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Pipeline Leak Detection Using Particle Filters

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Abstract: While most of the available Leak Detection Systems (LDS) can detect pipeline leaks, leak localization is still an unresolved problem. The main reason for this problem is the limited number of sensors installed in long pipelines. Because of lack of measurements, precise leak location cannot be easily determined. This study uses particle filter as soft-sensor to estimate the states at intermediate locations with the available end-point measurements. The residuals between the estimated states and the real measurements at the intermediate locations are then used for leak detection and localization. The proposed method can improve the leak localization accuracy by proper use of the intermediate pressure measurements. Simulation results show the efficacy of the proposed method.

Keywords: Pipeline, Leak Detection, Particle Filter, State Estimation, Leak Localization.

1. INTRODUCTION

Pipelines are now commonly used to transport hydrocarbon fluids over long distances from the production site to the end-user [Chis, 2007], [Sivathanu, 2003]. The fluids are often flammable, toxic, corrosive and hazardous to the environment. Early detection and localization of leaks is therefore of utmost importance [Chis, 2007], [Sivathanu, 2003].

Existing pipeline leak detection techniques can be classified as internal monitoring based methods and external monitoring techniques [Stafford et al., 1996], [Zhang et al., 2013]. Internal methods monitor internal pipeline parameters by using already installed sensors; on the other hand, external methods work on the principle of physical detection of escaping fluid [Zhang et al., 2013]. External methods can usually detect the leak location more precisely but they are more costly and cannot be retrofitted on old pipelines in most cases. So, external methods are not used for continuous monitoring. On the other hand, internal methods can be used for continuous monitoring with less cost. But most of them cannot detect the leak location precisely. In particular, small leaks are difficult to detect. If the leak detection methods are sensitized to detect small leaks, there is always the issue that the operators are overwhelmed by too many false alarms [Al-Rafai and Barnes, 1999].

Internal methods can be broadly divided into two groups : 1) model based methods and 2) data driven methods [Zhang et al., 2013]. For the model based methods, a dynamic pipeline flow model is required [Geiger, 2006], [Geiger, 2005]. The disadvantages of model based methods are that the models are composed of nonlinear partial differential equations and hence a closed form solution is not available. Furthermore, a lot of uncertainties are involved due to changes or imprecise information on fluid properties such as (fluid density and viscosity), fluctuation in ambient and process conditions, changes in pipeline properties such as scaling, roughness and grade changes. On the other hand, data driven methods [Zhang et al., 2013] do not require any model but they rely on the statistical analysis of the steady state archived data of the pipeline system. However the disadvantages are that this method cannot detect leaks under transient conditions and a lot of a-priori information plus data is required which may not always be available. Data driven methods also fail to localize the leak location when the leak size is small (less than 5 percent of nominal flow rate).

Since data driven (statistical analysis) methods suffer from poor performance during transient conditions and are unable to detect small leaks and localize the leak a model based method (based on real time transient model) is considered for this study. To handle the uncertainties involved with the model, Monte Carlo simulation based particle filter algorithms [Arulampalam et al., 2002], [Ristic et al., 2004] are used to estimate unmeasured states and filter measured values using the available measurements. The residual errors are used for leak detection. In a previous study, a particle filter algorithm was used to detect leaks in a gas pipeline in a simulation environment [Liu et al., 2005].

This study considered a more complex pipeline (with compressible fluid transport and elevation changes) model than the available models in the literature. This study focuses on both gas and liquid transport pipeline. The efficacy of the proposed method is shown via simulation results for leak detection and localization. Fluid leak is detected by comparing the particle filter estimated states with the available intermediate pressure measurements.

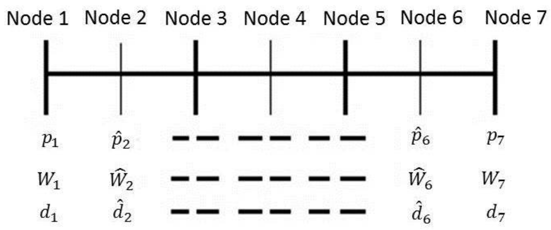

Fig. 1. Soft-Sensing approach in pipe-line leak detection.

