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Dynamic Operational Optimization of Air Source Heat Pump Heating System with the Consideration of Energy Saving

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Abstract: It is quite important to improve the energy saving of air source heat pump heating system through optimal operation and system engineering method. In this work, the nonlinear dynamic process of the heat pump heating process was established and the integral optimal objective function was then formulated to balance the control accuracy and energy saving. To solve the differential and algebraic optimization problems (DAOPs) efficiently, simultaneous method was used to discretize the problem by collocation of finite elements. Computing results show that potential energy saving of more than 16% can be achieved under permitted accuracy error of zone temperature control, and the ambient air temperature and electricity price have significant effect on the optimal operation and performance of the heat pump heating system. Base on the computing results and analysis of operational parameters, a strategy to dynamically adjust the weight of objective function to balance the control accuracy and energy cost was proposed for the propose of further energy saving, computing results show that: if we dynamically adjusted the weight of objective function with this method, about 4.7% of further energy cost saving can be obtained. Our research is of significant meaning for the optimal operation of heat pump heating system under dynamical conditions.

Keywords: dynamic; operational optimization; heat pump; energy saving;

1. INTRODUCTION

With the emerging of energy squeeze and environment pollution, heat pump techniques that are characterized as multiple advantages of high efficiency energy-saving and non-pollution provide the way out for building heating. For the reason of the outstanding performance of it and the continuous improvement of the control techniques, air source heat pump is rapidly accepted by more and more users. The application of air source heat pump is absolutely adaptive in a large area of China [Ju et al. (1996)]. The climate of the South of Yangtze River in China shows to be hot in summer and cold in winter, where the number of days that daily average temperature is blow 5℃ is about 45 to 60 [Wang and Tan (2004)]. How to complete the optimal operation of heat pump and save energy and minimize cost turns to be an important problem. To solve this problem, researchers analyzed and improved heat pump system on several fronts. For the application and economy, Feng et al. (2005) analyzed the low-temperature adaptability and investigating economy of heat pump system in different area of China. For the model level, Shi and Xue (2007) review the development of heat pump model. Gilles et al. (2002) simplified the models without distortion which still capture the control relevant dynamics. For the optimization, Clara et al. (2012) analyzed different cases of heat pump model and put out optimization-based control strategies. These work lays a solid foundation for the research of this paper.

As the coefficient of performance of air source heat pump is significantly affected by ambient temperature and the temperature of inlet and outlet of water cycle, the actual energy consumption of heat pump is different at various ambient temperatures and zone temperature requirements. For the reason of swing of the electricity price in a day, operation cost of heat pump is not same in different time. Considering ambient temperature, zone temperature requirement, electricity price and the dynamic process of operation of heat pump, to optimize the control of heating system in multiple objectives has very important significance.

2. SYSTEM ANALYSIS

The fundamental principle of heat pump heating system shown Fig.1 is that the heat pump extracts energy from air by the way of compressing to make the temperature of refrigerant rise. Then the refrigerant transfers the heat to the water in pipeline, which makes the water temperature turn from the inlet T_{wr} to the outlet T_{ws}.

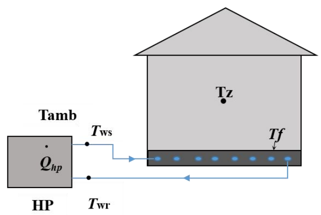


Fig.1. Schematic diagram of heat pump heating system.

The hot water with the temperature T_{ws} is transported to residential building by pipeline to rise the zone temperature, completing the goal of heating. For the system shown in Fig.1, the dynamic process of heating can be defined as：

$$\frac{dT_{ws}}{dt} = \frac{\dot{m}_w c_w}{C_{ws}} T_{ws} + \frac{\dot{m}_w c_w}{C_{ws}} T_{wr} + \frac{\dot{Q}_{hp}}{C_{ws}} \tag{1}$$

$$\frac{dT_{wr}}{dt}=\frac{\dot{m}_w c_w (T_{ws}-T_{wr})}{C_{wr}}+\frac{\kappa_{wz}(T_f-T_{w,r})}{C_{wr}} \quad (2)$$

