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Inclusion of Long-term Production Planning/Scheduling into Real-time Optimization

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Abstract: This work focuses on making the best possible decision at the RTO level, when it is not economically viable to have implemented a full Production scheduling and business planning optimization. It attempts to merge some of the longer-term decisions that are done in the production scheduling and inventory management into the RTO, thereby minimizing the total cost of implementations while attempting to get some of the benefits that a full production/inventory scheduling activity would bring. In the current work a decision on inventory levels is done within RTO by solving the optimization problem over a longer horizon and by augmenting the objective function for RTO with inventory cost based on historical average of marginal cost. The objective function in RTO is based on minimization of costs, and minimization of the proposed objective function leads to an overall reduction of long term marginal cost. A case study is presented in which average marginal cost is considered greater and lower than the current cost of production and shows that the long term marginal cost reduces over a period of time.

Keywords: Real-time Optimization, Scheduling and Planning, Inventory Management, Marginal Cost

1. INTRODUCTION

In industrial setting different levels of optimization problems are solved to make a decision. Five different levels of optimization are shown in Figure 1 as: PID Control, MPC, RTO, Production Scheduling and Business Planning (Darby *et al.*, 2011). However, doing all the optimizations is not economically viable and hence only some of the levels can be implemented. But to take advantage of some of the longer-term decisions made in Production Scheduling and Business Planning in the RTO level; RTO optimization can be augmented with relevant costs and solved over a longer horizon.

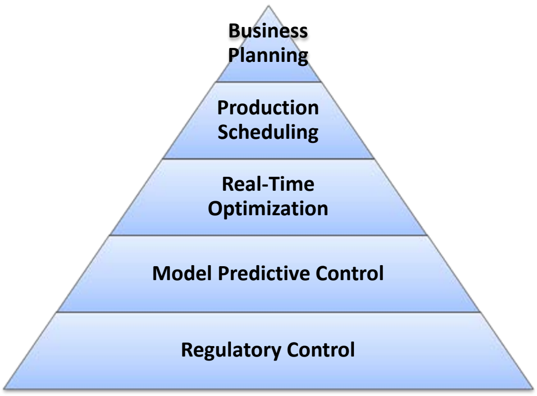

Figure 1: Different Levels of Optimization

In the current work RTO is used to make decisions on plant production rates given a system of multiple plants making same product and varying plant efficiency with respect to cost. This kind of system usually is given direction about daily/weekly production targets from the business operations. However, sometimes these targets change and it becomes impossible for the system of plants to meet them. To plan for this uncertainty it becomes important to carefully decide on inventory levels when keeping infinite inventory is not an option. The challenge in increasing/decreasing the inventory levels is compounded by the fact that cost of production is highly fluctuating depending on when the extra product is produced. The cost of production can fluctuate due to various reasons, viz. contractual terms especially caused by the cost of utilities and raw material. In this work a methodology is presented to account for changing production cost over a longer horizon, while deciding on the inventory levels. Another important factor which changes the cost of production is that the RTO is performed over a system of plants, which means each plant has its individual costs which varies depending on the efficiency. So in this work two decisions are made i) which asset to use and ii) when to produce, to capture two levels of optimization. Rawlings and Amrit, 2009 have proposed combining RTO and MPC level by using an economic objective function, similarly in the current work, decision on inventory level is made within RTO by using an economic objective function valid over a longer horizon.

Xenos *et al.*, 2015, have proposed an integrated RTO scheme for a network of compressors, whereby they decide solve load sharing for each compressor in short-term and scheduling and planning in long-term. The integration scheme solves two separate optimization problems for short and long-term.

Section 2 describes the problem in detail and develops the objective function to combine the two levels of optimization in order to decide on the inventory levels. Section 3 and 4 discusses the impact of marginal production cost on inventory and how the modified objective function results in reducing the long term marginal costs.

