

Robust Optimization of Competing Biomass Supply Chains Under Feedstock Uncertainty

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Abstract: Supply chain optimization for biomass-based power plants is an important research area due to greater emphasis on renewable power energy sources. This paper develops a robust quantile-based approach for stochastic optimization under uncertainty, which builds upon scenario analysis. We apply our approach to address the problem of analyzing competing biomass supply chains subject to stochastic demand and supply.

Keywords: renewable energy systems, robust estimation, convex optimization, uncertainty, stochastic approximation

1. INTRODUCTION

Presently, fossil fuels such as oil, coal and natural gas are the prime energy sources of the world. However, it is anticipated that these sources of energy will be depleted within the next 50 to 100 years (Hughes and Rudolph, 2011; Kerr, 2012; Saidur et al., 2011). The world is consuming more fossil fuel energy than is being discovered and the reserves of energy that can be cheaply mined have reached peak production (Hughes and Rudolph, 2011; Kerr, 2012; Saidur et al., 2011). Moreover, the expected environmental damages, such as global warming, acid rain and urban smog due to the production of emissions from the combustion of fossil fuels have compelled the world to reduce carbon emissions and shift towards utilizing sustainable and renewable energy sources (Saidur et al., 2011). Biomass has been recognized as a promising alternative energy source, since it is both renewable and CO_2 neutral (Rauch and Gronalt, 2010). However, renewable energy production from biomass faces many challenges due to uncertainty of its demand and continuous supply (Ebadian, 2013; Yue et al., 2014). The purpose of this study, is to develop and apply a stochastic optimization model to analyze the impact of inter-power plant competition for the available feedstock on cost structures.

In the past, optimization based approaches for the design of biomass supply chain networks assumed that the operational characteristics, and hence the design parameters are deterministic (Alam et al., 2012; Lin et al., 2014). How-

ever, critical parameters, such as customer demand, prices, and resource capacities are usually uncertain. Sources of uncertainty in decision outcomes are typically due to three different conditions: (1) lack of knowledge, such as the quality characteristics of available biomass feedstocks; (2) noise, such as measurement errors or incomplete data; and (3) events that have not yet occurred, such as future energy demand or feedstock supply shortages (Ravindran, 2007). Uncertainty creates a range of concerns regarding the volatility of one decision versus another.

Common approaches for optimization under uncertainty assign a probability distribution to the unknown model parameters and attempt to minimize the expected value of an objective function subject to constraints, which are satisfied on average or with high probability (Lee, 2014; Shin et al., 2008; McLean and Li, 2013). Use of the expected value for selecting the best solution explicitly assumes that the decision maker is primarily interested in the average behaviour of the performance metric and is not concerned with features of its distribution, such as quantiles or variance (Batur and Choobineh, 2010).

In this paper, we propose a robust method for solving optimization problems under uncertainty based on scenario analysis. We define a robust solution as one that remains feasible and sufficiently optimal across the majority of realizable scenarios. In Section 2, we provide a brief discussion of scenario analysis and potential advantages over the more familiar mean-based stochastic optimization approach. In Section 3, we present our quantile-based scenario analysis (QSA) approach for stochastic optimization. In Section 4,

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the motivating problem of this paper is formulated as a QSA problem. Finally, results are presented in Section 5, followed by a conclusion in Section 6.

2. STOCHASTIC OPTIMIZATION

This section intends to provide the reader with a short introduction to stochastic optimization and describes two popular approaches to solving such problems. For more details, the reader may consult the work of Birge and Louveaux (2011).