$$C_f\frac{dT_f}{dt}=\kappa_{wf}(T_{w,r}-T_f)+\kappa_{fz}(T_z-T_f) \quad (3)$$

$$C_b\frac{dT_z}{dt}=\kappa_{fz}(T_f-T_z)+\kappa_b(T_{amb}-T_z) \quad (4)$$

where， T_{ws}, T_{wr}, T_f, T_z and T_{amb} represent respectively the supply water temperature， the return water temperature, the floor temperature, the zone temperature and the ambient temperature. m_w is the mass flow rate of the water circuit and c_w[J/kg.K] the specific heat capacity of water. $C_{w,s}$ and $C_{w,r}$ represent the thermal capacity of the water at temperature T_{wr} and the water at temperature T_{ws}, K_w, K_{fz} and K_{wz} represent overall heat exchange coefficient correspondingly. Q_{hp} [W] is the thermal power delivered by the heat pump, related to the operation frequency of compressor. dT_{ws}/dt, dT_{wr}/dt, dT_f/dt and dT_z/dt are the derivative of respectively T_{ws}, T_{wr}, T_f and T_z. The specific parameter values of this model are listed in Table 1.

Table 1 Parameter values of the model

Item	Value	Item	Value
$C_{w,s}$(J/K)	1.15×10^5	$\dot{m}_w$ (kg/s)	0.26
$C_{w,r}$ (J/K)	5.2×10^5	$K_{w,f}$(W/K)	1150
C_f(J/K)	5.6×10^7	$K_{f,z}$(W/K)	6120
C_b(J/K)	2.6×10^7	K_b(W/K)	280

For energy saving of heat pump, the heating specific energy consumption relates to the electricity consumption and performance of heat pump [Sakellari et al. (2006)], while the energy efficiency of heat pump is expressed by the coefficient of performance (COP):

$$\dot{Q}_{hp}(t)=P_{hp}(t)COP(t) \quad (5)$$

Where P_{hp} represents the input power (compressor power). In dynamic process, the COP is relevant to the temperatures, which can be expressed as follow [Radu et al. (2011)]:

$$\begin{aligned}COP(t)&=F(T_{amb},T_{ws})=Cop_0+\\ &\alpha_1T_{amb}+\alpha_2T_{ws}+\alpha_3T_{amb}{}^2+\alpha_4T_{ws}{}^2+\alpha_5T_{amb}T_{ws}\end{aligned} \quad (6)$$

The heat pump heating system controls the compressor operation frequency to determine the supply water temperature T_{ws}. When the ambient temperature fluctuates, the building heat losses change. So in relatively steady state, the heat delivered by heat pump matches the building heat loss expressed as follow:

$$\dot{Q}_{b,ss}=\kappa_b(T_{z,ref}-T_{amb}) \quad (7)$$

$$T_{ws}{}^*=T_{z,ref}+\frac{\beta\dot{Q}_{b,ss}}{\kappa_{wf}+\kappa_{fz}} \quad (8)$$

Where, $Q_{b,ss}$ is the building heat loss, $T_{ws}{}^*$ represents the reference supply water temperature and β (β>1) the ratio of the actual heat loss and the heat released to the zone. For the heat pump, the relation of the delivered heat, operation frequency and temperatures can be obtained by data-fitting [Liang et al. (2014)]:

$$\begin{aligned}\dot{Q}_{hp}&=h_0+b_1T_{amb}+b_2T_{ws}+b_3f_{re}+b_4T_{ws}^2+b_5T_{ws}^2+\\ &b_6f_{re}^2+b_7T_{amb}T_{ws}+b_8T_{amb}f_{re}+b_9T_{ws}f_{re}\end{aligned} \quad (9)$$

where， f_{re} is the operation frequency of the compressor and b_0, b_1...b_9 are the coefficients of the equation.

3. OPTIMIZATION OBJECTIVES AND SOLUTION ANALYSIS

As the main function of the floor heating system with air source heat pump is to meet the thermal comfort requirement in buildings, the reference zone temperature normally ranges from 15-25℃. The reference zone temperature is assumed to be $T_{z,ref}$ and the actual zone temperature is $T_z(t)$. As a general objective of control is to make the actual zone temperature approach to the reference zone temperature and to minimize the control error, which can be formulated by:

$$Min\ J_T=(T_z(t)-T_{z,ref})^2 \quad (10)$$

On the other hand， the operation cost of the system is in consideration. So squeezing the energy cost in the promise of a certain control accuracy and achieving the optimal control of the operation is also desirable. The electricity cost is mainly embodied in the electricity consumption of the system and directly relative to the electricity price in different period. So the cost objective is:

$$MinJ_e=e_p(t)P_{hp}(t) \quad (11)$$

Where e_p (t) is the electricity price at different moments. The operation power and COP of heat pump and the building heat loss are closely related to ambient condition(such as temperature), while the ambient condition changes with daily weather, which makes the system balance the cost objective and the thermal discomfort objective depending on the ambient condition. So in a certain period [0 t_{end}], an objective function is given as follow including both electricity cost and control accuracy.