2. PROBLEM DESCRIPTION

2.1 System of Plants

Consider a system of plants and each plant produces same product Pr_j , represented by $P_i, i \in [1,2,\dots,\ n_{plants}]$. It is important these plants meet daily and weekly production targets set by business personal, possibly arising from a higher level of optimization. In order to meet these in optimal way, a RTO is used to decide how much to produce at each plant given their minimum and maximum capacities. The objective here can be either to maximize revenue or minimize cost. In most cases, lets consider the objective is to minimize cost, then the problem becomes as follows:

$$\min_{F_i(k)} \left(\sum_{i=1}^{n_{plants}} Plant\ Cost_i + \sum_{j=1}^{n_{penalty\_cons}} PenaltyCost_j \right) \qquad (1)$$

Subject to

$$\sum_{i=1}^{n_{plants}} F_i(k) = Production_{target}(k)\ \forall k = [1,2,3 \dots p] \qquad (2)$$

$$F_{min} \le F_i \le F_{max} \qquad (3)$$

$$|F_i(k) - F_i(k-1)| \le \Delta F_{max} \qquad (4)$$

Where, $Plant\ Cost_i$ is material and utility cost and $PenaltyCost_j$ is a term to account for some of the soft constraints, especially ramp constraints, $F_i(k)$ is production rate at each plant i bounded by $F_{min}$and F_{max} , and sum all the production $F_i(k)$ should meet the daily target $Production_{target}$, k is the time step in the prediction horizon p, which is a constant, ΔF_{max} is the maximum allowable change in production rate at every plant. Thus the daily production target is a combined set-point for the system of plants and it should be satisfied at every time step k in the prediction horizon p. Above optimization (1) can easily be changed to include discrete decision variables to account for equipment switch on/off. However in the current formulation discrete variables have been ignored. Current formulation also assumes that if a plant is on then it should atleast run at the minimum rate and that the optimizer does not have the option to turn on/off the plant. The solution of Eq. 1, is expected to run the cheaper plants first and then the more expensive plants. However due to the minimum rate constraint (Eq. 3) even the more expensive plant will be producing some part of the product. Currently the order of k varies between 15-60 minutes and p varies between 24 h to 1 week depending on the type of system, however it stays constant for a given system.

Another key aspect of evaluating the objective function is the plant models; which relates the amount of raw materials and utility is required to run the each of the plant i at $F_i(k)$. These plant models can be developed either once or updated in real-time parallel to the RTO.

2.2 RTO with inventory Decision

In Eq. *1*, the system of plants is producing at the level of daily target, however if there is a sudden change in the target which is impossible to meet even if all the plants run at maximum capacity. In that scenario, it becomes important to decide on the inventory levels. In order to decide on these levels cost related to developing the inventory needs to be included in Eq. *1*. If the inventory is decided to be increased by some level ΔV in the entire prediction horizon, p, then the cost of using the inventory can be shown as follows:

$$\begin{aligned} &Inventory\ Cost = \overline{MP} * \Delta V \\ &\Delta V = \textstyle\sum_{k=1}^{p} \sum_{i=1}^{n_{plants}} (F_i(k) - Production_{target}(k)) \\ &or\ \Delta V = V_f - V_i \end{aligned} \qquad (5)$$

where, $\overline{MP}$ is the historical average marginal cost of developing the inventory in the past n_{MC}, days, and ΔV is result of change in final and initial volume of the inventory V_f and V_i respectively. Inventory cost really represents the cost of building the inventory, however the effect of this cost is different depending on whether the inventory is being depleted or filled. Then Eq. *1* can be augmented with *5*, leading to *6*, where V_f is bounded by V_{min} and V_{max}.

$$\min_{F_i(k),\ V_f} \sum_{i=1}^{n_{plants}} Plant\ Cost_i + \sum_{j=1}^{n_{penalty_{cons}}} PenaltyCost_j - \overline{MP} * (V_f - V_i) \qquad (6)$$

