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Infinite-Dimensional Observer for Process Monitoring in Managed Pressure Drilling ⋆

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Abstract: Utilizing flow rate and pressure data in and out of the mud circulation loop provides a driller with real-time trends for the early detection of well-control problems that impact the drilling efficiency. This paper presents state estimation for infinite-dimensional systems used in the process monitoring of oil well drilling. The objective is to monitor the key process variables associated with process safety by designing a model-based nonlinear observer that directly utilizes the available information coming from the continuous-time online process output measurements at the topside of the well. The observer consists of a copy of the plant plus output injection terms where the gain is computed analytically in terms of the Bessel function of the first kind. The design is tested using data from a real-field drilling commissioning test in the North Sea by Statoil Oil Company. The results show that the design estimates the flow and pressure accurately.

Keywords: Process Monitoring, State Estimation, Distributed Parameter Systems, Nonlinear Systems, Control Applications.

1. INTRODUCTION

Advanced process monitoring and control in oil well drilling require information on the entire state variable. Unfortunately, the information of the state variables is limited by the number of sensors and the time delay in processing and measurements. In many cases, the measurements are available only at the topside and the downhole of the well. The state estimation problem is, given this limited number of measurement, to compute the best estimate of the flow and pressure at the current time.

During oil well drilling, a carefully designed drilling fluid is pumped into the drill string, through the drill bit, and up the annulus between the drill string and the sidewall of the well. The objectives are to ascertain the downhole pressure environment limits, to accordingly maintain a certain hydraulic pressure gradient along the length of the well profile, and to clean the well from the cuttings. Based on the pressure balance between the well section and the reservoir, drilling techniques can be divided into two types: First, over-balanced drilling (OBD), if the pressure at the well is intentionally set up higher than the reservoir pore pressure. Thus, the circulation fluid flows into the reservoir formation, and second, under-balanced drilling (UBD), if the pressure at the well is intentionally set up lower than the reservoir pore pressure. Thus, the reservoir fluid flows into the well annulus and up to the surface. OBD is considered to be simpler and cheaper than its UBD counterpart because it requires smaller crews and less equipment to handle the reservoir fluids. Due to the prospect of drilling increasing in complexity, a precise pressure control technology in OBD, called managed pressure drilling (MPD), is introduced.

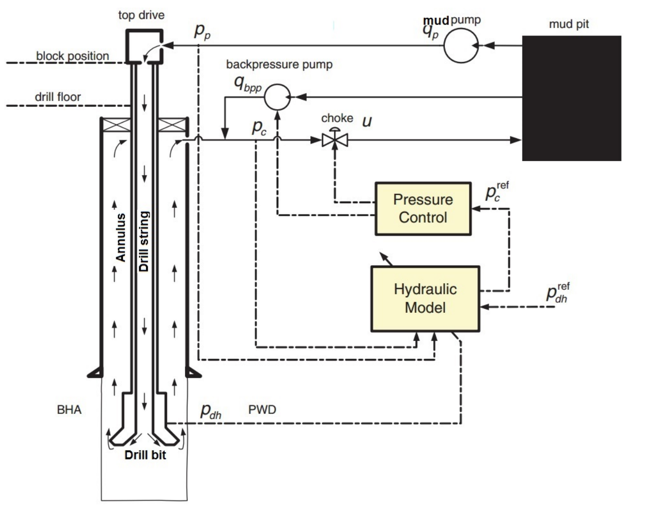

Fig. 1. Schematic of an automated MPD system (Kaasa et al., 2012).

1.1 Managed Pressure Drilling

The MPD control system usually consists of two main components (see **Fig**. 1):

- the hydraulics well model that estimates the downhole pressure, and,
- a feedback control algorithm that automates the choke manifold to maintain the desired choke pressure.

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One of the main challenges in MPD is that the number of measurements is very limited. Measurements are usually taken at two locations during drilling; a downhole pressure measurement (also known as pressure while drilling or PWD) and topside pressure and flow measurements. Although the downhole pressure measurement is very important to know, it is unreliable due to slow sampling, transmission delays, and loss of communication. The only reliable measurement is the one that located at the topside of the well. Therefore, there is an incentive to utilize the flow rate and pressure data of the drilling fluid circulation at the topside of the well, as a process monitoring tool, which can be used to provide a driller with real-time trends for the early detection of well-control problems that impact the drilling efficiency. Furthermore, the real-time process monitoring system can also be used to mitigate safety risks, minimize the formation damage, and help to achieve the drilling target faster.

