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Distributed Model Predictive Control Based on Nash Optimality for Large Scale Irrigation Systems ⋆

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Abstract: Irrigation system is a large scale system, consisting of many interacting channels, and spanning vast geographical areas. In practice, the water level is expected to the desired reference values for safety and efficiency. In this paper, the model of irrigation system is modeled by the principle on conservation of mass. Because of the strong coupling between neighboring pools in irrigation system and large disturbance from the environment, a non-cooperative distributed model predictive control (NDMPC) algorithm based on Nash optimality is proposed for the large scale irrigation system to design the controller for setting points regulation. Finally, a simulation example is given to demonstrate the effectiveness of the proposed algorithm.

Keywords: Distributed model predictive control, Irrigation systems, Nash optimality, Large scale systems.

1. INTRODUCTION

Water is one of the most vital elements in human life. Among other things, it is used for drinking, agriculture, transportation, recreation, and energy production. Irrigation systems consist of water bodies, such as lakes and reservoirs, connected by natural water channels and man-made channels. The water flows in the rivers and pools can be manipulated by pumps and gates (Agriculture and Water, 2010). Water is drawn from the reservoir and distributed through the main channel and many secondary channels to farms. Along the channels, mechanical gates are installed to regulate the flow. A stretch of water between two neighboring gates is called a pool. An irrigation system is usually largely gravity-fed(i.e. there is no pumping); to satisfy water-demands from farms and to decrease water wastage, the water levels in the pools should be regulated to certain set points (Li and Schutter, 2011; Rostalski et al., 2008). Since most farms sit at the downstream ends of pools, it is more important to control downstream water levels by controlling the gate opening. To avoid excessive communication load for large scale irrigation systems, decentralized control(there is no communication between any local controller) and distributed control(communication deliver between neighboring local controllers) are preferred to centralized control. Moreover, an irrigation channel is a system presenting strong coupling between pools, i.e. the flow into a pool is equal to the flow out of the neighboring upstream pool. With local gate opening controlling, when offtakes occur at a down stream pool (Litrico and Fromion, 2003, 2005).

There are lots of results for large scale irrigation system. In Cantoni et al (2007)., the irrigation system model had been described detailedly and a closed-loop centralized control algorithm and a decentralized control algorithm of open-water channels had been proposed from the perspective of large scale irrigation network management. In Bolea et al (2014)., the irrigation systems were suitably represented for control purposes by using linear parameter-varying (LPV) models, and the system parameters can be easily identified from input-output data by means of classical identification techniques, moreover, a decentralized controller is designed for the irrigation system. Because of the expensive communication cost and huge computation burden, the centralized control algorithm is confronted with great difficulties in practical applications. In the decentralized control, there is no information transfer between local controllers, which lead to a poor system performance of irrigation system. In Farina et al (2011)., a distributed moving horizon estimation (MHE) method was proposed for discrete-time nonlinear systems decomposed into coupled pools with non-overlapping states.

In Negenborn et al (2006)., a novel DMPC algorithm was presented based on Lagrange dual theory for transport systems to handle couplings between subsystems. An iterative DMPC algorithm was developed based on Nash optimality for large scale power system to tackle the state coupling between subsystems, and the relevant computation convergence and the nominal stability condition were presented for unconstrained DMPC (Li et al., 2005; Zhang and Li, 2007). Considering different forms of collaboration, the DMPC can be classified into two categories: 1) Distributed algorithms where each local controller minimizes a global cost function (cooperating algorithms). In Stewart et al (2010)., a cooperative linear DMPC strategy was presented for any finite number of subsystems and a

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stabilizability condition was also presented. A cooperative DMPC algorithm was investigated to provide an approach based on negotiation, cooperation and learning techniques (Javalera et al., 2010). In Morosan et al (2010)., a cooperative DMPC strategy had been proposed for building temperature regulation using electrical convectors. 2) Distributed algorithms where each local controller minimizes a local performance index (non-cooperating algorithms). A NMPC algorithm was investigated to deal with optimal control of large-scale power systems (Camponogara et al., 2002). In Mercangoz and Doyle (2007), a NDMPC algorithm was proposed for liquid level control system with four tanks.