$$J=K\int_0^{t_{end}}e_p(t)P_{hp}(t)+(1-K)\int_0^{t_{end}}(T_z(t)-T_{z,ref})^2 \quad (12)$$

Here K represents the weighting factor of the two objectives. For K=0, the optimal control profile only squeezes the energy cost under the given constraints. For K=1, the heat pump operation minimizes the thermal discomfort cost, regardless of the corresponding energy cost. In order to get excepted result, the value of K can be chosen depending on the tolerance to temperature tracking error set by users to satisfy both the thermal comfort requirement of user and decrease the energy cost. So the thermal comfort requirement should be introduced into system model equations, which can be:

$$\left|T_{z,ref}-T_z\right|\le\Delta \quad (13)$$

For the whole system, despite the clear ambient temperature change in a day, the change between adjacent days indicates to be quasi-periodicity to some extent. So for the convenience of study, optimization objective function (12) takes 24 hours as the cycle and makes the terminal and initial states the same. The temperature $TT=[T_{ws}, T_{wr}, T_f, T_z]^T$ is defined to reflect the main temperature states of heating. So the terminal and initial temperature states of the system should be:

$$\mathrm{TT}(0)=\mathrm{TT}(24) \tag{14}$$

And these states satisfy the constraint below：

$$\mathrm{TT}_{lb} \le \mathrm{TT}(t) \le \mathrm{TT}_{ub} \tag{15}$$

The problem that takes (12) as objective function and (1-9) and (13-15) as constraints is named as Opt1. Since the optimal problem includes a set of strong nonlinear DAEs (Differential and Algebraic Equations) and many constraints, it is fairly tough to solve this kind of problem by direct ways. Currently, DAEs optimization problems are solved by various strategies that apply nonlinear programming (NLP) solvers to the DAE model [Biegler et al. (2002), (2007)]. To solve the Opt1, simultaneous method was used to convert the DAEs into an NLP by approximating state and control profiles by a family of polynomials on finite elements. Here, we use the following monomial basis representation for the differential profile, which is popular for Runge-Kutta discretizations [Biegler (2010)]:

$$z(t)=z_{i-1}+h_i\sum_{q-1}^{KK}\Omega_q\left(\frac{t-t_{i-1}}{h_i}\right)\frac{dz}{dt_{t,q}} \tag{16}$$

Here z_{i-1} is the value of the differential variable at the beginning of element i, h_i is the length of element i, $dz/dt_{i,q}$ denotes the value of its first derivative in element i at the collocation point q, and Ω_q is a polynomial of order KK, satisfying

$$\Omega_q(0)=0 \quad q=1...,KK \tag{17}$$

$$\Omega_q^{'}(\rho_r)=\delta_{q,r} \quad q=1...,KK \tag{18}$$

Where ρ_r is the location of the rth collocation point within each element. Continuity of the differential profiles is enforced by:

$$z(t)=z_{i-1}+h_i\sum_{q-1}^{KK}\Omega_q(1)\frac{dz}{dt_{t,q}} \tag{19}$$

Based on a number of studies of Larry group, radau collocation points were selected in this study because they allow constraints to be set at the end of each element and to stabilize the system more efficiently if high index DAEs are present. In addition, the control and algebraic profile are approximated using a Lagrange basis representation which takes the form:

$$y(t)=\sum_{q=1}^{KK}\psi_q\left(\frac{t-t_{i-1}}{h_i}\right)y_{i,q} \tag{20}$$

$$u(t)=\sum_{q=1}^{KK}\psi_q\left(\frac{t-t_{i-1}}{h_i}\right)u_{i,q} \tag{21}$$

Here y_i, q and $u_{i,q}$ represent the values of the algebraic and control variables respectively in element i at collocation point q, t is the value satisfy $t_{i-1}\le t\le t_i$, ψ_q is the Lagrange polynomial of degree KK satisfying:

$$\psi_q(t)=\prod_{r=1}^{KK}\frac{(t-t^r)}{(t^q-t^r)}=\prod_{r=1}^{KK}\frac{(t-t^r)/h_i}{(t^q-t^r)/h_i}=\prod_{r=1}^{KK}\frac{(t-t^r)/h_i}{(\rho^q-\rho^r)} \tag{22}$$