In this paper at first a detail problem description is given in section 2 along with the solution methodology. In section 3 model selection for particle filter and modeling of a leak is described. Section 4 deals with some of the important features about the simulation methodology. Lastly, simulation results are described in section 5.

2. METHODOLOGY AND PROBLEM DESCRIPTION

Figure 1 gives a rough idea about applying the soft-sensing approach in pipeline leak detection. For simplicity, we assume that the pipeline operates in an isothermal mode of operation. So, the energy equation and temperature measurements are neglected in the current study.

In most of the cases, measurements are available at the ends of the pipeline. In the case of Figure 1, pressure, mass flow rate and density of both node 1 (p_1, W_1, d_1) and node 7 (p_7, W_7, d_7) are available. We need at least 4 of these 6 inlet-outlet measurements, of which 2 will be used as the boundary conditions for the continuity and momentum equations and 2 will be used as measurements with which the estimates of the particle filter will be compared to update the estimated states.

Our goal is to use the particle filter to estimate the pressure, mass flow rate and densities (states) of the intermediate nodes (denoted by 'hat symbol' ($\hat{}$) in Figure 1). For example, let W_1 and p_7 be the boundary conditions and p_1 and d_1 are two available measurements. The particle filter is then developed to estimate the rest of the 17 unknown states from the non-linear model of the fluid transport system.

The main advantage of the particle filter is that it can estimate the unknown states quite accurately even if the model is highly non-linear and the noise is non-Gaussian. The main difference between the model predicted method and particle filter estimated method is that, in case of particle filter, the model predicted states are updated with some of the available measurements. So, we have more confidence on particle filter estimated states rather than the model predicted states. Of course if we had some more true measurements in hand (suppose nodes 3 and 5), the accuracy of the estimated states will increase. This is often the case when some intermediate nodes are measured in real life specially pressure at the intermediate pump and valve stations. Using these data and the power of particle filter we can generate estimated data states at every 10 kilometers or even smaller intervals.

Now, if a leak occurs, both the true end data (nodes 1 and 7) and true intermediate data (nodes 3 and 5) will show some discrepancy from normal condition. By capturing these discrepancies (deviation), a leak can be detected. When leak occurs in a section, the subsequent pipeline sections (length of pipeline between two subsequent measurements) will also show the signature of this deviation. So, by comparing the available intermediate pressure measurements with the particle filter estimated pressure at those intermediate nodes, this method can isolate the section where the possible leak was generated.

To best of our knowledge, the available leak detection methods can localize leak within a range of 50 km. Often pressure measurements are available in each 20-30 km interval of a long pipeline. The proposed method can localize leak within a range of 20-30 km when real pressure measurements are available in each 20 or 30 km segment.

3. MODEL SELECTION FOR PARTICLE FILTER

Any state estimator such as a particle filter works in two steps. In the first step, it predicts the unknown states with the help of the model of the process. In the second step, it corrects the predicted states with the help of available measurements and regenerates the initial state for the next set of simulations [Arulampalam et al., 2002]. The difference between a particle filter and other classical state observer such as a Kalman filter or an extended Kalman filter is that, the update stage of the classical state observers are done with the help of update-equation and the calculation of Kalman gain is a crucial part for this step. Whereas in case of particle filter, some particles (initial conditions) are chosen from an appropriate prior probability distribution. After each simulation, weights of each particle are calculated based on the error likelihood. Then in the resampling stage, particles with negligible weights are discarded and particles with higher weight are again distributed to generate the same number of particles as the first run. These new particles will then be used as the initial state for the next simulation. In each simulation, prediction of unknown states is carried out by using the resampled (posterior) particles from the earlier step as initial states and all the predicted states are kept stored.

The corresponding estimated states are then compared with the available measurements and weights are calculated based on the likelihood of error. The final updated states after each simulation is the average of the posterior distribution of all the particles.