In MPD, the process monitoring system is done via the hydraulic well models. In many cases, this hydraulic model is the limiting factor to achieve accuracy of the system. Therefore, many researches have been put into developing advanced hydraulics well models. An important point when developing a hydraulic well model is that the model should be able to capture the annular dynamic of the wellbore, because in general, the interest is not only to monitor the annular downhole pressure. For well-control purposes, the pressure limits should be respected in all parts of the wellbore; thus, it is preferable to use distributed models instead of lumped models.

1.2 Previous Works

Many studies have been conducted to develop advanced hydraulic well models in order to capture many aspect of the drilling hydraulics, e.g., Kaasa et al. (2012); Petersen et al. (2001); Bjorkevoll et al. (2000); Lage et al. (1999); Rommetveit and Vefring (1991). For many simulation models, unfortunately, the overall accuracy is reduced by the least accurate term. The uncertainty may come from the unmeasured reservoir outflux and the friction coefficients along the well. Therefore, calibrations are vital components of any real-time hydraulics model in order to predict the downhole pressure with high accuracy. Model-based monitoring systems for drilling are also received many attentions, e.g., Hauge et al. (2012, 2009, 2013); Landet et al. (2013); Hasan (2015). However, most of them used lumped models which only estimate the downhole pressure, whilst the interest is to know the annular pressure along the wellbore.

A possible method for calibrations of distributed parameter systems using only boundary measurements is the backstepping method (Krstic and Smyshlyaev, 2008). Originally developed for parabolic PDEs, the backstepping method has been used as control and observer designs for some exotic PDEs such as the Ginzburg-Landau equation (Aamo et al., 2005) and the Schrodinger equation (Krstic et al., 2011). The idea is to use a Volterra integral transformation to transform the original system into a target system. The stability of the target system is usually known beforehand, or at least is easier to prove. For some cases, the gain for both the controller and the observer, can be computed analytically in terms of the Bessel function (Smyshlyaev and Krstic, 2005) or the Marcum Q-function (Vazquez and Krstic, 2013). It should be mentioned that in oil and gas control problems, the backstepping method has been used for the gas conning problem (Hasan et al., 2010, 2012, 2013), slugging control (di Meglio, 2011), and drilling automation (Hasan, 2014; Hasan and Imsland, 2014).

1.3 Organization and Contributions of this Paper

In this paper, we present an infinite-dimensional observer for process monitoring in oil well drilling. We start with presenting a nonlinear hydraulic model based on mass and momentum conservation in section 2. Section 3 contains the main contribution of this paper, where we design an infinite-dimensional observer relying only on one boundary measurement. This problem, where the reliable measurement is the one that taken at the top of the well, is a typical problem in oil well drilling. The design uses a Volterra integral transformation to transform the original error system into a target system that is stable in the sense of Lyapunov. The transformation is invertible so that the stability of the target system can be translated into the stability of the original error system. Therefore, the estimates are guaranteed to converge to the actual values. Furthermore, the observer gain can be computed analytically by solving a Goursat-type first-order hyperbolic system, where the solution is written in terms of the Bessel function of the first kind. The design is tested against commissioning test data taken from a real-field in the North Sea by Statoil Oil Company. The test descriptions and the results are presented in section 4. Finally, section 5 contains conclusions and recommendations.

2. THE MUD FLOW MODEL

The mud flow model is used to estimate the annular pressure and to provide the choke pressure set point for the MPD feedback control system. In general, the model should be able to capture many aspects of the drilling mud hydraulics, but, it should not be so complex that it requires expert knowledge to set up and calibrate. Simplifications can be made to the model, for instance, by removing unnecessary dynamics, such that the model includes only the dominant dynamics of the system or by lumping together parameters that are not possible to calibrate from existing measurements.

2.1 Basic Assumptions

The main assumption for the mud flow model is to consider the drilling mud fluid as a viscous fluid, so that the flow obeys fundamental relations such as the equation of state, the mass and the momentum conservation, and the energy conservation equation. The following are assumed in our mud flow model:

- the flow can be treated as a one-dimensional flow along the flow path,
- the flow is radially homogeneous,
- the flow is incompressible,
- the mud density is constant, and
- the dependence on temperature is negligible.