And at collocation points:

$$\psi_q(\rho_r)=\delta_{q,r}=\begin{cases}1, q=r\\0, q\ne r\end{cases} \quad (q,r=1,...,KK) \tag{23}$$

The differential variables are required to be continuous throughout the time (or space) horizon, while the control and algebraic variables are allowed to have discontinuities at the boundaries of the elements. By the method of collocation the optimal problem with DAEs is transferred into NLP. Large scale NLP solvers such as IPOPT [Biegler and Zavala (2009)] are very suitable for the solution.

4. CASE STUDY

To analyze the performance of cost saving and temperature control with the method above, this paper takes a situation of air source heat pump heating in a certain area of Zhejiang as the object of study. Fig.2 shows the typical daily ambient temperature of this area in winter. Fig.3 depicts the fitted curves of air source heat pump COP. The time-of-use (TOU) electricity price in the area can be seen in Fig.4 (a). The reference zone temperature is set to be 20℃. According to the reality of the operation of heat pump system, the upper and lower bound of the state temperatures are identified to be TT_{lb}=[20,25,12,11], TT_{ub}=[65,50,35,25]. The optimization problem is solved with the simultaneous optimization technology, which split 24 hours into 48 control units to make the system regulate the input power. Then 10 finite elements are allocated in every unit and every finite element has 3 collocation points. And then the IPOPT solver is used under the platform of GAMS to solve the discretized NLP. In this way, the optimal operation and internal states of system meeting corresponding requirement is obtained.

4.1 Optimization results with difference weighting factors K

The optimization objective function includes two conflicting parts. The first part is operation electricity cost，namely the economy objective that aims to save money. The second part is consideration of the thermal discomfort, which is the thermal comfort objective to minimize the gap between the actual zone temperature and the reference zone temperature. The bigger the value of K is, the more attention the optimization pays to value the thermal comfort. On the contrary, the optimization will squeeze the actual electricity cost.

Square root of the second part $\int_0^{t_{end}}(\mathrm{T}_z(\mathrm{t})-\mathrm{T}_{z,ref})^2$ is identified to be the thermal discomfort. While the value of K changes from 0 to 1, the purpose of optimization varies. So the choice of the value of weighting factor will significantly influence thermal discomfort and actual electricity cost.

To make sure the balance of values of the two objectives, the optimization objective function is normalized. The normalized function is as follow:

$$J=(1-K)\int_0^{t_{end}}[e_p(t)Q_{hp}(t)/COP(\mathrm{t})+ \alpha K\int_0^{t_{end}}(T_z(t)-T_{z,ref})^2 \tag{24}$$

Here α is a constant obtained by computing, and when its value is 0.03, the objectives turn to be balanced well.

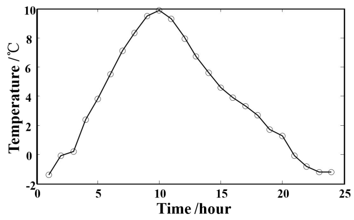


Fig.2. The typical daily ambient temperature of a certain area of Zhejiang in winter.

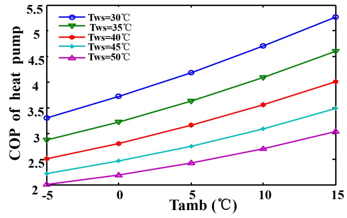
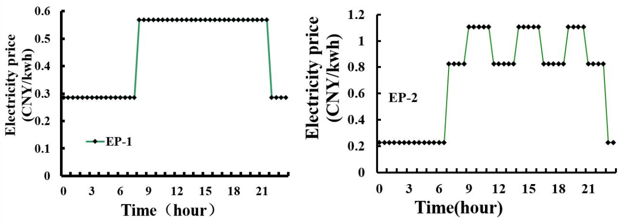


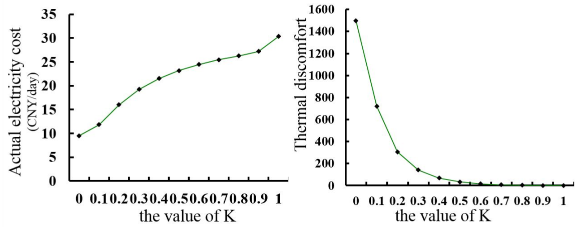
Fig.3.The fitted curves of air source heat pump COP.