If the model in the prediction stage is not good, then no state estimator will perform well, no matter how many particles are chosen for the particle filter. That is why this study investigates the validity and accuracy of the model at first. In the literature, almost all the model based pipeline leak detection methods were studied on the basis of simpler version of pipeline models [Verde, 2001], [Nagel et al., 2012], [Uilhoorn, 2014]. In this work we consider a slightly complex model, to study if increasing the model complexity can improve the prediction. In this initial study we have not included the energy balance equation, instead we increased the complexity of our momentum equation by considering the liquid compressibility and elevation change term. These two terms were neglected in many of the previous literature. But we believe that these two terms can improve the prediction accuracy of our model and make our model more general both for liquid and gas. This is one of the main improvement that has been done in this study.

Since analytical solution of the coupled partial differential equations (continuity and momentum equations) system is not easily accommodated in a particle filter, these equations are discretized by the method of characteristic [Thomas, 1999]. They are solved for computing the mass flow rate and pressure at each of the nodes explicitly. The discretized model is the one that is used in this particle filter study. While discretizing the Courant-Friederich-Lewy(CFL) condition [Uilhoorn, 2014] was kept satisfied by taking $c_{son} = \frac{\Delta x}{\Delta t}$. Details of the model appear below :

$$\frac{\partial p}{\partial t} + \frac{C_{son}^2}{A}\frac{\partial W}{\partial x} = 0 \tag{1}$$

$$\frac{1}{A}\frac{\partial W}{\partial t} + \frac{\partial p}{\partial x}(1 - \frac{\nu^2}{C_{son}^2}\frac{W^2}{A^2}) + 2\nu\frac{W}{A^2}\frac{\partial W}{\partial x} + \frac{2\nu W|W|}{A^2 D}f + \frac{g}{\nu}sin\theta = 0 \tag{2}$$

The discretized equations are :

$$\begin{aligned} p_{i,j} = &\frac{1}{2}(p_{i-1,j-1}+p_{i+1,j-1})+\frac{C_{son}}{2A}(W_{i-1,j-1}-W_{i+1,j-1}) \\ &+ \Delta t\frac{C_{son}}{2}\frac{2f}{A^2D}(\nu_{i+1,j-1}W_{i+1,j-1}|W|_{i+1,j-1} \\ &- \nu_{i-1,j-1}W_{i-1,j-1}|W|_{i-1,j-1})+ \\ &\Delta t\frac{C_{son}}{2}gsin\theta(\frac{1}{\nu_{i+1,j-1}} - \frac{1}{\nu_{i-1,j-1}}) \end{aligned} \tag{3}$$

$$\begin{aligned} W_{i,j} = &\frac{1}{2}(W_{i-1,j-1} + Wi + 1, j - 1)+ \\ &\frac{A}{2C_{son}}(p_{i-1,j-1} - p_{i+1,j-1}) \\ &+ \Delta t\frac{A}{2}\frac{2f}{A^2D}(\nu_{i+1,j-1}W_{i+1,j-1}|W|_{i+1,j-1} \\ &+ \nu_{i-1,j-1}W_{i-1,j-1}|W|_{i-1,j-1})+ \\ &\Delta t\frac{A}{2}gsin\theta(\frac{1}{\nu_{i+1,j-1}} + \frac{1}{\nu_{i-1,j-1}}) \end{aligned} \tag{4}$$

where,
p = pressure (Pa)
C_{son} = velocity of sound in the pipe fluid $(\frac{m}{s})$
A = cross-sectional area of the pipe (m^2)
W = mass flow rate $(\frac{kg}{s})$
f = pipeline friction factor (Fanning friction factor)
D = pipeline diameter (m)
$\triangle t$ = sample time (s)
ν = specific volume $= \frac{1}{d}$ $(\frac{m^3}{kg})$
d = fluid density $(\frac{kg}{m^3})$
g = gravitational acceleration $(\frac{m}{s^2})$
θ = angle of elevation from the flat land $(degree)$
i = number of space node
j = number of time node

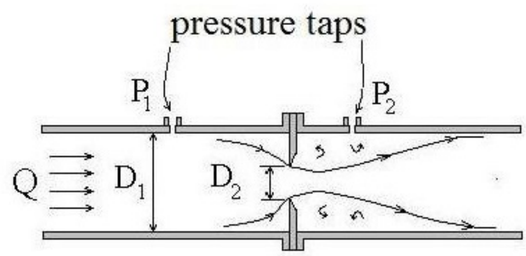

Fig. 2. Schematic diagram of an orifice meter.