2.1 Mean-based Approach

The basic stochastic programming problem is given by:

$$\begin{aligned} & \underset{\mathbf{x}}{\text{minimize}} && F_0(\mathbf{x}) = E_{\omega}(f_0(\mathbf{x}, \omega)) \\ & \text{subject to} && F_i(\mathbf{x}) = E_{\omega}(f_i(\mathbf{x}, \omega)) \leq 0, \quad i = 1, 2, \dots, m \end{aligned}$$

where the objective function, $f_0(\mathbf{x}, \omega)$, and constraint functions, $f_i(\mathbf{x}, \omega)$, depend on the optimization variables \mathbf{x} and the values of the random parameters $\omega \in \Omega$, which has a discrete or continuous joint probability distribution, denoted F_{ω} , that is either known or can be estimated from data. Thus, $f_0(\mathbf{x}, \omega)$ and $f_i(\mathbf{x}, \omega)$, $i = 1, 2, \dots, m$, are explicit functions of the optimization variables, for a given realization of the random parameters ω . The goal is to choose \mathbf{x} , such that the constraints are satisfied on average and the objective is small on average over the probability space Ω . If the $f_i(\mathbf{x}, \omega)$ are convex in \mathbf{x} for each $\omega \in \Omega$, then the F_i are convex and hence the stochastic programming problem is convex.

A limitation of this approach is that it does not provide any assurance on how well the solution will perform (in terms of minimizing the objective function and satisfying the constraints) for different realizations of the random parameters ω . The scenario analysis approach described below addresses this issue.

2.2 Scenario Analysis Approach

As described in the seminal paper by Dembo (1991), the multi-scenario stochastic optimization problem is given by:

$$\begin{aligned} & \underset{\mathbf{x}}{\text{minimize}} && J(\mathbf{x}, \omega) = \mathbf{c}_{\omega}^T \mathbf{x} \text{ (uncertain objective)} \\ & \text{subject to} && \mathbf{A}_{\omega} \mathbf{x} \leq \mathbf{b}_{\omega}, \text{ (uncertain constraints)} \\ & && \mathbf{A}_d \mathbf{x} \leq \mathbf{b}_d, \text{ (deterministic constraints)} \end{aligned}$$

To develop the multi-scenario formulation, we define a scenario as a particular realization of the uncertain data \mathbf{A}_{ω} , \mathbf{c}_{ω} , and \mathbf{b}_{ω} , represented by \mathbf{A}_{ω_i} , \mathbf{c}_{ω_i} , and \mathbf{b}_{ω_i} , where $i \in I$, and I is an index set whose members label (or index) specific members of the set Ω . Note that for each realized scenario, $\omega_i \in \Omega$, the above problem reduces to a deterministic subproblem. It is evident that the solution of a single scenario poses no difficulty. On the other hand, solving each subproblem does not provide a definite way of determining what a reasonable solution to the original stochastic problem should be.

A fundamental issue in scenario analysis is how to combine the solutions from different scenarios to form a single reasonable solution to the underlying stochastic problem. As described by Dembo (1991), a common coordination

model that combines the scenario solutions into a single feasible solution is given by:

$$\begin{aligned} & \underset{\mathbf{x}}{\text{minimize}} && \sum_{i \in I} (\|\mathbf{c}_{\omega_i}^T \mathbf{x} - \mathbf{v}_{\omega_i}\|^2) P(\omega = \omega_i) \\ & && + \sum_{i \in I} (\|\mathbf{A}_{\omega_i} \mathbf{x} - \mathbf{b}_{\omega_i}\|^2) P(\omega = \omega_i) \quad (1) \\ & \text{subject to} && \mathbf{A}_d \mathbf{x} \leq \mathbf{b}_d, \end{aligned}$$

where $\mathbf{v}_{\omega_i} = J(\mathbf{x}_i, \omega_i)$ and \mathbf{x}_i is the optimal solution of scenario ω_i . The coordinating model incorporates the random constraints into the objective function as a penalty. This model attempts to track the scenario solutions as closely as possible while still maintaining feasibility. In short, the scenario optimization approach to stochastic programming proceeds in two stages:

- Compute a solution to the deterministic problem for each scenario.
- Solve a coordinating model to find a single, feasible policy.

The problem referred to in stage (a) could be a linear, nonlinear or mixed-integer programming problem (Dembo, 1991). Alternatively, it could consist of a system of equations with stochastic coefficients or be any function dependent on stochastic parameters (Dembo, 1991). Nevertheless, for any assumed scenario, this problem is deterministic and can be solved using known algorithms (Dembo, 1991).