(a) (b)

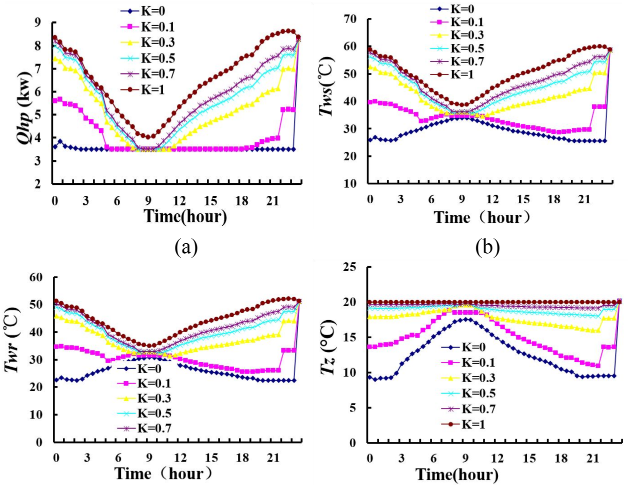
Fig.4. Time-of-use electricity price EP-1 (a) and EP-2 (b).

Disparate values of the weighting factor is adopted and the tolerance of zone temperature tracking error is undesigned. In this situation, solving the optimal control problem, the results are shown in Fig.5-Fig.6. Fig.5 (a) shows that the actual electricity cost of daily heating changes substantially as weight factor varies. When K=1, the cost is more than 30CNY per day. While K=0, it decreases to less than 10 CNY per day. Of course, this is the result of the sacrifice of thermal comfort, which can be seen from Fig.5 (b). Table 2 provides the maximum difference between the actual zone temperature and reference zone temperature, the thermal discomfort， the electricity cost and the cost-saving ratio. Even though the cost-saving ratio reaches an amazing level of 68.66%, meantime the maximum temperature difference also comes to an unacceptable value of 11.02℃ and the thermal discomfort is 38.68. But when the weighting factor ranges from 0.3 to 1, this compromise, despite the sacrifice of thermal comfort, demonstrates desired cost-saving performance, the temperature difference displayed in Table 2 is acceptably limited and the thermal discomfort is tolerable. For example, when K=0.7, the thermal discomfort is only 7 and the maximum temperature difference is merely 0.85℃. The cost， by contrast, decreases from 30.34 to 25.47 CNY. The larger the value of K is, the larger the thermal power becomes, as depicted in Fig.6. At midnight, the ambient temperature is low and the value of COP is small. In order to ensure a preferable zone temperature, higher supply water temperature is needed which means more thermal power requirement. With the climbing of ambient temperature, the thermal power declines the same as the supply water temperature. When the ambient temperature touches the peak, the thermal power and supply water temperature come to respective valley points.



(a) (b)

Fig.5. Actually electricity cost (a) and thermal discomfort (b) with different weighting factors.



(a) (b)

(a) (b)

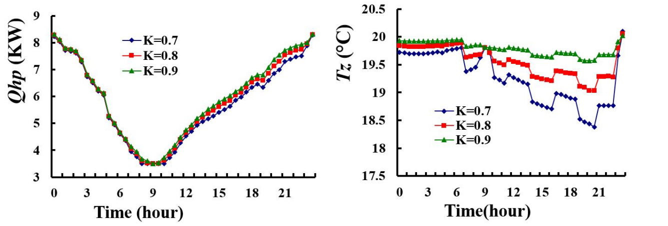
Fig.6. Optimal thermal power (a), supply water temperature (b), return water temperature (c) and zone temperature profiles (d) with different weighting factors.