Subject to $\quad V_{min} \le V_f \le V_{max}, Eq. 2-4 \qquad (7)$

In eq. 6, if V_f increases at the end of prediction horizon, then the cost of producing extra volume in the prediction horizon is already included in the $\sum Plant\ Cost_i$ and $\sum Penalty\ Cost_j$, and inventory cost represents the cost of building the inventory in past, however the inventory is being built in present. The effect of inventory cost is to compare the current marginal cost MP with historical average of marginal cost, $\overline{MP} * \Delta V$. If the optimizer decides to fill the inventory, then the current marginal cost is lower than $\overline{MP}$, and this results in reducing the objective function. Similarly if the optimizer decides to lower the inventory then it is purely done because currently it is expensive to build the inventory. This behaviour is shown in Figure 2, where point A represents when $MP < \overline{MP}$ and point C represents when $MP > \overline{MP}$. Point B represents the scenario when $MP \sim \overline{MP}$ in the prediction horizon, p.

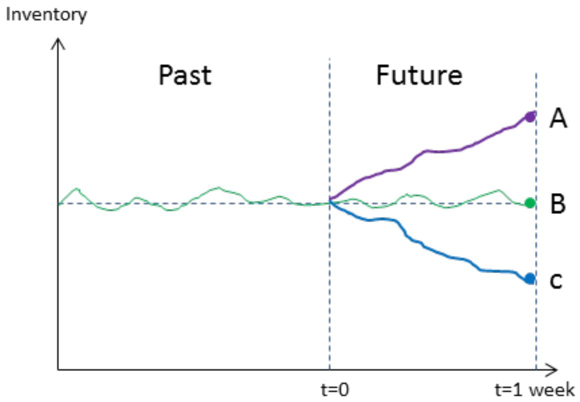

Figure 2: Effect of Marginal Price on Inventory Levels

2.3 Marginal Price Calculation

Performance of Problem 6 is very sensitive to $\overline{\mathrm{MP}}$. To determine $\overline{\mathrm{MP}}$ a naïve approach can be used, whereby the production target $Production_{target}$, is perturbed by $\Delta Pr * Production_{target}$, (where ΔPr is the % of change from $Production_{target}$), in Eq. 1 which results in optimal cost C_2. Then if C_1 represents the cost without perturbation and ΔPr is fixed as 10% or 20% of $Production_{target}(k)$, Current Marginal Cost MP can be computed as follows:

$$MP = \frac{C_2 - C_1}{\Delta Pr * Production_{target}} \tag{8}$$

$$\overline{MP} = \sum_{i=1}^{n_{MC}} MP_i \, , \Delta Pr = 10\% \; or \; 20\%$$

When MP is computed using fixed perturbation method, it is observed that this results in discontinuous first-order derivative as shown in Figure 3. Points A and B in Figure 3 are points of discontinuity for first-order derivative.

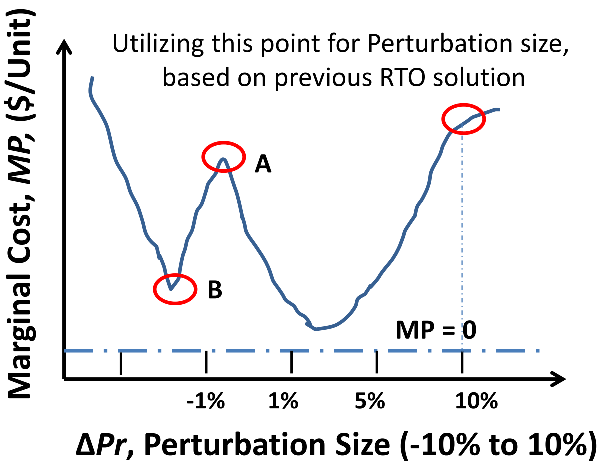

Figure 3: Marginal Cost MP vs Perturbation Size ΔPr

As a result, the naïve approach is modified to remove the discontinuities. If the demand and plant conditions do not change significantly between successive optimizer runs, then it is safe to assume that the solution at previous time step $k-1$, can be used to determine perturbation size at current time step, ΔPr_k. In this case, the solution for inventory change ΔV_{k-1} from Problem 6 is used to determine the perturbation for MP calculation, as shown in 9, where Δt is the length of prediction horizon p as ΔV is the inventory change over the prediction horizon p. This removes the problem of discontinuity as ΔPr_k allows to calculate marginal price averaged over the expected change in inventory.