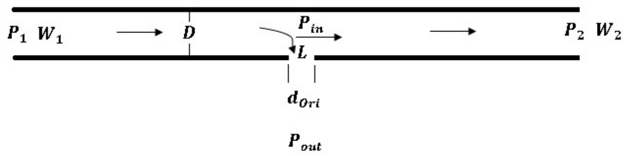

Fig. 3. Schematic diagram of a Leak in a Pipeline.

Fluid compressibility was simulated by using the following equations (Hayward [1967]) :

$$\frac{1}{\nu_{i,j}} = \frac{\frac{1}{\nu_0}\overline{K_{i,j}}}{\overline{K_{i,j}} - p_{i,j}} \tag{5}$$

where,
$\overline{K_{i,j}} = \frac{p_{i,j}V_0}{V_0 - V_{i,j}}$ = bulk modulus of the liquid,
V_0 = volume of the liquid at zero pressure (p_0) and
$V_{i,j}$ = volume of the liquid at pressure $(p_{i,j})$.

3.1 Modeling Leak

In this study, a leak is modeled by modified orifice equation. Figure 2 is an example of an orifice meter that is used to measure the flow rate in a pipeline. The orifice equation is as follows [Daugherty and Franzini, 1977] :

$$Q = C_d\frac{\pi}{4}D_2^2\sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}} \tag{6}$$

where,
D_2 = Orifice diameter
C_d = Discharge coeficient
ρ = Fluid density
β = Diameter ratio $(\frac{D_2}{D_1})$
D_1 = Pipe inside diameter
P_1 = Upstream Pressure
P_2 = Downstream Pressure
Q = Volume flow rate

Equation 6 can be modified to give the mass flow rate rather than the volumatric flow rate :

$$\dot{m} = C_d\rho\frac{\pi}{4}D_2^2\sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}} \tag{7}$$

Now from Figure 3, we can see that there are two significant differences between the orifice flow and a pipeline leak - a) in case of orifice, the direction of flow inside the orifice and the pipeline is same, whereas in case of leak they are perpendicular (Figure 3), b) for the same reason, the diameter ratio for leak will be very negligible since,in this case the whole pipe length could be several kilometer while the orifice diameter (d_{Ori} in Figure 3) could be only few millimeter to few centimeter.

So, some modifications were necessary before applying the orifice equation in case of a leak. It is also assumed that in this case, $P_{out} = P_{atm}$.

From Figure 3, we take $P_{in} \approx \frac{P_1+P_2}{2}$. Now we can define our leak rate as the following :

$$\dot{m}_L = C_L\sqrt{(P_{in} - P_{atm})} \tag{8}$$

where, C_L = Leak coefficient. Clearly, its value depends on the leak diameter which is unknown while detecting a leak. Equation 8 can be further simplified by noting that $P_{in} - P_{atm} = P_{in,gauge}$ as follows :

$$\dot{m}_L = C_L\sqrt{(P_{in,gauge})} \tag{9}$$

The discretized equation with leak can be written as follows :

$$\begin{aligned} p_{i,j} = {} & \frac{1}{2}(p_{i-1,j-1}+p_{i+1,j-1})+\frac{C_{son}}{2A}(W_{i-1,j-1}-W_{i+1,j-1}- \\ & \dot{m}_{Li,j}) + \triangle t\frac{C_{son}}{2}\frac{2f}{A^2D}(\nu_{i+1,j-1}W_{i+1,j-1}|W|_{i+1,j-1} \\ & - \nu_{i-1,j-1}W_{i-1,j-1}|W|_{i-1,j-1})+ \\ & \triangle t\frac{C_{son}}{2}gsin\theta(\frac{1}{\nu_{i+1,j-1}} - \frac{1}{\nu_{i-1,j-1}}) \end{aligned} \tag{10}$$