The incompressible flow assumption means that only the spatial density transients in the flow are neglected. Furthermore, because the thermal liquid expansion coefficient is usually small, the density changes due to temperature changes are negligible.

2.2 Model Derivation

The derivation of the simplified mud flow model is based on White (2007) and is outlined as follows. For a single-phase and a one-dimensional flow in the annulus, the mass conservation is given by:

$$\rho_t(z,t) = -\frac{1}{A} w_z(z,t), \tag{1}$$

where ρ denotes the mud density, A is the cross-sectional area of the annulus, and w is the mass flow rate. Furthermore, t is the instantaneous time, and $z \in [0, l]$ is the spatial coordinate along the flow path beginning from the downhole $z = 0$ to the topside $z = l$. The subscripts t and z denote the partial derivatives with respect to t and z, respectively. Using the definition of the bulk modulus $\beta = \rho p_\rho$, gives:

$$p_t(z,t) = -\frac{\beta}{A} q_z(z,t), \tag{2}$$

where q is the volumetric flow rate. The bulk modulus describes the stiffness of the mud and is the reciprocal of the mud compressibility, i.e., $c = 1/\beta$.

The second relation can be obtained from the momentum balance equation as follows:

$$\begin{aligned} w_t(z,t) = &-Ap_z(z,t) - A\frac{\partial}{\partial z}\int \rho v^2 \,\mathrm{d}A - F_c(z,t) \\ &-Ag\sin\tau(z). \end{aligned} \tag{3}$$

The friction force acting on the volume, F_c, is an important component of the mud flow model in addition to the hydrostatic force. The frictional pressure drop depends on the geometry of the annulus, the frictional coefficient, the mud velocity, and the viscosity of the mud. The frictional pressure drop in the drill string is typically nonlinear with respect to the flow, while in the annulus it is typically linear, see e.g., Landet et al. (2013). In this paper, we use an empiric model for the frictional force as follows:

$$F_c(z,t) = f_1 q(z,t) + f_2 q(z,t)^2, \tag{4}$$

where f_1 and f_2 denote the frictional coefficients. The frictional coefficients are the uncertain parameters of the system. Therefore, there is an incentive to calibrate the mud flow model with respect to these parameters. A simple recursive least-squares algorithm based on the topside measurement can be implemented to obtain the value of these parameters. Assuming the integral term is sufficiently small, and the density is constant such that $w_t = \rho q_t$, the flow rate equation is given by:

$$\begin{aligned} q_t(z,t) = &-\frac{A}{\rho} p_z(z,t) - \frac{f_1}{\rho} q(z,t) - \frac{f_2}{\rho} q(z,t)^2 \\ &-Ag\sin\tau(z). \end{aligned} \tag{5}$$

The boundary conditions are given by:

$$\begin{aligned} q(0,t) &= q_d(t), \\ p(l,t) &= p_c(t), \end{aligned} \tag{6}$$

where q_d denotes the mud rate at the downhole of the well, while p_c denotes pressure at the topside (choke) of the well. It should be mentioned that in this paper, all well and fluid parameters are assumed to be known. An online estimation algorithm for estimating the drilling parameters such as density and bulk modulus can be found in Kaasa et al. (2012).

2.3 Coupled Hyperbolic Systems

For the ease of the observer design, the mud flow model (2) and (5), together with the boundary conditions (6) can be simplified into a 2×2 quasi-linear hyperbolic system. The hydrostatic head can be removed from the momentum equation by defining:

$$\bar{p}(z,t) = p(z,t) - \rho g \left(l - \int_0^z \sin\tau(s)\,\mathrm{d}s \right). \tag{7}$$

The resulting system can be diagonalized using the following Riemann's coordinate transformation:

$$\begin{aligned} \bar{w}_1(z,t) &= \frac{1}{2}\left(q(z,t) + \frac{A}{\sqrt{\beta\rho}} \bar{p}(z,t) \right), \\ \bar{w}_2(z,t) &= \frac{1}{2}\left(q(z,t) - \frac{A}{\sqrt{\beta\rho}} \bar{p}(z,t) \right). \end{aligned} \tag{8}$$

