-1)^i R_i \tau_c e^{-(T_{Ni} - \theta)/\tau_c} \right] + \sum_{i=0}^{N-2} \left[\sum_{j=i+1}^{N-1} \left(\alpha_j 2 R_i \tau_c e^{-(T_{ji} - \theta)/\tau_c} \right) \right], \quad (9)$$

where $R_0, R_1, ..., R_{n-1}$ is the corresponding value to the variation of the setpoint amplitude between two transitions, and the index α is given by:

$$\alpha_j = \begin{cases} 1, & if \quad |i-j| \quad is \quad odd \\ -1, & if \quad |i-j| \quad is \quad even. \end{cases}$$
(10)

Proof.

For a pulse reference signal the control error equation is given in (8), and the IAE is given by:

$$IAE = IAE_{(T_0 \le t < \theta)} + IAE_{(\theta \le t < T_1)}$$

$$+ IAE_{(T_1 \le t < T_1 + \theta)} + IAE_{(T_1 + \theta \le t < T_2)},$$

$$(11)$$

and it follows that:

$$IAE = (R_0 + R_1)(\theta + \tau_c) - 2R_0\tau_c e^{-(T_1 - \theta)/\tau_c}$$
(12)
+2R_0\tau_c e^{-(T_2 - \theta)/\tau_c} - 2R_1\tau_c e^{-(T_{21} - \theta)/\tau_c}.

The same result obtained in (12) can be obtained using (9).

For a pulse reference signal one can observe two intervals between setpoint transitions:

$$e(t) = \begin{cases} e_0(t), \ T_0 \le t < T_1, \\ e_1(t), \ T_1 \le t < \infty. \end{cases}$$
(13)

and

$$IAE = \sum_{i=0}^{1} \left[R_i(\theta + \tau_c) + (-1)^i R_i \tau_c e^{-(T_{4i} - \theta)/\tau_c} \right] (14) + \sum_{i=0}^{0} \left[\sum_{j=i+1}^{1} \left(\alpha_j 2 R_i \tau_c exp^{-(T_{ji} - \theta)/\tau_c} \right) \right],$$

from which it follows that

Copyright © 2015 IFAC

$$IAE = (R_0 + R_1)(\theta + \tau_c) - 2R_0\tau_c e^{-(T_1 - \theta)/\tau_c}$$
(15)
+2R_0\tau_c e^{-(T_2 - \theta)/\tau_c} - 2R_1\tau_c e^{-(T_{21} - \theta)/\tau_c}.

4. FREQUENCY-DOMAIN PERFORMANCE ASSESSMENT

In this paper, the closed-loop is also evaluated from the gain and phase margins point of view. These margins are obtained experimentally using a relay-based experiment performed in the closed-loop system. Relay-based experiment to evaluate closed loop characteristics can be found in de Arruda and Barros (2003).

4.1 Relay-Based Gain and Phase Margins Estimation

The relay-based experiment is a combined of two welldefined relay experiments, named here as Gain Margin and Phase Margin experiments. In this section, the relay experiments are revised and the estimation of gain and phase margins with corresponding critical and crossover frequencies are shown.

Gain Margin Experiment The standard relay test presented in Aström and Hägglund (1995) is used to estimate the critical point and frequency. It can be shown (see Schei (1994)) that if this relay test is applied to a closed-loop T(s), the limit cycle occurs at the critical frequency of the L(s), i.e $L(j\omega_c) = G(j\omega_c) C(j\omega_c)$.

The estimation of critical frequency $\hat{\omega}_c$ is obtained from the frequency of the limit cycle. $G(j\hat{\omega}_c)$ is estimated computing the DFT of one period of the process input u and output y when the relay oscillation is present and steady. With the knowledge of C(s), can compute $C(j\hat{\omega}_c)$. The closed-loop gain margin is computed as

$$\hat{A}_m = \frac{1}{|L(j\hat{\omega}_c)|} = \frac{1}{|G(j\hat{\omega}_c) C(j\hat{\omega}_c)|}.$$
(16)

Phase Margin Experiment The relay feedback structure applied for crossover frequency point estimation of the loop transfer function is presented in Fig. 4.



Fig. 4. Phase Margin Experiment.

It can be shown (see de Arruda and Barros (2003) and Schei (1994)) that if this relay feedback structure is applied to a closed-loop T(s), the limit cycle occurs at the crossover frequency L(s), i.e $|L(j\hat{\omega}_q)| \approx 1$.