$$\begin{aligned} W_{i,j} = {} & \frac{1}{2}(W_{i-1,j-1} + Wi + 1, j - 1)+ \\ & \frac{A}{2C_{son}}(p_{i-1,j-1} - p_{i+1,j-1}) \\ & + \triangle t\frac{A}{2}\frac{2f}{A^2D}(\nu_{i+1,j-1}W_{i+1,j-1}|W|_{i+1,j-1} \\ & + \nu_{i-1,j-1}W_{i-1,j-1}|W|_{i-1,j-1})+ \\ & \triangle t\frac{A}{2}gsin\theta(\frac{1}{\nu_{i+1,j-1}} + \frac{1}{\nu_{i-1,j-1}}) - \dot{m}_{Li,j} \end{aligned} \tag{11}$$

4. SIMULATION PREPARATION

In this study, the simulated pipeline is of 37 km in length with 16 inch inlet diameter. For the simulation purpose, the pipeline was divided into 19 equal divisions (20 nodes). Figure 4 is a schematic of the nodes. We have total 60 variables which include pressure, mass flow rate and density at each of these nodes.

Here, the boundary conditions were mass flow rate at node 1 ($W_{(1,j)}$) and pressure at node 20 ($p_{(20,j)}$). We also had two measurements as pressure at node 1 ($p_{(1,j)}$) and density at node 1 ($d_{(1,j)}$). It is also assumed that 3 other pressure measurements are available at nodes 5,10 and 15.

There were two significant mismatches introduced between the simulated pipe system and prediction model of particle filter (PF) estimation. Unequal initial condition was the first mismatch. Process parameter uncertainty was the second mismatch. Process parameter uncertainties were simulated by adding Gaussian white noise with process parameters.

In this study, the pipeline was simulated with the combination of equations 3, 4 and 5. Kerosene was chosen as the working fluid for these simulations. So, properties of kerosene were used in equations 3 and 4.

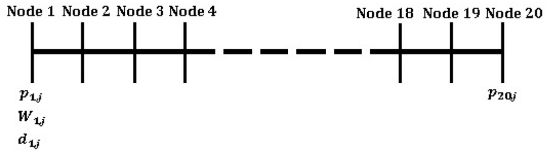

Fig. 4. Schematic diagram of the process to be used in the simulation.

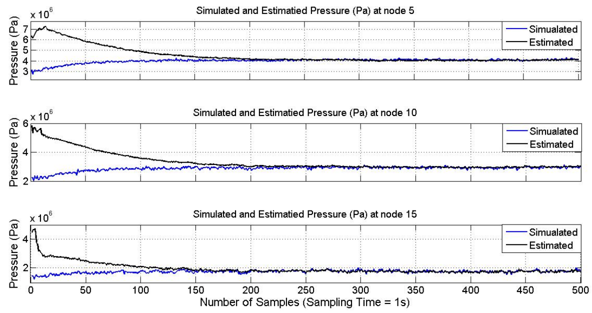

Fig. 5. Simulated and Particle Filter estimated pressures (Pa) for nodes 5, 10 and 15.

Although in the real case, the mass flow rate is not constant under leak free conditions, for simplicity we assume it to be as constant with some noise. Similar simplifications were made for pressure and density. We also avoided unpredictable or sudden changes in our simulated model.

In this study it is found that the friction factor (f) is the most crucial tuning parameter for the simulated model to converge with the true pressure profile. Relatively large roughness is assumed to consider the old pipelines commonly seen in reality.

The next step was to add noise to the simulated data, which will be used as measurement and boundary value for the particle filter.