A limitation of the coordination model described above, which constitutes stage (b) of the scenario analysis approach, is that it assumes that both the objective function and the aggregated penalties are in the same scale and/or units of measure. Therefore, each penalty must be weighted according to their relative importance before incorporating them into the objective function. In practice, this is not straightforward, and the solutions obtained are highly sensitive to the penalty weight assigned to each constraint type. Another limitation of the coordination model described above is that it assumes that the objective function does not depend on the problem constraints being satisfied. In many situations, the computed value of the objective function may not be achievable if the problem constraints have not been satisfied. In such case, the computation of $\|\mathbf{c}_{\omega_i}^T \mathbf{x} - \mathbf{v}_{\omega_i}\|^2$ in Eq. (1) does not accurately represent the optimization problem.

Although, the coordination model attempts to find a solution that performs, on average, close to the optimal of each scenario, a more accurate and comprehensive comparison of the performance distribution associated with each candidate solution is desirable.

Example Consider a town with a demand of 5 units of water. The town receives water from a river authority at no cost. The authority has water in the amount $\omega = 1, 2$, or 3 units with probabilities 0.5, 0.4, and 0.1, respectively. The authority can purchase additional water at \$5 per unit. If the authority has excess water, it is released to down stream users at \$2 per unit. The authority must select the number of units to purchase in a manner that minimizes the cost while satisfying the demand. The natural water supply ω is uncertain and the number of

units to purchase, x , must be made beforehand. The scenario analysis approach to this problem first computes the cost associated with purchasing x units of water under each possible scenario, while satisfying the constraint that 5 units are supplied to the town. The optimal solution of each scenario are identified by an asterisk in Table 1. The coordinating model in Eq. (1) selects the solution with the lowest expected combined loss across the scenarios. For example, the expected loss of the solution $x = 2$ is $0.5(10 - 20)^2 + 0.4(10 - 15)^2 + 0.1(10 - 10)^2 + 0.5(3 - 5)^2 + 0.4(4 - 5)^2 + 0.1(5 - 5)^2 = 60$. Similarly, the expected loss for solutions, $x = 3$ and $x = 4$, are 14, and 8, respectively. The coordinating model in Eq. (1) attempts to find a solution that is most similar to the optimal solution for each scenario and equally penalizes both positive and negative deviations. For example, the candidate solution $x = 2$ is heavily penalized, because it has $J = 10$ under scenario $\omega = 1$, which is half of the achieved cost, $J = 20$, under the optimum, $x = 4$, of that scenario.

Table 1. Deterministic Scenario Solutions

ω_i	$P(\omega = \omega_i)$	x	$J(x, \omega_i)$	$(\omega_i + x - 5)$	v_{ω_i}	b_{ω_i}
1	0.5	4*	20	0	20	5
		3	15	-1	-	-
		2	10	-2	-	-
2	0.4	4	18	1	-	-
		3*	15	0	15	5
		2	10	-1	-	-
3	0.1	4	16	2	-	-
		3	13	1	-	-
		2*	10	0	10	5

3. QUANTILE-BASED SCENARIO ANALYSIS (QSA) APPROACH

Our approach examines the performance of a solution \mathbf{x}_i , for a given scenario, $\omega_i \in \Omega$, across other scenarios in Ω . Specifically, we evaluate the performance of \mathbf{x}_i at a scenario ω_j , $i, j \in \mathbf{I}$, based on two criteria: (1) its objective function value, $J(\mathbf{x}_i, \omega_j)$, and (2) a penalty function, $C(\mathbf{x}_i, \omega_j)$, expressing the aggregated impact of all unsatisfied constraints. If the constraints cannot be aggregated in a sensible manner, then separate penalty functions may be included.