The setpoint $y_r(t)$ is the excitation applied to the closed loop T(s). $G(j\hat{\omega}_g)$ is estimated computing the DFT of one period of the process input u and output y when the relay oscillation is present and steady. With the knowledge of C(s), can compute $C(j\hat{\omega}_g)$. The closed-loop phase margin is computed as

$$\hat{\phi}_m = \pi + \angle L(j\hat{\omega}_g) = \pi + \angle \left(G\left(j\hat{\omega}_g\right)C\left(j\hat{\omega}_g\right)\right).$$
(17)

393

4.2 Frequency-Domain Characterization of the IMC-PI

Consider the basic definitions of the gain and phase margins, the following set of equations is obtained:

$$\angle G(j\omega_c) C(j\omega_c) = -\pi, \qquad (18)$$

$$|G(j\omega_c) C(j\omega_c)| = \frac{1}{A_m},$$
(19)

$$|G(j\omega_g) C(j\omega_g)| = 1, \qquad (20)$$

$$\angle G\left(j\omega_q\right)C\left(j\omega_q\right) + \pi = \phi_m. \tag{21}$$

Lemma 3. Using the same procedure presented in Ho et al. (2001) analytical relations between τ_c , A_m , ϕ_m and ω_g are defined:

$$\phi_m = \frac{\pi}{2} - \omega_g \theta, \qquad (22)$$

$$\omega_g = \frac{1}{\tau_c + \theta},\tag{23}$$

$$A_m = \omega_c \left(\tau_c + \theta \right), \tag{24}$$

$$0 = \frac{\pi}{2} - \omega_c \theta. \tag{25}$$

Proof. For details see Acioli Junior and Barros (2011)

Solving (25) gives a constant $\omega_c \theta = \alpha = \frac{\pi}{2} = 1.5708$. Consider $\tau_c = \beta \theta$ into (22)-(24). They can be rewritten as

$$\omega_g \theta = \frac{1}{(1+\beta)},\tag{26}$$

$$\phi_m = \frac{\pi}{2} - \frac{1}{(1+\beta)},\tag{27}$$

$$A_m = \alpha \left(1 + \beta\right). \tag{28}$$

From (27) and (28), gain and phase margins for the IMC-PI design can be related

$$\phi_m = \frac{\pi}{2} \left(1 - \frac{1}{A_m} \right). \tag{29}$$

Equation (29) gives the IMC-PI design achievable margins and Figure 5 shows the curve for the above relationship. Using (28) the parameter β can be related to A_m

$$\beta = \frac{2A_m}{\pi} - 1. \tag{30}$$



Fig. 5. Gain and Phase margins for IMC-PI design

Once β is defined for the IMC-PI design, only gain and phase margin combinations along the curve can be obtained. On the other side, defining $A_m(\phi_m)$, β and $\phi_m(A_m)$ are also defined.

Copyright © 2015 IFAC

5. PROPOSED EXCITATION SIGNAL FOR PERFORMANCE ASSESSMENT

The excitation to be used here is assumed to be generated by a combination of a pulse signal and a relay-based experiment. Through the use of this excitation signal the objective is to provide a comprehensive assessment in time and frequency domains aspects of the PI control loop performance. Such excitation is generated by combining in time a pulse signal with another signal obtained by the use of a relay-based experiment. The proposed excitation signal is shown in Figure 6.



Fig. 6. Proposed Excitation Signal

Through analysis of the pulse response during the first time interval of the excitation, from time t = 0 to $t = T_1$ (see Figure 6), is possible to extract data such as the delay and the time constant of the closed-loop. Time $t = T_1$ is the instant when the closed-loop output y reaches the value corresponds to 63% of the reference.

Thereafter the system response analysis will be directed to other aspects of interest such as stability and robustness by evaluating the phase and gain margin indices. The closedloop phase and gain margins are estimated using the Phase Margin and Gain Margin Experiment (see section 4.1) data, respectively. These experiments correspond to the second (from the time $t = T_1$ to $t = T_2$) and third (from the time $t = T_2$ to $t = T_3$) time intervals (see Figure 6), which are multiples of the oscillation periods.

Thus, the analysis of the characteristics aspects of system response to the various components of the proposed excitation signal applied enable a fairly complete performance and robustness assessment of the PI control loop.

The proposed excitation can be viewed as a sequence of steps, so that the computation of the IAE can be done with the results showed previously. This makes it possible to perform a comparison between the performance of a evaluated controller and a standard IMC response.

6. SIMULATION EXAMPLES

In this section two representative simulation examples are shown. White noise with variance 0.001 is added.

6.1 Example 1

The process is a FOPDT model given by

$$G_1(s) = \frac{1.167}{8.33s+1}e^{-0.95s},\tag{31}$$

394

and the initial PI controller is $C_1 = 0.812 \left(1 + \frac{1}{2.83s}\right)$.

The PI control closed-loop performance is evaluated using a proposed excitation signal (described in section 5) response shown in Fig. 7. A FOPDT model is estimated according to the identification technique proposed in Acioli Junior et al. (2009). In this case, it is given by

$$\hat{G}_1(s) = \frac{1.166}{8.303s+1}e^{-0.8s}.$$
 (32)



Fig. 7. Proposed Excitation Signal Response - Ex. 1

The computed IAE is obtained using (9). This value and the experimental IAE for the proposed excitation response are shown in Table 1.

Table 1. IAE - Example 1 $IAE_{computed}$ 31.42 $IAE_{experimental}$ 66.39

The closed-loop gain and phase margins are estimated according to the described in section 4.1. The estimated margins for the initial loop are shown in Table 2.