The particle filter was used to estimate the unknown states. For brevity, in this study we have included the result for nodse 5, 10 and 15 to check the efficiency of particle filter. Pressure, mass flow rate and densities of these two nodes were investigated. In all these simulations, total 500 samples were simulated and the sampling frequency is 1s. So the total simulation time was 500s.

5. RESULTS

5.1 Validating Model

The simulation results are shown in Figures 5 to Figure 7. One crucial choice in the particle filter is to select the optimal number of particles. Generally the more the number of particles used, the better the accuracy of estimation but at the cost of more computational effort [Modisette et al., 2013]. All the simulations of this study were done by taking 50 particles.

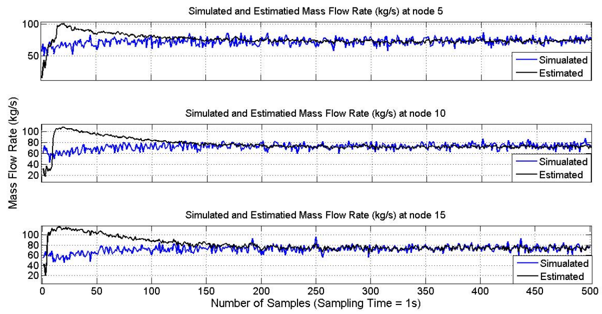

Fig. 6. Simulated and Particle Filter estimated mass flow rates ($\frac{kg}{s}$) for nodes 5, 10 and 15.

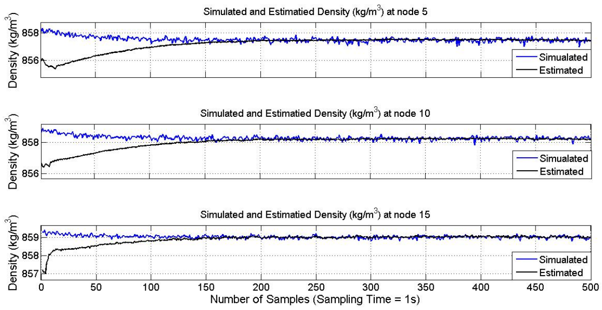

Fig. 7. Simulated and Particle Filter estimated densities ($\frac{kg}{m^3}$) for nodes 5, 10 and 15.

Figure 5 compares the estimated pressure with the simulated pressure at nodes 5, 10 and 15. It should be noted from Figure 5 that though there was a huge difference in the initial states between the simulation model and the estimation model, the particle filter was able to track the original (simulated) pressure asymptotically. It should also be noted that due to model uncertainty or noise in the original (simulated states) states, exact estimation of the states is not possible.

Figure 6 shows the comparison of simulated and estimated mass flow rates at nodes 5, 10 and 15. It is seen that though there were differences in initial conditions; the particle filter was able to track the true mass profile.

Figure 7 shows the comparison of simulated and estimated densities at nodes 5, 10 and 15. It is again clear that though there were differences in initial conditions; the particle filter was able to track the true(simulated) density profile.

By summarizing the above results, it can be said that with a good model, the particle filter can capture the dynamics of the process quite efficiently.

5.2 Leak Detection and Localization

In this section, a leak was introduced in the simulated pipeline system. For simplicity, the value of $\dot{m}_L$ was kept as 7 kg/s, which is approximately 10 percent of the nominal mass flow rate. The idea of leak detection and localization is based on the deviation method. A leak was not introduced in the particle filter model. So, the particle filter model is able to first generate the behavior of the pipeline under normal operating conditions. Whereas, due to leak the behavior of the simulated pipeline system will change.

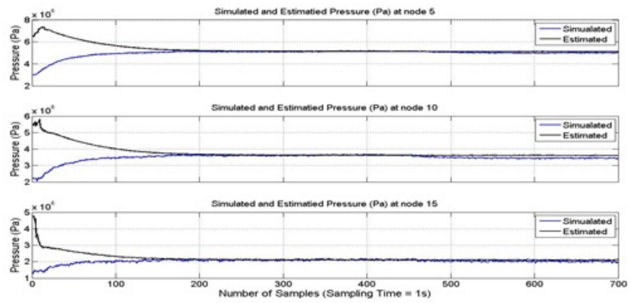

Fig. 8. Simulated (with leak at node 7) and Particle Filter estimated pressures (Pa) for nodes 5, 10 and 15.