There are also some results about DMPC for large scale irrigation system. In Alvarez et al (2013)., a NDMPC algorithm had been proposed to the control of an accurate model of an actual irrigation system in Spain, but the cooperative DMPC algorithm increases much of the computation burden to the local controller. A cooperating DMPC algorithm based on Lagrange dual theory was proposed for irrigation system (Negenborn et al., 2009), but the proposed algorithm was complicated with slow converging speed. To the best of authors' knowledge, although some of results have been available about DMPC for irrigation system, there are still some problems needing much further investigation, such as handing the strong coupling of the pools and large disturbance from environment in irrigation system, and reducing computation burden and communication cost in controller design. Besides, the irrigation system performance under the controlling of the existence algorithms is not satisfactory and lots of algorithms are still too complicated. NDMPC based on Nash optimality is a powerful approach to design controller with low computation burden and communication cost, fast converging speed and preferable system performance. In view of the above discussion, we are motivated to study NDMPC based on Nash optimality for the large scale irrigation systems.

2. MODELING OF THE IRRIGATION SYSTEM

An irrigation system consists of a main channel and several secondary channels, see Fig. 1. An irrigation channel is in fact a string of pools, and Fig. 2 shows the sideview of pools and the illustration of associated variables. With in the context of control design for set point regulation and load disturbance rejection, the following model is given based in principle on conservation of mass

$$\begin{aligned} \dot{x}_i(t) = x_i(t) + \frac{1}{\alpha_i}u_i(t) \\ - \frac{1}{\alpha_i}\sum\nolimits_{h\in\bar{i}} v_{i,h}(t) + d_i(t) \end{aligned} \tag{1}$$

$$\begin{aligned} s.t.\quad & x_i(t) \in X_i \\ & u_i(t) \in U_i \end{aligned}$$

where $x_i(t)$ is the water level of pool i, $i \in (1, ..., n)$, α_i is the surface area of pool i, $\bar{i}$ is the set of neighboring downstream pool i, $u_i(t)$ is the control decision of pool i (the flow from the neighboring upstream pool j to the pool i), $v_{i,h}(t)$ is the flow from the pool i to the neighboring downstream pool h with $v_{j,i}(t) = u_i(t)$ and $v_{i,h}(t) = u_h(t)$, $d_i(t)$ is the disturbance. X_i, U_i is the feasible domain of $x_i(t), u_i(t)$.

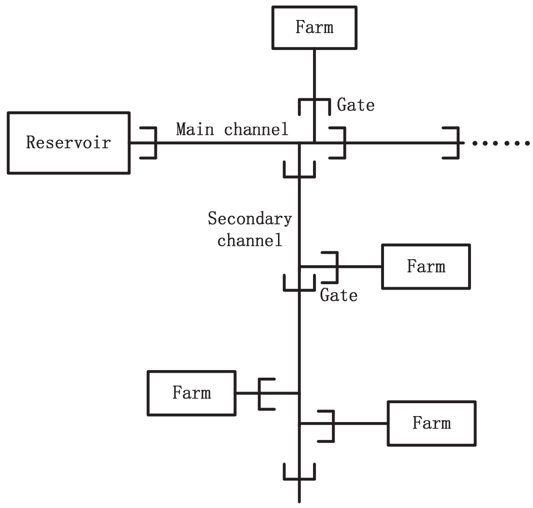

Fig.1. Irrigation system.

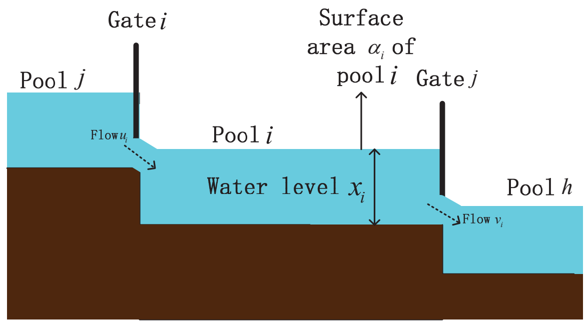

Fig.2. Illustration of pool i and associated variables

Discretize system (1) with the sampling time T, the discrete time state space model can be described as follows

$$\begin{aligned} x_i(k+1) = x_i(k) + \frac{T}{\alpha_i}u_i(k) \\ - \frac{T}{\alpha_i}\sum\nolimits_{h\in\bar{i}} v_{i,h}(k) + Td_i(k) \end{aligned} \tag{2}$$