Table 2. Robustness Measures - Example 1

	$\hat{A_m}$	$\hat{w_c}(rad/s)$	$\hat{\phi_m}$	$\hat{w_g}(rad/s)$
Initial Loop	12.21	1.43	50.75°	0.18
IMC Design $\beta = 2$	4.71	1.96	70.90°	0.17

6.2 Example 2

The process is given by

$$G_2(s) = \frac{(2s+1)}{(10s+1)(0.5s+1)}e^{-s}.$$
(33)

The initial PI controller is $C_2 = 1.68 \left(1 + \frac{1}{13.53s}\right)$.

The proposed excitation signal response is shown in Fig. 8. The estimated model is given by

$$\hat{G}_2(s) = \frac{0.999}{8.694s + 1}e^{-0.6s}.$$
(34)

The computed and experimental IAE are shown in Table 3. The estimated margins are shown in Table 4.

Table 3. IAE - Example 2

$IAE_{computed}$	27.54
$IAE_{experimental}$	51.04

Copyright © 2015 IFAC



Fig. 8. Proposed Excitation Signal Response - Ex. 2

Table 4. Robustness Measures - Example 2

	$\hat{A_m}$	$\hat{w_c}(rad/s)$	$\hat{\phi_m}$	$\hat{w_g}(rad/s)$
Initial Loop	4.13	2.24	102.46°	0.13
IMC Design $\beta = 3$	6.28	2.62	75.68°	0.42

7. CONCLUSIONS

In this paper a methodology for performance assessment of IMC PI control loops was proposed. A combined excitation signal for reference changes was proposed to experimentally evaluate time and frequency domains aspects of the PI control closed-loop performance. Simulation examples illustrated the capabilities of the techniques. The further development of this work focuses on using performance assessment information that were experimentally evaluated for closed-loop PI redesign.

REFERENCES

- Acioli Junior, G. and Barros, P.R. (2011). Closed-loop evaluation and pi controller redesign satisfying classical robustness measures. 504–509. IECON 2011 - 37th Annual Conference on IEEE Industrial Electronics Society, Melbourne, Australia.
- Acioli Junior, G., Santos, J., and Barros, P.R. (2009). On simple identification techniques for first-order plus time-delay systems. 605–610. 15th IFAC Symposium on System Identification, Saint-Malo, France.
- Åström, K.J. and Hägglund, T. (1995). *PID Controllers: Theory, Design and Tuning.* Instrument Society of America, Research Triangle Park, North Carolina.
- Barroso, H.C. and Barros, P.R. (2012). Performance assessment and redesign of pi controllers with pulse excitation. IFAC Conference on Advances in PID Control, Brescia, Italy.
- de Arruda, G.H.M. and Barros, P.R. (2003). Relay based gain and phase margins PI controller design. *IEEE Transactions on Inst. and Meas.*, 1548–1553.
- Desborough, L. and Harris, T. (1992). Performance assessment measures for univariate feedback control. *The Canadian Journal of Chemical Engineering*, 70, 1186– 1197.
- Harris, T. (1989). Assessment of closed loop performance. The Canadian Journal of Chemical Engineering, 67, 856–861.
- Harris, T.J., Seppala, C.T., and Desborough, L.D. (1999). A review of performance monitoring and assessment

techniques for univariate and multivariate control systems. Journal of Process Control, 9, 1–17.

- Ho, W.K., Lee, T.H., Han, H.P., and Hong, Y. (2001). Self-tuning imc-pid control with interval gain and phase margins assignment. *IEEE Transactions on Control* Systems Technology, 9(3), 535 – 541.
- Huang, B. and Shah, S.L. (1999). Performance assessment of control loops: theory and applications. Springer Verlag, London.
- Huang, H. and Jeng, J. (2002). Monitoring and assessment of control performance for single loop systems. *Ind. Eng. Chem. Res.*, 41, 1297–1309.
- Jelali, M. (2006). An overview of control performance assessment technology and industrial applications. *Control EngineeringPractice*, 5(14), 441–466.
- Jeng, J.C., Huang, H.P., and Lin, F.Y. (2006). Modified relay feedback approach for controller tuning based on assessment of gain and phase margins. *Ind. Eng. Chem. Res*, 45(12), 4043–4051.
- Qin, S.J. (1998). Control performance monitoring a review and assessment. Computers in Chemical Engineering, 23, 173–186.
- Rivera, D.E., Morari, M., and Skogestad, S. (1986). Internal model control.4.pid controller design. *Industrial* and Engineering Chemistry Process Design and Developments, 25, 252–265.
- Schei, T. (1994). Automatic tuning of pid controller based on transfer function estimation. *Automatica*, 30(12), 1983–1989.
- Skogestad, S. (2003). Simple analytic rules for model reduction and pid controller tuning. *Journal of Process Control*, 13, 291–309.
- Veronesi, M. and Visioli, A. (2009). Performance assessment and retuning of pid controllers. *Industrial and Engineering Chemistry Research*, 48, 2616–23.