The unused pressure measurement of nodes 5, 10 and 15 are used to detect and localize the leak. There are two test cases : a) a leak was introduced at node 7 at 450 sample time (450 seconds) and b) a leak was introduced at node 13 at 450 sample time (450 seconds). The deviations were determined by an index called mean absolute deviation (MAD). MAD is the mean of absolute deviations between the real measurements and particle filter estimated normal condition. When there is no leak, the deviation between the estimated value and the real measurement is very low. This deviation is larger around the leak point. So, the leak point is in between two nodes with larger deviation values. The unit of MAD index is Pa since we are comparing real and estimated pressure measurements. Leak rate can be determined by comparing the mass flow rates of the end node. Generally mass flow rate is available at the end node of the pipeline.

Figures 8 to 11 show the results for these test cases. From Figure 8, it is clear that since the leak is introduced at node 7, there is no change in the pressure at node 5. But there are changes in pressure of nodes 10 and 15. Figure 9 is the zoomed version of Figure 8. This also confirms the persistent deviation (i.e., the difference between estimation and real measurement) at node 10 and node 15. So, from this observation, we can say that leak has occured at $t = 450$ seconds and in between node 5 and 10. Although, exact leak location (node 7) is not yet possible with this method, this method can at least isolate the section of the long pipeline which is suffering from a leak. Generally, true measurements are available in each 20-30 km segment. So we need to look for leak in the section which is suspected to have a leak. This is a clear improvement over the existing leak detection methods which can localize leaks with an accuracy of 50 km.

Similar results have been found from Figures 10 and its zoomed version (Figure 11). Here, there is no deviation in measured (simulated) and estimated pressure at nodes 5 and 10, the deviation has occured at node 15. So, it can be inferred, that leak has occured between node 10 and 15.

6. CONCLUSION

This paper studies the application of particle filter for pipeline leak detection. Simulation results show that the

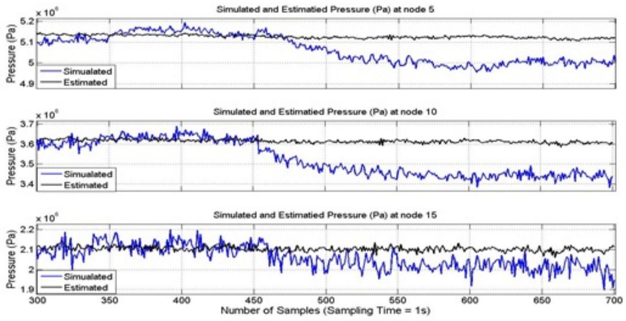

Fig. 9. Zoomed in version of Figure 8.

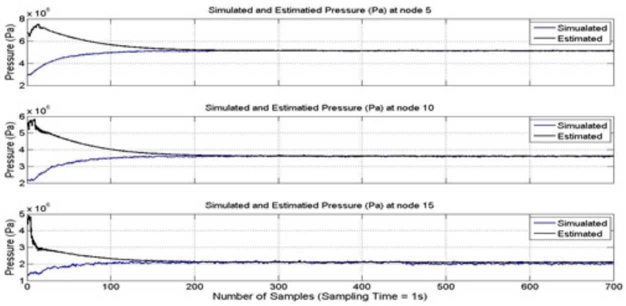

Fig. 10. Simulated (with leak at node 13) and Particle Filter estimated pressures (Pa) for nodes 5, 10 and 15.