Finally, defining $w_1(x,t) = \bar{w}_1(xl,t)$ and $w_2(x,t) = \bar{w}_2(xl,t)$, (2) and (5) can be written as

$$\begin{aligned} w_{1t}(x,t) = &-\frac{1}{l}\sqrt{\frac{\beta}{\rho}} w_{1x}(x,t) - \frac{f_1}{2\rho}\left(w_1(x,t) + w_2(x,t)\right) \\ &- \frac{f_2}{2\rho}\left(w_1(x,t) + w_2(x,t)\right)^2, \\ w_{2t}(x,t) = &\frac{1}{l}\sqrt{\frac{\beta}{\rho}} w_{2x}(x,t) - \frac{f_1}{2\rho}\left(w_1(x,t) + w_2(x,t)\right) \\ &- \frac{f_2}{2\rho}\left(w_1(x,t) + w_2(x,t)\right)^2. \end{aligned} \tag{9}$$

Defining $\mathbf{w} = [w_1\ w_2]^\intercal$, the above system together with the boundary conditions, can be written in a more compact form as follow:

$$\begin{aligned} \mathbf{w}_t(x,t) &= \boldsymbol{\Sigma}\mathbf{w}_x(x,t) + \mathbf{C}\mathbf{w}(x,t) + \mathbf{f}(\mathbf{w},x), \\ w_1(0,t) &= -w_2(0,t) + q_d(t), \\ w_2(1,t) &= U(t), \end{aligned} \tag{10}$$

where

$$\begin{aligned} \boldsymbol{\Sigma} &= \mathrm{diag}[-\epsilon_1\ \epsilon_2]^\intercal = \mathrm{diag}\left[-\frac{1}{l}\sqrt{\frac{\beta}{\rho}}\ \ \frac{1}{l}\sqrt{\frac{\beta}{\rho}} \right]^\intercal, \\ \mathbf{C} &= \begin{pmatrix} 0 & c_1 \\ c_2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -f_1/2\rho \\ -f_1/2\rho & 0 \end{pmatrix}, \\ \mathbf{f}(\mathbf{w},x) &= \begin{pmatrix} -\frac{f_1}{2\rho} w_1(x,t) - \frac{f_2}{2\rho}\left(w_1(x,t) + w_2(x,t)\right)^2 \\ -\frac{f_1}{2\rho} w_2(x,t) - \frac{f_2}{2\rho}\left(w_1(x,t) + w_2(x,t)\right)^2 \end{pmatrix}. \end{aligned} \tag{11}$$

In this hyperbolic form, the flow and pressure are linear combinations of the new states w_1 and w_2. Therefore, estimating w_1 and w_2 are equal to estimating p and q. Furthermore, in the new system (10), the spatial coordinate is $x \in [0, 1]$. Thus, the downhole is located at $x = 0$, while the topside is located at $x = 1$.

3. INFINITE-DIMENSIONAL OBSERVER DESIGN

As mentioned previously, the only reliable measurement is the one that located at the topside of the well, in this case, at $x = 1$ in the $\mathbf{w}$ coordinate. Thus, we assume $w_1(1, t)$ is measured. In this section, we present an observer design for the mud flow well model (10) with $w_1(1, t)$ as the only measurement. The observer consists of a copy of the plant plus output injection terms in the domain as follow:

$$\begin{aligned} \hat{\mathbf{w}}_t &= \boldsymbol{\Sigma}\hat{\mathbf{w}}_x + \mathbf{C}\hat{\mathbf{w}} + \mathbf{f}(\hat{\mathbf{w}}, x) + \mathbf{p}(x)\tilde{w}_1(1, t), \\ \hat{w}_1(0, t) &= -\hat{w}_2(0, t) + q_d(t), \\ \hat{w}_2(1, t) &= U(t), \end{aligned} \tag{12}$$

where $\mathbf{p}(x)$ denotes the observer gain. In Vazquez et al. (2011), if $\mathbf{f} = 0$ (linear case), the observer gain $\mathbf{p}(x)$ is given by:

$$\mathbf{p}(x) = \begin{pmatrix} -\epsilon_1 P^{uu}(x, 1) \\ -\epsilon_1 P^{vu}(x, 1) \end{pmatrix}. \tag{13}$$

where the kernels P^{uu} and P^{vu} satisfy the following first-order hyperbolic PDEs:

$$\begin{aligned} \epsilon_1 P^{uu}_x(x, \xi) + \epsilon_1 P^{uu}_\xi(x, \xi) &= -c_1 P^{vu}(x, \xi), \\ \epsilon_1 P^{uv}_x(x, \xi) - \epsilon_2 P^{uv}_\xi(x, \xi) &= -c_1 P^{vv}(x, \xi), \\ \epsilon_2 P^{vu}_x(x, \xi) - \epsilon_1 P^{vu}_\xi(x, \xi) &= c_2 P^{uu}(x, \xi), \\ \epsilon_2 P^{vv}_x(x, \xi) + \epsilon_2 P^{vv}_\xi(x, \xi) &= c_2 P^{uv}(x, \xi), \end{aligned} \tag{14}$$