$$\begin{aligned} MP &= \frac{C_2 - C_1}{\Delta Pr * Production_{target}}, \\ \Delta Pr_k * Production_{target} &= \Delta V_{k-1} \end{aligned} \tag{9}$$

Figure 4 shows the highly fluctuating behaviour of MP over time.

3. ADVANTAGES OF INCLUDING MARGINAL COST

As we discussed previously, when past averaged unit marginal cost is higher than current marginal cost, optimizer will try to produce more liquid for the next T time buckets since liquid production is cheaper compared to previous time. However, if current marginal cost is higher than averaged historical marginal cost, optimizer will try to reduce liquid production for the next T time buckets. Therefore theoretically over long term, this continuous optimization by including marginal cost will drive the averaged marginal cost decrease until stabilized at the optimal value, which means we could obtain lower unit production cost by this way.

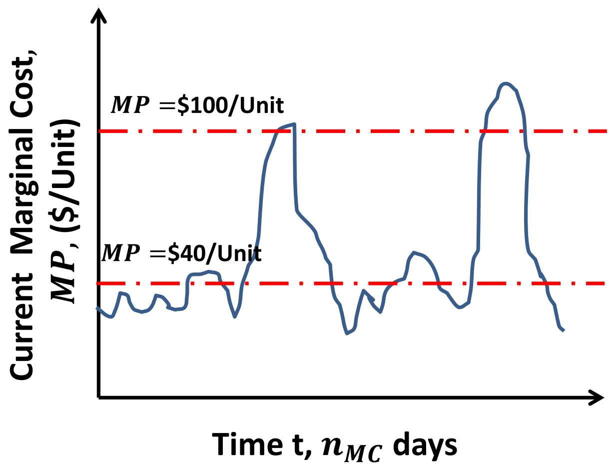

Figure 4: $\overline{MP}$ over past n_{MC}days

Take a simple case as an example to illustrate this point. A simplified objective for optimization including marginal cost will be:

$$\begin{aligned} &\min Prod * CurCost - (Prod - Dmd) * AvgCost \\ &= \min Prod * (CurCost - AvgCost) + Dmd * AvgCost \end{aligned} \tag{10}$$

Where $Prod$ is the decision variable of production amount, Dmd is customer demand, $CurCost$ is the calculated current marginal cost value, and $AvgCost$ is past averaged marginal cost. Given the range for decision variable $Prod$ is [0, 200], this monotonic function without any constraint will force decision variable either goes to its upper bound 200 or its lower bound 0. Two simple cases of this optimization are shown in Table 1 and Table 2. Case A illustrates the optimization, when initial averaged marginal cost is higher than current marginal cost, and due to optimization the averaged marginal cost drops over the Time Bucket T.

Case A: initial averaged cost is 120$/Unit, initial tank inventory is 800Unit, the target demand is 100Unit, and optimization is started when averaged marginal cost is higher than current marginal cost. With the above mentioned objective, optimal production amount will be either 200 when current production cost is lower than averaged marginal cost; or 0 when current production cost is higher than averaged marginal cost. Therefore over several time buckets, the averaged marginal cost is decreasing, which mean the unit cost of product is decreasing.