The proposed quantile-based scenario analysis (QSA) approach is as follows. For each scenario solution \mathbf{x}_i , $i \in \mathbf{I}$, we define the random variable $J(\mathbf{x}_i, \omega)$ with a cumulative distribution function (CDF) denoted by $F_{\mathbf{x}_i}$, and the random variable $C(\mathbf{x}_i, \omega)$, with a CDF denoted by $G_{\mathbf{x}_i}$. The CDF's $F_{\mathbf{x}_i}$ and $G_{\mathbf{x}_i}$ convey the overall objective and constraint performance of the solution \mathbf{x}_i , respectively. In the event that more than one penalty function is needed, then each penalty function would have an associated CDF $G_{\mathbf{x}_i k}$, for $k = 1, 2, \dots, m$, where m is the number of required penalty functions, which could not be combined in a reasonable manner. We consider coordination models based on different functionals of $F_{\mathbf{x}_i}$ and $G_{\mathbf{x}_i k}$, such as:

$$\begin{aligned} & \underset{i \in \mathbf{I}}{\text{minimize}} && F_{\mathbf{x}_i}^{-1}(q_0), \quad 0 < q_0 < 1 \\ & \text{subject to} && G_{\mathbf{x}_i k}^{-1}(q_k) \leq \gamma_k, \quad 0 < q_k < 1, \end{aligned} \quad (2)$$

where q_0 and q_k are given quantiles, and γ_k , $k = 1, 2, \dots, m$ are appropriate tolerances. A quantile is a value taken

from the inverse of the CDF of a random variable and is a general way of describing a point in the distribution where a certain proportion of numbers is less than a given reference point, $0 < q < 1$. For example, the median corresponds to the 0.5 quantile. In practice, the CDF's $F_{\mathbf{x}_i}$ and $G_{\mathbf{x}_i k}$ are unknown and therefore must be estimated using Monte Carlo simulation. Assuming that we are able to sample from the distribution, F_{ω} , of scenarios, then $F_{\mathbf{x}_i}$ and $G_{\mathbf{x}_i k}$ may be estimated using the algorithm outlined in Figure 1.

The QSA method can be used to obtain solutions with distinct probabilistic characteristics without having to perform any additional optimization, once the solutions to the sampled scenario problems have been calculated in stage (a) of the scenario analysis approach.

Step 1 Generate n independent and identically distributed samples, ω_i , $i \in \mathbf{I} = \{1, 2, \dots, n\}$ from F_{ω} .

Step 2 For each sampled scenario, ω_i , solve the deterministic subproblem of interest. For example,

$$\begin{aligned} & \underset{\mathbf{x} \in \mathbf{X}}{\text{minimize}} && J(\mathbf{x}, \omega_i) = \mathbf{c}_{\omega_i}^T \mathbf{x} \\ & \text{subject to} && \mathbf{A}_{\omega_i} \mathbf{x} \leq \mathbf{b}_{\omega_i}, \\ & && \mathbf{A}_d \mathbf{x} \leq \mathbf{b}_d, \end{aligned}$$

Denote \mathbf{x}_i as the solution to the optimization subproblem for scenario ω_i .

Step 3 For each solution, \mathbf{x}_i , $i = 1, 2, \dots, n$ compute:

$$\begin{aligned} h_{ij} &= J(\mathbf{x}_i, \omega_j), \quad \text{for } j = 1, 2, \dots, n \\ c_{ijk} &= C_k(\mathbf{x}_i, \omega_j), \quad \text{for } j = 1, 2, \dots, n \end{aligned}$$

where $k = 1, 2, \dots, m$. For example, $h_{ij} = \mathbf{c}_{\omega_j}^T \mathbf{x}_i$ and $c_{ij1} = \|\mathbf{A}_{\omega_j} \mathbf{x}_i - \mathbf{b}_{\omega_j}\|^2$.

Step 4 Compute the empirical distribution functions:

$$\begin{aligned} \hat{F}_{\mathbf{x}_i}(t) &= \frac{1}{n} \sum_{j=1}^n I(h_{ij} \leq t), \quad \text{for } i = 1, 2, \dots, n \\ \hat{G}_{\mathbf{x}_i k}(t) &= \frac{1}{n} \sum_{j=1}^n I(c_{ijk} \leq t), \quad \text{for } i = 1, 2, \dots, n \end{aligned}$$

where $k = 1, 2, \dots, m$ and $I(\cdot)$ is the indicator function of a logical statement.