with boundary conditions:

$$\begin{aligned} P^{uu}(0, \xi) &= -P^{vu}(0, \xi), \\ P^{uv}(x, x) &= \frac{c_1}{\epsilon_1 + \epsilon_2}, \\ P^{vu}(x, x) &= -\frac{c_2}{\epsilon_1 + \epsilon_2}, \\ P^{vv}(0, \xi) &= -P^{uv}(0, \xi). \end{aligned} \tag{15}$$

We utilize the results in Vazquez et al. (2011) and Vazquez et al. (2012), where the backstepping method was used to design an observer for the 2×2 linear and quasi-linear hyperbolic PDEs. To this end, let us define the error function as $\tilde{\mathbf{w}} = \mathbf{w} - \hat{\mathbf{w}}$. Subtracting the plant system (10) by the observer (12), the observer error system is given by:

$$\begin{aligned} \tilde{\mathbf{w}}_t &= \boldsymbol{\Sigma}\tilde{\mathbf{w}}_x + \mathbf{C}\tilde{\mathbf{w}} + (\mathbf{f}(\tilde{\mathbf{w}} + \hat{\mathbf{w}}) - \mathbf{f}(\hat{\mathbf{w}})) \\ &\quad - \mathbf{p}(x)\tilde{w}_1(1, t), \\ \tilde{w}_1(0, t) &= -\tilde{w}_2(0, t), \\ \tilde{w}_2(1, t) &= 0. \end{aligned} \tag{16}$$

The task is to show the error system (16) converges to its equilibrium. First, we use the following Volterra integral transformations of the second kind:

$$\begin{aligned} \tilde{\mathbf{w}}(x, t) &= \tilde{\boldsymbol{\psi}}(x, t) - \int_x^1 \mathbf{P}(x, \xi)\tilde{\boldsymbol{\psi}}(\xi, t)\, \mathrm{d}\xi, \\ \hat{\mathbf{w}}(x, t) &= \hat{\boldsymbol{\psi}}(x, t) + \int_0^x \mathbf{L}(x, \xi)\hat{\boldsymbol{\psi}}(\xi, t)\, \mathbf{d}\xi, \end{aligned} \tag{17}$$

where the transformation kernels are given by:

$$\begin{aligned} \mathbf{P}(x, \xi) &= \begin{pmatrix} P^{uu}(x, \xi) & P^{uv}(x, \xi) \\ P^{vu}(x, \xi) & P^{vv}(x, \xi) \end{pmatrix}, \\ \mathbf{L}(x, \xi) &= \begin{pmatrix} L^{uu}(x, \xi) & L^{uv}(x, \xi) \\ L^{vu}(x, \xi) & L^{vv}(x, \xi) \end{pmatrix}, \end{aligned} \tag{18}$$

to transform (16) into the following system:

$$\begin{aligned} \tilde{\boldsymbol{\psi}}_t(x, t) &= \boldsymbol{\Sigma}\tilde{\boldsymbol{\psi}}_x(x, t) + \mathbf{F}[\tilde{\boldsymbol{\psi}}, \hat{\boldsymbol{\psi}}](x, t), \\ \tilde{\psi}_1(0, t) &= -\tilde{\psi}_2(0, t), \\ \tilde{\psi}_2(1, t) &= 0. \end{aligned} \tag{19}$$

It was shown using successive approximation method in Vazquez et al. (2011) that such kernels (18) are exists. Thus, the transformation is invertible, which means the properties of the target system (19) can be translated into the original system (16). The only problem is that, this target system is depend on $\hat{\boldsymbol{\psi}}$. Therefore, we assume the process is bounded, i.e., there exists $M > 0$, such that $\boldsymbol{\psi} \leq M$. Thus, the nonlinearity term $\mathbf{F}$ is depend only on the error function $\tilde{\boldsymbol{\psi}}$. Utilizing the results in Vazquez et al. (2011), the system (19) is locally exponentially stable in the $\mathbb{H}^2$-norm. Thus, the estimate $\hat{\mathbf{w}}$ converge to the actual value.