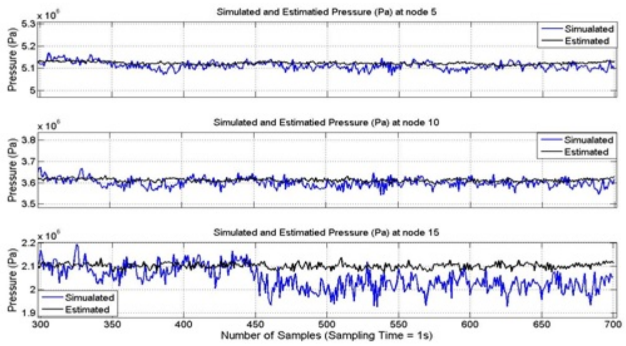

Fig. 11. Zoomed in version of Figure 10.

particle filter works very well to capture the dynamics of the real system and it can serve the purpose of soft-sensor.

The proposed leak detection system can detect leak efficiently. Although leak localization is not yet exact, it can at least isolate the smallest possible section of the pipeline which is suspected to have a leak. Work is going on to improve the leak location accuracy and to make this method suitable even for smaller leaks as low as 1 percent of the nominal mass flow rate.

Our ongoing efforts are underway to evaluate this method on data without and with leaks from a real pipeline system. More complex model as well as the energy conservation equation will be also employed for the purpose of better estimation of the dynamics of the real pipeline, which is always challenging.

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REFERENCES

Al-Rafai, W. and Barnes, R. (1999). Underlying the performance of real-time software-based pipeline leak-detection systems. *Pipes and Pipelines International*, 44(6), 44–51.

Arulampalam, M.S., Maskell, S., Gordon, N., and Clapp, T. (2002). A tutorial on particle filters for online nonlinear/non-gaussian bayesian tracking. *Signal Processing, IEEE Transactions on*, 50(2), 174–188.

Chis, T. (2007). Pipeline leak detection techniques. *Annals. Computer Science Series, 5th Tome, 1st Fasc.-2007.*

Daugherty, R.L. and Franzini, J.B. (1977). *Fluid mechanics, with engineering applications.* McGraw-Hill NY.

Geiger, G. (2006). State-of-the-art in leak detection and localization. *Oil Gas European Magazine*, 32(4), 193.

Geiger, I.G. (2005). Principles of leak detection. *Fundamentals of leak detection. KROHNE oil and gas.*

Hayward, A.T.J. (1967). Compressibility equations for liquids: a comparative study. *British Journal of Applied Physics*, 18(7), 965.

Liu, M., Zang, S., and Zhou, D. (2005). Fast leak detection and location of gas pipelines based on an adaptive particle filter. *International Journal of Applied Mathematics and Computer Science*, 15(4), 541.

Modisette, J.P. et al. (2013). State estimation of pipeline models using the ensemble kalman filter. In *PSIG Annual Meeting.* Pipeline Simulation Interest Group.

Nagel, T., Zhang, C., and Liu, S. (2012). Kalman filter based leak localization applied to pneumatic systems. In *Control Automation Robotics & Vision (ICARCV), 2012 12th International Conference on*, 1777–1782. IEEE.

Ristic, B., Arulampalam, S., and Gordon, N.J. (2004). *Beyond the Kalman filter: Particle filters for tracking applications.* Artech House.

Sivathanu, Y. (2003). Technology status report on natural gas leak detection in pipelines. *Prepared for US Department of Energy, National Energy Technology Laboratory by Yudaya Sivathanu EnUrga Inc.*

Stafford, M., Williams, N., and Britain, G. (1996). *Pipeline leak detection study.* HSE Books.

Thomas, P.J. (1999). *Simulation of industrial processes for control engineers.* Butterworth-Heinemann.

Uilhoorn, F.E. (2014). State-space estimation with a bayesian filter in a coupled pde system for transient gas flows. *Applied Mathematical Modelling.*

Verde, C. (2001). Multi-leak detection and isolation in fluid pipelines. *Control Engineering Practice*, 9(6), 673–682.

Zhang, J., Hoffman, A., Murphy, K., Lewis, J., and Twomey, M. (2013). Review of pipeline leak detection technologies. In *Pipeline Simulation Interest Group Annual Meeting, Prague, Czech Republic, 16 19 April, 2013.* Pipeline Simulation Interest Group.