Fig. 1. Algorithm for Estimating $F_{\mathbf{x}_i}$ and $G_{\mathbf{x}_i k}$

4. QSA FORMULATION OF THE COMPETING BIOMASS SUPPLY CHAINS PROBLEM

To demonstrate the QSA approach for energy supply chain design and optimization under uncertainty, we discuss a supply chain network design problem for supplying forest biomass feedstock to three competing power plants within a study region. We assume that the power plant locations are given, and that they utilize only one type of feedstock, namely forest harvest residue (FHR), which includes tops and branches and unmerchantable wood left after stand harvesting. Each power plant is associated with an annual

energy demand expressed in terms of British thermal units (BTU).

The study region is 400 km × 400 km and is composed of 4 km × 4 km (16 km²) forest grid cells. Each grid cell is associated with an annual technical availability of FHR biomass, which is computed as a product of the harvesting factor and the theoretical availability of FHR at each grid cell. The harvesting factor is a value between 0 and 1, which evaluates the harvesting capability of a grid cell. The FHR biomass available at each grid cell is associated with a wet basis moisture content value. The wet basis moisture content is used to describe the water content of biomass and is defined as the percentage equivalent of the ratio of the weight of water to the total weight of the biomass. When it comes to biomass, the higher the moisture content the lower the net energy content. The energy content of a wet ton of biomass was calculated using Eq. (3), which expresses the relationship between lower heating value, l , and wet basis moisture content, m , as an average for wood in terms of BTU per ton (Roise et al., 2013).

$$l(m) = (8660 - 9712m) \times 2240 \quad (3)$$

The constants 8660 and -9712 in Eq. (3) are in units of BTU per lb and represent the estimated average higher heating value of wood (across several species), and the effect of the moisture content on the lower heating value of wood, respectively. The multiplication factor 2240 is the conversion factor from lb to long ton.

The cost of procurement of a green ton of biomass was calculated based on processing and transportation costs. The processing cost includes harvesting, grinding/chipping and piling of FHR. The transportation cost from a forest cell to a power plant was assumed proportional to the distance between them. The vehicle considered for transporting forest biomass feedstock is a tractor with a 53-foot semitrailer and a payload of 40.55 tons (t). A fixed cost of \$5.24 per green ton (gt) due to load/unload overhead, and a delay of 2.5 hours per trip was assumed. The charge-out rate for a biomass truck with operator was fixed at \$85 per hour (h). An average driving speed of 60 km/h was assumed. With this information, the cost of transporting forest biomass from each road cell of the study region to the three power plants was established.

The objective is to determine the optimal FHR catchment area of each power plant that minimizes a specific quantile, q_0 , of the distribution of the procurement (harvesting, processing, and transportation) cost, subject to the availability of forest biomass in each depleted forest cell and the energy demand of each power plant. The FHR catchment area of a power plant consists of the set of forest cells from which FHR will be procured as well as the quantity of FHR to procure from each forest cell.

The following 5 parameters were considered to have significant uncertainties or variability: biomass availability per forest cell; harvesting factor per forest cell; processing cost per forest cell; average FHR moisture content per forest cell; energy demand at each power plant. The uncertain parameters were modelled as Gaussian and are summarized in Table 2. The FHR availability at each forest cell depends on the scenario. Therefore, the solution obtained for a given scenario may not satisfy the FHR availability constraint at each forest cell when applied to a different

Table 2. Normally Distributed Input Data

Input	Mean	SD
Biomass Availability per Cell (gt)	4990	280
Moisture Content per Cell (%)	51	2.6
Demand per Power Plant (MBTU)	852,096	16,000
	1,065,120	20,000
	1,278,144	24,000
Harvesting Factor (%)	61	5
Processing Cost (\$ per gt)	26	3

scenario. In the event that a solution requests more FHR than is available at a given forest cell, then the available FHR at that forest cell is divided among the power plants in proportion to their requested amounts.