Remark from (11), $\epsilon_1 = \epsilon_2$ and $c_1 = c_2$. Let us denote these parameters as ϵ and c, respectively. The observer gain (13) can be obtained by solving the following first-order hyperbolic system:

$$\begin{aligned} \epsilon P^{uu}_x(x, \xi) + \epsilon P^{uu}_\xi(x, \xi) &= -c P^{vu}(x, \xi), \\ \epsilon P^{vu}_x(x, \xi) - \epsilon P^{vu}_\xi(x, \xi) &= c P^{uu}(x, \xi), \end{aligned} \tag{20}$$

with boundary conditions

$$\begin{aligned} P^{uu}(0, \xi) &= -P^{vu}(0, \xi), \\ P^{vu}(x, x) &= -\frac{c}{2\epsilon}. \end{aligned} \tag{21}$$

Utilizing the result in Vazquez and Krstic (2013), the solutions for (20)-(21) are given by:

$$\begin{aligned} P^{vu}(x, \xi) &= -\frac{1}{2\epsilon}\left\{ c I_0\left[\frac{|c|}{\epsilon}\sqrt{\xi^2 - x^2}\right] \right. \\ &\qquad \left. -|c|\sqrt{\frac{\xi - x}{\xi + x}} I_1\left[\frac{|c|}{\epsilon}\sqrt{\xi^2 - x^2}\right] \right\}, \\ P^{uu}(x, \xi) &= \frac{1}{2\epsilon}\left\{ c I_0\left[\frac{|c|}{\epsilon}\sqrt{\xi^2 - x^2}\right] \right. \\ &\qquad \left. -|c|\sqrt{\frac{\xi + x}{\xi - x}} I_1\left[\frac{|c|}{\epsilon}\sqrt{\xi^2 - x^2}\right] \right\}, \end{aligned} \tag{22}$$

where I_n denotes the Bessel function of the first kind. Substituting these equation into (13), we obtain explicit expressions for the observer gain $\mathbf{p}(x)$.

4. TESTING, RESULTS, AND DISCUSSIONS

The commissioning test was undertaken in the North Sea by Statoil Oil Company. One of the objectives is to provide data for the validation and development of the mud flow well models. The simple schematics well setting for this experiment is given in **Fig**. 2 below.

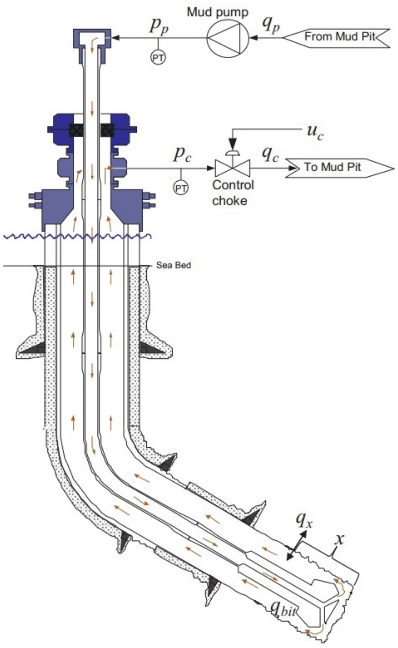

Fig. 2. A Simple schematic of oil well drilling system (Courtesy of Statoil).

In this experiment, drilling mud is pumped down from the mud pit through the drill string through the drill bit, up the annulus, and back to the mud pit. At the top of the well, a pressure sensor and a Coriolis flow meter are installed to measure the topside pressure and mud flow, respectively. These measurements are used in our observer design algorithm. Remark that, from (7) and (8), we have:

$$w_1(1,t) = \frac{1}{2}\left(q(l,t) + \frac{A}{\sqrt{\beta\rho}}p(l,t)\right), \tag{23}$$

where $q(l,t)$ and $p(l,t)$ denotes the topside flow and pressure which are measured. The boundary measurement $w_1(1,t)$ is plugged into the observer (12), where $\mathbf{p}(x)$ is computed analytically using (22), to generate the estimate of $\mathbf{w}(x,t)$. Once $\hat{\mathbf{w}}(x,t)$ is obtained, the flow and pressure can be obtained using the following formulas:

$$\begin{aligned}\hat{q}(z,t) &= \hat{w}_1\left(\frac{z}{l},t\right) + \hat{w}_2\left(\frac{z}{l},t\right),\\ \hat{p}(z,t) &= \frac{\sqrt{\beta\rho}}{A}\left(\hat{w}_1\left(\frac{z}{l},t\right) - \hat{w}_2\left(\frac{z}{l},t\right)\right)\\ &\quad + \rho g\left(l - \int_0^z \sin\tau(s)\,\mathrm{d}s\right).\end{aligned} \tag{24}$$

Remark that the solution to our state estimation problem is based on the mathematical mud flow model, which consists of PDEs and boundary conditions. Thereby, model parameters are assumed to be known. These parameters are used not only in solving the PDEs, but also in computing the gain $\mathbf{p}(x)$.