Table 2 Maximum temperature difference, electricity cost, saving rate and thermal discomfort with different K

K	maximum temperature difference (℃)	electricity cost (CNY)	saving rate (%)	thermal discomfort
0	11.02	9.51	68.66	38.68
0.1	9.05	11.86	60.91	26.84
0.2	5.86	16.04	47.13	17.49
0.3	4.03	19.23	36.62	11.87
0.4	2.79	21.52	29.07	8.27
0.5	1.99	23.2	23.53	5.8
0.6	1.33	24.46	19.38	4.01
0.7	0.85	25.47	16.05	2.64
0.8	0.5	26.26	13.45	1.56
0.9	0.22	27.2	10.35	0.7
1	0	30.34	0	0

4.2 Optimal operation with different TOU electricity price

Electricity price is a direct factor that determines the actual electricity cost. For inspecting the effect of different electricity price on optimal operation, this paper introduces another industrial electricity price policy and constant electricity price. Another time-of-use (TOU) electricity price (EP-2) can be seen in Fig.4 (b). Fig.7 depicts the thermal power and zone temperature based on the electricity price EP-2 with the value of K respectively 0.7, 0.8 and 0.9. The resulting profiles show that the thermal power swings with the electricity price. At the condition of same *K* and period, high electricity price corresponds to low thermal power and vice versa. The relation between zone temperature and electricity price is same as the thermal power. What is more, the larger the weighting factor is, the smaller the effects of electricity price on thermal power and zone temperature are, which is because large *K* means little attention to economy objective.



(a) (b)

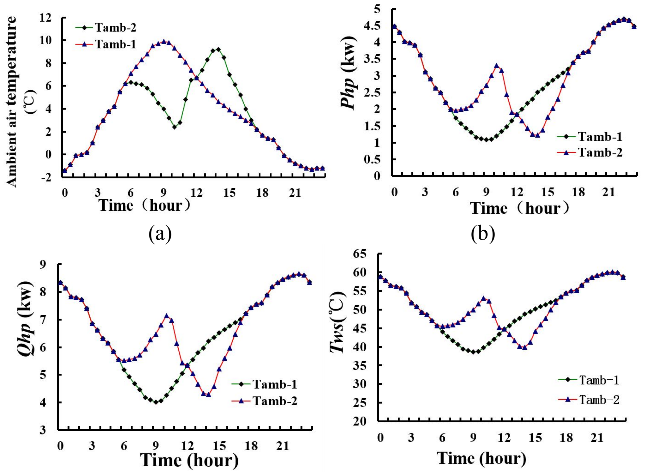
Fig.7. Thermal power curves (a) and zone temperature profiles (b) with the electricity price EP-2.

4.3 Analysis of optimal operation with ambient temperature fluctuation

Ambient temperature is an important condition for the running of air source heat pump, which directly impacts coefficient of performance (COP). To obtain the overall optimal performance, the corresponding energy consumption at different ambient temperature varies obviously. For inspecting the effect of ambient temperature on optimal objectives, the ambient temperature in Fig.2 is changed. Then the change of energy consumption is obtained by optimal solving. Table 3 lists energy costs when the ambient temperature overall rises and drops 1℃ and the K is 1. The results reveal that ambient temperature has an important effect on energy cost with optimal objective. One degree rises of the temperature cases 10.78% cost saving and one degree drop leads to 11.39% cost increasing. The optimal control trajectory and related states got by optimal saving show outstanding performance when the temperature distribution changes in a whole day. Fig.8 lays out the heat pump input and corresponding output power of two different ambient temperature profiles and the supply water temperature. When the ambient temperature is high, the requirement of the input power is comparatively low. Otherwise, it would be high and lead to more energy cost. As the optimal input power changes distinctly with different ambient temperature profiles, to complete the totally optimal objective of the system needs to regulate the input power according to the fluctuation of the ambient temperature.

Table 3 The effect of ambient temperature on optimal electricity cost

K=1	One degree rise	Tamb-1	One degree drop
Electricity Cost (CNY/day)	27.07	30.34	33.78
Rate of change (%)	-10.78	0	11.39



(a) (b) (c) (d)

Fig.8. Two different ambient temperature curves in the area (a), the profiles of respectively the thermal power (b), the input power (c) of heat pump and the supply water temperature (d) with the ambient temperature in Fig.8 (a).