For this problem, it is not appropriate to compare solutions in terms of total procurement cost, because the demand is not fixed between scenarios. Typically a larger demand will incur a greater procurement cost. The delivered cost of FHR, assuming a transport distance of 70 km, is in the range of \$38 to \$42 per green ton (Hall et al., 2007). However, if the material is to be used as a fuel, its value lies in its energy content and not in its weight. The energy content of FHR is in the range of \$4.50 to \$4.90 per MBTU at 50% moisture content (Hall et al., 2007). In order to make valid comparisons between scenario solutions, the procurement cost should be made relative to the amount of energy produced. To this end, the total procurement cost of each solution and scenario combination is divided by the corresponding amount of energy produced and is in units of dollar per MBTU.

In accordance with the coordination model described in Eq. (2), we seek a solution that minimizes the overall procurement cost, subject to being highly likely to satisfy at least $(1 - \gamma_1) \times 100\%$ of the energy demand, where γ_1 is some small number less than one (e.g. 0.05). Precisely how likely this constraint is satisfied is determined by the choice of the quantile q_1 in Eq. (2). Therefore, we define the QSA penalty function for this problem as follows:

$$C_1(\mathbf{x}_i, \boldsymbol{\omega}_j) = \frac{1}{3} \sum_{m=1}^3 \left(\frac{d_{mi} - y_{mij}}{d_{mj}} \right)^+, \quad (4)$$

where d_{mi} is the energy demand of the m^{th} power plant in the i^{th} scenario, and y_{mij} is the energy produced by the m^{th} power plant when applying the i^{th} solution to the j^{th} scenario. Here, the exponent (+) indicates that only positive values of the parenthetical expression are considered. Equation (4) calculates the overall percentage of unfilled demand across the three power plants for a given scenario and solution.

5. RESULTS

The QSA optimization model was implemented using the R system for statistical computing (Team et al., 2012). A total of 1000 scenarios were simulated by sampling from the appropriate Gaussian distribution of each random parameter included in the model. The demand from each of the three power plants were modelled as a Multivariate Gaussian distribution with a correlation of 0.50 and standard deviations as shown in Table 2. The remaining parameters were modelled as independent Gaussian random variables as summarized in Table 2. Spatial variogram

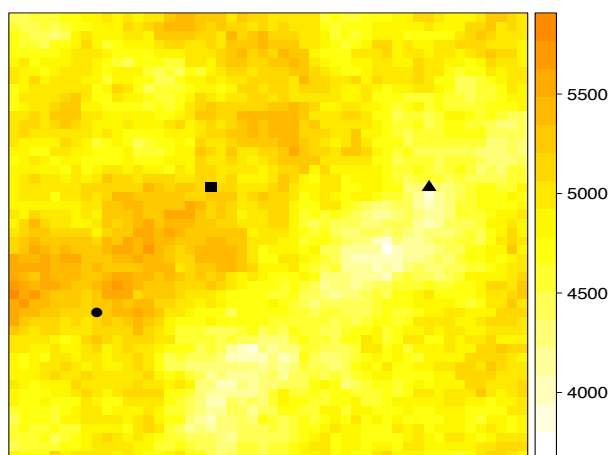


Fig. 2. FHR Availability (gt)

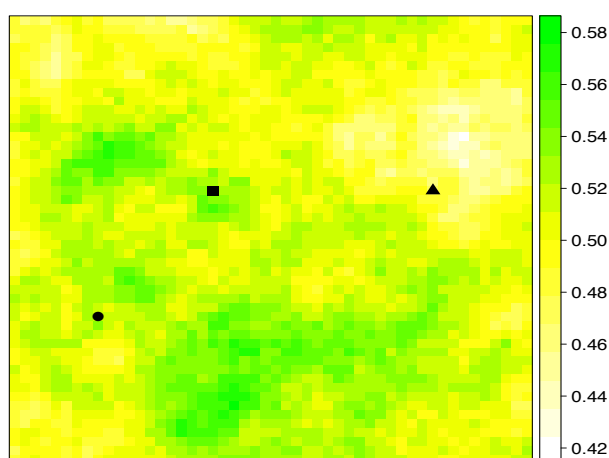


Fig. 3. FHR Moisture Content (% Wet Mass Basis)