$$\begin{aligned} s.t.\quad & x_i(k) \in X_i \\ & u_i(k) \in U_i \end{aligned}$$

3. NDMPC BASED ON NASH OPTIMALITY ALGORITHM DESIGN

3.1 Local performance index of pool

Considering the strong coupling between neighboring pools, communication constrains between pools and large disturbance in irrigation system, the NDMPC based on Nash optimality algorithm will be an efficient approach to handle these problems in controlling irrigation system. The structure of the NDMPC is shown in Fig 3. Because of the spacial structure of irrigation system, there is only control decision coupling between neighboring pools, and the direction of the information transfer is one-way, from downstream to upstream. Based on Nash optimality algorithm, the solution of NDMPC is computed in iterative form. In order to describe the algorithm clearly, the system model of pool can be rewritten as follows:

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$$\begin{aligned} x_i(k+1) &= x_i(k) + B_i u_i(k) \\ &\quad - \frac{T}{\alpha_i}\sum\nolimits_{h\in \bar{i}} u_h(k) + T d_i(k) \end{aligned} \tag{3}$$

$$\begin{aligned} s.t. \quad & x_i(k) \in X_i \\ & u_i(k) \in U_i \end{aligned}$$

where $B_i = \frac{T}{\alpha_i}$, $u_h(k) = v_{i,h}(k)$.

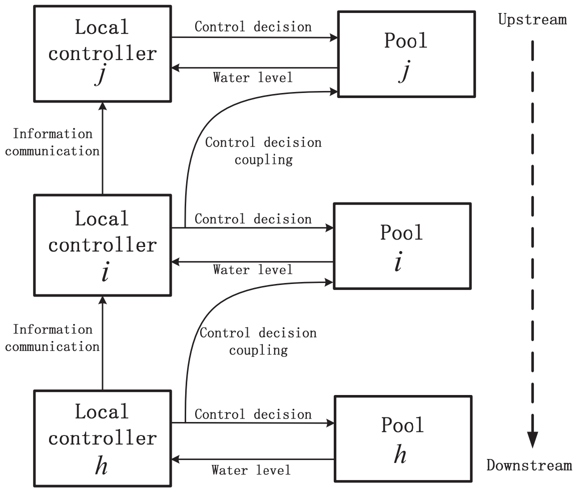

Fig.3. Illustration of NDMPC structure in irrigation system

According to (3), each local controller computes the local control decision based on neighborhood communication, and the optimize problem for local performance index is defined as follows

$$\min_{\substack{u_i(k|k)\\ \vdots \\ u_i(k+M|k)}} \quad J_i(k) \tag{4}$$

where

$$\begin{aligned} J_i(k) &= \sum\nolimits_{s=1}^{P} \left\| x_i(k+s|k) - x_{i,r}(k+s) \right\|_{Q_i} \\ &\quad + \sum\nolimits_{s=0}^{M-1} \left\| u_i(k+s|k) \right\|_{R_i} \end{aligned} \tag{5}$$

$$x_i(k+s|k) = x_i(k) + \hat{\mathbf{B}}_i \hat{\mathbf{u}}_i(k) - \sum_{h\in \bar{i}} \hat{\mathbf{B}}_i \hat{\mathbf{u}}_h(k) \tag{6}$$

and $\hat{\mathbf{u}}_i(k) = [u_i(k|k), \cdots, u_i(k+s-1|k)]^T$, $\hat{\mathbf{B}}_i = \underbrace{[B_i, \cdots, B_i]^T}_{s\times B_i}$. when $s \succ M$, then $s = M$. $x_i(k+s|k)$ and $u_i(k+s|k)$ are the predictive water level and control decision, respectively, at time instant $k+s$ based on the water level at time instant k of pool i. $x_{i,r}$ is the set point of water level for pool i. Q_i and R_i are the weight coefficients matrices of water level and control decision with appropriate dimension, P is the prediction horizon, M is the control horizon and $M \le P$. The objective of the each local controller is to regulate the pool's water level to the set point while keeping the performance index minimal and satisfying the above constraints. So the local control decisions $u_i(k)$ is computed by solving local optimization $\min J_i(k)$ with constraints in the NDMPC based on Nash optimality. The form of the local controller is given

$$u_i(k) = k_{i,1} x_i(k) + k_{i,2} \tag{7}$$

where $k_{i,1}$ and $k_{i,2}$ is the local controller gains of pool i.