To solve the infinite-dimensional observer (12), first, using finite difference, the system is changed into a set of ODEs. In our implementation, this ODEs is solved using a Runge-Kutta method with a variable time step for efficient computation. The case considered here is the stepping of the mud pump test.

The objective of this experiment is to show that the observer is able to capture the dynamic of the system accurately. This is done by stepping up and down the flow rate of the mud pump. During this experiment, the MPD choke and the backpressure pump are isolated, i.e., no control is applied. Here are the procedures:

- Establish circulation and set choke opening
  - Ramp up the mud pump to 1 l/min
- Step down the mud pump in 0.05 l/min steps to full stop
  - Wait for a steady pressure after each step
- Start the mud pump at the minimum flow rate possible
  - Wait for a steady pressure (not described in DOP)
- Step up the mud pump in 0.05 l/min steps to 1 l/min
  - Wait for a steady pressure after each step

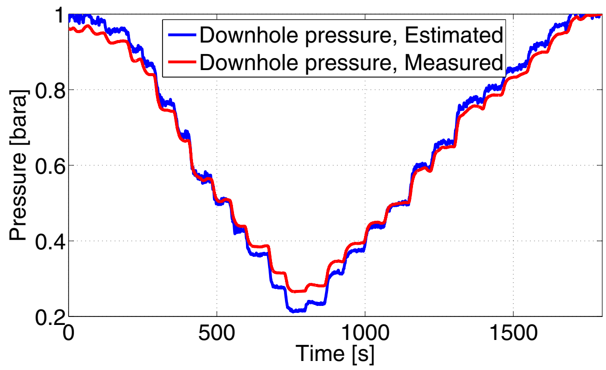

Fig. 3. The normalized downhole pressure.

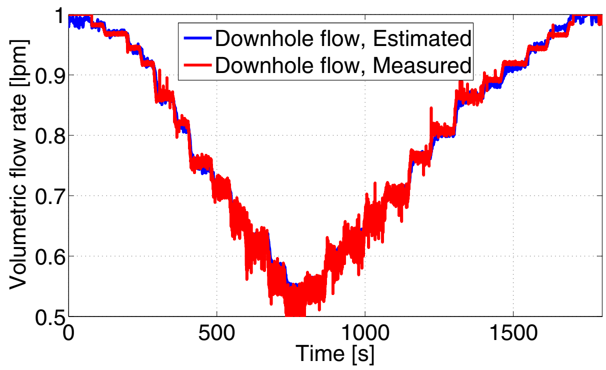

Fig. 4. The normalized downhole flow rate.

To validate our test, a Coriolis flow meter and a pressure sensor are also installed at the downhole of the well. Using our observer design (12), where the gain is given by (13), the downhole pressure estimation show in **Fig**. 3. It can be observed that the observer accurately estimates the actual value of the downhole pressure for both steady and transient periods. The deviation between $t = 600\,s$ and

$t = 700\,s$ is due to our frictional model (4). Furthermore, it can be observed from **Fig**. 4 that the topside flow estimation matches with the topside flow measurement.

5. CONCLUSIONS AND RECOMMENDATIONS

We have presented infinite-dimensional observers for process monitoring in managed pressure drilling. The design, which is based on the backstepping method, relies only on the available information that comes from continuous-time online process output measurements at the top of the well. One of the sticking features of the backstepping method is that the observer gain can be computed analytically, in this case, in terms of the Bessel function of the first kind. The performance of the observer is satisfactory when it is tested against real-field drilling data. In this experiment, the flow of the drilling mud from the drill bit is assumed to be equal to the drilling mud flow from the mud pump. However, in reality, this quantity is unknown because some of the drilling mud goes into the formation. Therefore, future work should include this unknown parameter in the calculations. The problem is becoming an adaptive observer design problem.

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