models were fitted using the R package “sp” to represent the theoretical availability of biomass in green tons (gt) at each forest cell, as well as their average wet basis moisture content (Bivand et al., 2008). The spatial distribution of the available FHR and its corresponding average moisture content among the forest cells is shown in Figures 2 and 3, respectively. The R package “Rglpk” was used to solve the resulting deterministic linear optimization scenario subproblems (Hornik et al., 2012).

We restrict attention to solutions that have a 90% probability of satisfying at least 95% of the demand across scenarios. Among these, we are interested in the solution that minimizes the worst possible procurement cost across the scenarios 75% of the time. Therefore, we chose $q_0 = 0.75$, $q_1 = 0.90$ and $\gamma_1 = 0.05$ as the parameter values of the QSA coordination model for the competing biomass supply chains problem. In other words, we seek a solution whose empirical procurement cost distribution has the smallest 0.75 quantile and whose overall percentage of unfilled demand is less than 5.0% at the 0.90 quantile of its corresponding empirical distribution, adjusting for the FHR biomass availability at each forest cell.

The optimal catchment areas identified by the solution are illustrated in Figure 4. As may be observed from Figures 2, 3, and 4, the selected catchment areas tend to favour forest cells that have a lower than average expected moisture

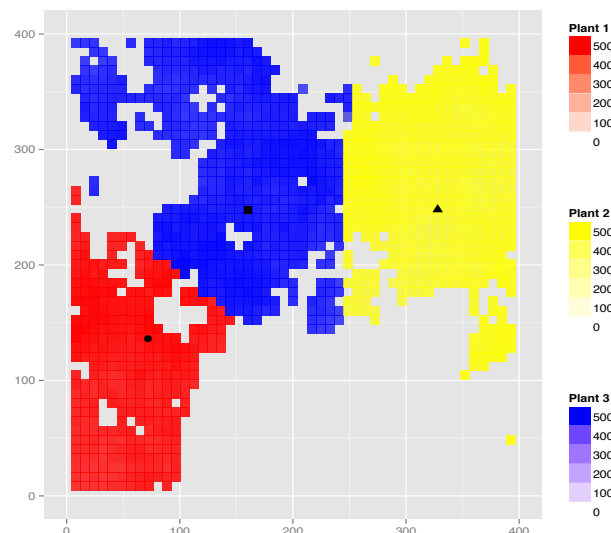


Fig. 4. Optimal Catchment Areas (gt)

Table 3. Quantile Estimates

Quantile	Estimate	95% CI
$q_0 = 0.75$	\$4.85	(\$4.81, \$4.89)
$q_1 = 0.90$	4.76%	(4.61%, 5.12%)

content, are more proximate to their assigned power plant, and have an above average expected availability of FHR. The empirical CDF of the procurement cost, \hat{F}_{x_i} , obtained by the solution, is shown in Figure 5. The x -axis of Figure 5 represents the cost of procurement in dollar per MBTU. The y -axis of Figure 5 describes the probability that the procurement cost will be found to have a value less than or equal to that of x . The 0.75 quantile of the procurement cost distribution of the solution is \$4.85 per MBTU. The empirical CDF of the penalty, \hat{G}_{x_i} , obtained by the solution, is shown in Figure 6. The x -axis of Figure 6 corresponds to the overall percentage of unfilled demand, which was calculated using Eq. (4). The y -axis of Figure 6 describes the probability that the overall percentage of unfilled demand will be less than or equal to that of x . The 0.90 quantile of \hat{G}_{x_i} is 4.76%, which satisfies the requirement that it be less than 5.0%. The horizontal red lines in Figures 5 and 6 correspond to the quantiles, q_0 and q_1 , of interest in the QSA coordination model in Eq. (2). Similarly, the vertical red line in Figure 6 corresponds to the tolerance level parameter, γ_1 , in Eq. (2). Table 3 includes approximate 95% confidence intervals for the quantiles, q_0 and q_1 , of interest (Millard, 2013). A computation time of 40 minutes was required by the QSA method to solve this problem running on a 2.4 GHz Intel Core i7 processor.