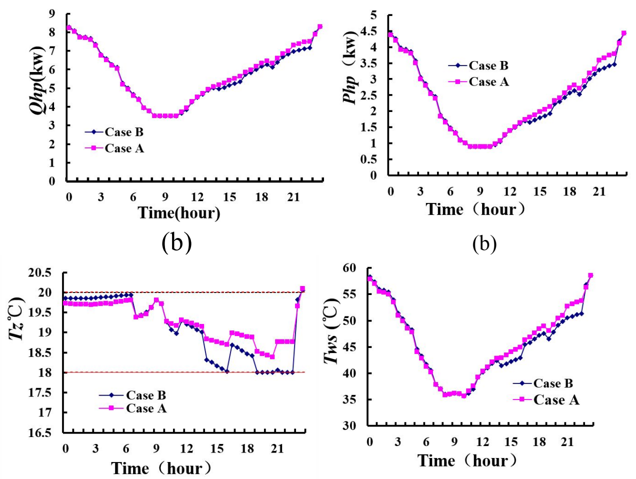
4.4 Analysis of optimization based on the dynamic adjustment of the weight of objectives

The analyses discussed in the previous paragraph indicate that the ambient temperature directly impact the optimization objective. For the energy cost, it means that the higher the ambient temperature is, the larger the COP is, namely more economic. On the other hand, the fluctuation of electricity price in different periods leads to the fact that to satisfy the optimal objective, the operation cost varies in different periods. In order to both minimize the zone temperature control error and get better cost-saving performance, a more advisable choice is that when the electricity price is comparatively low and the COP is relatively high, the optimization pays more attention to the control precision, namely do more to satisfy the requirement of zone thermal comfort. On the contrary, the optimization squeezes the energy cost of system further. According to the principle above, this paper adjusts the optimization objective function depicted in (12) to dynamically regulate the weighting factor between the energy cost objective and the thermal discomfort objective. The regulation executes once in every half hour for the expectation of better optimization performance. The adjusted objective function is:

$$J=(1-K)(e_p(t)/e_{p\min})\int_0^{t_{end}}[e_p(t)Q_{hp}(t)/COP(t)] \qquad (25)$$
$$+\alpha COP(t)K\int_0^{t_{end}}(T_z(t)-T_{z,ref})^2$$

Where $e_p(t)$ represents the dynamic regulation factor positively correlating to electricity price and $e_{p\min}$ the minimum value of electricity price in the whole period.

Based on the result analyses of Table3, the zone temperature tracking error is little and the performance of cost saving is desirable when the weighting factor ranges from 0.5 to 0.7. So to re-solve the optimization control problem, the weighting factor is set at 0.7 and the maximum difference between zone temperature and the reference zone temperature is required to no more than 2 ℃. The re-solving bases on the electricity price EP-1and the ambient temperature Tamb-1. Contrasting the new optimal results with the ones based on the former optimization objective function reveals that the adjusted optimization objective function leads to a better performance of cost saving. By the adjusting, the electricity cost changes from 36.56 to 34.85 CNY per day and the thermal discomfort from 4.26 to 5.77. Despite a little rise of the thermal discomfort, the zone temperature still meets the requirements and the system obtains a more cost saving close to 4.7%. The calculating result shows that the adjustment of the optimization objective function has visible performance. Fig.9 depicts the changes of input power and other related states. Case A represents the results based on optimization objective function (23) and Case B the results based on (25). As can be seen in Fig.9, the input power transforms apparently afternoon (after12 o'clock) when the optimization objective function alters. At the same time the zone temperature profile reflecting the thermal comfort is in the tolerance range despite the fluctuation.



(b) (b) (c) (d)

Fig.9 Comparison of the optimal results before and after the objective function adjusted

By the way, the total number of variables is 15554, the total number of equality constraints is 15307 and inequality constraints 52. Then, the computation time of the problem is about 59s.

5. CONCLUSION

As the fluctuation of ambient temperature and electricity price significantly influence the operation cost and the control performance of air source heat pump heating system, optimizing control of the heat pump heating system is meaningful through system optimization method. This paper, based on the basic dynamic process equations and the overall optimization objective, analyzes and studies the optimal operation control of the air source heat pump heating system for cost saving. At first, the paper analyzes the optimal operation control of the system with different weighting factors of two optimization objectives and the optimal results meeting a certain tolerance of thermal discomfort reveal that the optimization can achieve more than 16% cost saving by a little sacrifice of zone temperature control accuracy. Then, the paper studies the effect of electricity price and ambient temperature on the optimal operation control of the system and proposes a strategy that dynamically adjusts the objective function to squeeze cost. The calculating results shows that this strategy can lead to a further cost saving ratio which can be about 4.7%. The study of this paper has certain significance on directing the operation and cost saving of air source heat pump heating system. Based on this paper, the study about the operation of heat pump system with load fluctuation and internal heat storage will be in consideration as well as the relevant control techniques.

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