Table 1: Case A. Initial Averaged Marginal Cost > Current Marginal Cost

Time Bucket	1	2	3	4	5	6	7
Target Demand (Unit)	100	100	100	100	100	100	100
Current Cost ($/Unit)	110	105	98	99	98	120	110
Averaged Cost ($/Unit)	**119**	**118**	**116**	**114**	**113**	**113**	**112**
Optimal Production (Unit)	200	200	200	200	200	0	200
Tank Inventory (Unit)	900	1000	1100	1200	1300	1200	1300

Case B: initial averaged cost is 90$/Unit, initial tank inventory is 800Unit, the target demand is 100Unit, and we start the optimization when averaged marginal cost is lower than current marginal cost. Similarly, optimal production amount will be either 200 when current production cost is lower than averaged marginal cost; or 0 when current production cost is higher than averaged marginal cost; or pick a random number (here we use 100unit) when current marginal cost is equal to averaged marginal cost. Similar with Case A, averaged marginal cost is decreasing over several time buckets too.

Table 2: Case B. Initial Averaged Marginal Cost < Current Marginal Cost

Time Bucket	1	2	3	4	5	6	7
Target Demand (Unit)	100	100	100	100	100	100	100
Current Cost ($/Unit)	110	105	98	90	92	120	110
Averaged Cost ($/Unit)	**90**	**90**	**90**	**90**	**90**	**90**	**90**
Optimal Production (Unit)	0	0	0	100	0	0	0
Tank Inventory (Unit)	700	600	500	500	400	300	200

These two simple cases clearly illustrate that optimization with marginal cost will drive the averaged production cost decrease over long term optimization until stabilized at certain value. Although in the real condition, there may be additional constraints, such as tanks redline or a max capacity constraint, which means optimizer, cannot always select the upper bound or lower bound value. However, over long term optimization, the averaged marginal cost will decrease until stabilized at certain point.

4. OPTIMIZATION WITH MARGINAL COST FORECAST

A further extension for optimization with marginal cost is to include long term marginal cost forecasting into short term operation optimization. As shown in Figure 5, operational level optimization is in daily interval, and optimization horizon is one week. We can calculate daily instant production cost over time, shown as purple line. After certain time periods, weekly averaged marginal cost could be calculated, shown as green line. With enough historical weekly averaged data, future weekly production cost could be forecasted by ARIMA model or simple moving average (MA) approach. The forecasted future weekly production cost can

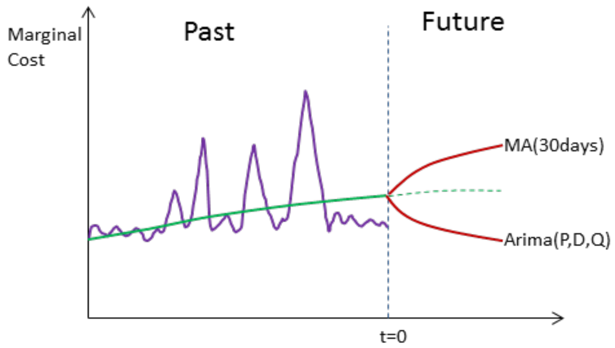

Figure 5: Marginal Cost Forecast using time series models

serve as an input parameter to daily operational optimization, which could help making decision whether daily operation should build or decrease inventory. A modified optimization objective is illustrated below, where the forecasted weekly marginal cost is added into this objective to make the right decision whether daily production should build or decrease inventory. When forecasted marginal cost of next week or even more future is higher than current production cost, optimizer will suggest building the inventory to save the long term cost, and vice versa. This type of approach has been used by Singh *et al.*, 2000 where forecasted values of the feedstock properties are used to optimize for gasoline blending operations.

$$\min Prod * CurCost - DeltaInv * NextWeekCost \qquad (11)$$

Similar with what we discuss for including the past averaged marginal cost into optimization, this could also drive the averaged production cost drop over even longer term.

5. CONCLUSIONS

In the current work it is proposed to account for some of the long-term decisions of Production Scheduling and Planning at the RTO level. RTO level objective function is augmented with costs from longer-horizon and this type of a model is applied to a RTO deciding on production rates for plants along with inventory levels (higher level optimization). The proposed objective function results in comparing the current marginal cost and historical average of marginal cost, which drives the overall long-term production cost to a lower value.

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