6. CONCLUSION

Current approaches for supply chain optimization under uncertainty define the best solution as the one with the largest or smallest expected performance metric, subject to satisfying a set of constraints. However, when other statistics or attributes of the objective performance metric are more appropriate for comparing solutions, the mean-based approaches are not able to appropriately accommodate the selection needs.

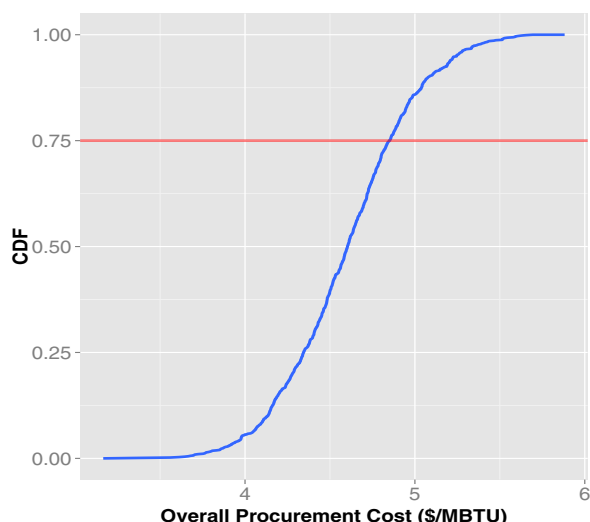


Fig. 5. \hat{F}_{x_i} of the Optimal Solution

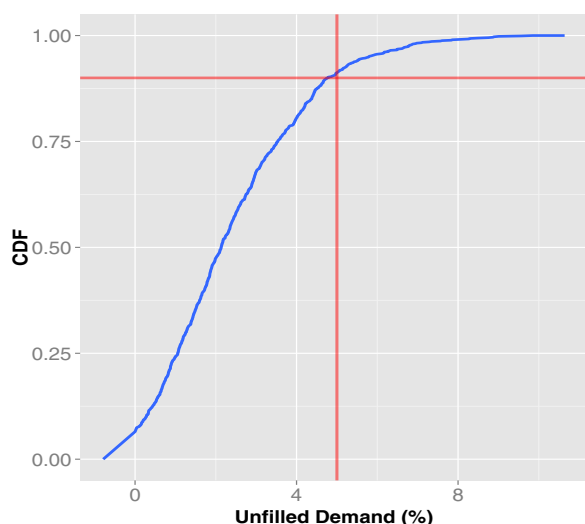


Fig. 6. \hat{G}_{x_i} of the Optimal Solution

We propose the use of quantiles as an alternative to the mean, as the basis of comparison when coordinating scenario solutions into a single feasible solution. Our approach computes the empirical distribution of the performance metric for each scenario solution. This provides a significant advantage over mean-based optimization approaches, as it enables the decision maker to efficiently identify a variety of solutions based on different quantiles of the performance metric without having to redo the optimization process. Moreover, the quantile is a robust measure of performance with respect to outliers, while the mean is not; for instance, a single large value is capable of influencing the mean whereas the median remains unchanged by an unusual high or low value.

We presented a robust quantile-based scenario analysis procedure (QSA) for optimization under uncertainty. We applied our approach to address the problem of analyzing a system of biomass supply chains, which are competing for the same feedstock, subject to stochastic demand and supply in addition to numerous constraints. In such circumstances, it is important to identify solutions that will,

with high probability, meet the energy demand and satisfy the problem constraints, for the majority of scenarios. The energy demand and feedstock supply were considered uncertain, but with known distributions. Our method was able to provide a solution that minimizes the specified 0.75 quantile of the procurement cost distribution, while satisfying at least 95% of the energy demand with a probability of 90%.

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