3.2 Algorithm design

The Nash optimality algorithm is one of the iterative algorithms, which is developed to seek the local optimal control decision for each subsystem at each sampling time. Each subsystem resolves its local optimization problem with the local optimal control decision of its neighbours available. Then each subsystem compares the newly computed control decision with that obtained in last iteration and checks if the terminal condition is satisfied. If the algorithm is convergent, all the terminal conditions of all the subsystems will be satisfied and the iteration will be terminated; otherwise, each subsystem then transfer the newly computed control decision to its neighbours and resolves its local problem with the updated values for neighbours. This optimization process will be repeated at the next sampling time. The overall control system will converge at Nash equilibrium. However, the Nash optimal solution to the local optimization problem may not equal to the global optimal control decision. In large scale irrigation systems, the goal of communication and exchanging solutions among neighboring local controllers is to achieve the optimal solutions of the pools in an iterative fashion. Based on Nash optimality, an online iterative algorithm for NDMPC is proposed to seek the local optimal control decision for each pool at each time instant. The algorithm is summarized as Algorithm 1.

Algorithm 1

S1 : Initialization:
At time instant k, each local controller measure its initial water level $x_i(k|k)$, initialize its predictive control decision $\hat{\mathbf{u}}_i^p(k) = [u_i(k|k), ..., u_i(k+M-1|k)]^T$, $p = 0$, here p is the iteration times.

S2 : Communication:
Each local controller transmits the predictive control decision $\hat{\mathbf{u}}_i^p(k)$ to neighbors. Simultaneously, local controller receive it's neighbors' predictive control decisions.

S3 : Local optimization:
Each local controller solves its local optimization problem objective (4) subject to (5) and (6) to derive its local optimal control decision $\hat{\mathbf{u}}_i^{p+1}(k)$.

S4 : Checking and updating:
Given error accuracy ε_i, if $\left\| \hat{\mathbf{u}}_i^{p+1}(k) - \hat{\mathbf{u}}_i^p(k) \right\| \le \varepsilon_i$, Set the local optimal control decision for time instant k with $u_i^*(k|k) = u_i^{p+1}(k|k)$, then go to S5. Else, go to the next iteration $p = p+1$, go to S2.

S5 : Receding horizon:
Let $[u_i(k+1|\, k+1), ..., u_i(k+M|k+1)] = [u_i(k+1|k), .., u_i(k+M-1|k), u_i(k+M-1|k)]$. Move horizon to the next sampling time $k = k+1$, go to S2, and repeat the above steps.

3.3 Stability analysis

The model of the overall irrigation system can be described as

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \\ &= (\mathbf{I} + \mathbf{B}\mathbf{K_1})\,\mathbf{x}(k) + \mathbf{K_2} \end{aligned} \tag{8}$$

$$\begin{aligned} s.t. \quad & \mathbf{x} \in \mathbf{X} \\ & \mathbf{u} \in \mathbf{U} \end{aligned}$$

where, $\mathbf{K_1} = [k_{1,1} \ ,..., \ k_{n,1}]^T$, $\mathbf{K_2} = [k_{1,2} \ ,..., \ k_{n,2}]^T$, $\mathbf{x} = [x_1 \ ,..., \ x_n]^T$, $\mathbf{u} = [u_1 \ ,..., \ u_n]^T$, $X = [X_1 \ ,..., \ X_n]^T$, $\mathbf{U} = [U_1 \ ,..., \ U_n]^T$. $\mathbf{I}$ is the identity matrix with appropriate dimension.

$$\mathbf{B} = \begin{bmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nn} \end{bmatrix}, \quad b_{ij} = \begin{cases} \dfrac{T}{\alpha_i} & i = j, \\ -\dfrac{T}{\alpha_i} & j \in \cancel{i}, \\ 0 & else. \end{cases}$$

According to the above analysis, the theorem of system stability can be summarized as follows

Theorem 1. The nominal stability of the irrigation system can be guaranteed if only if the following condition

$$\begin{aligned} &|\lambda_m (\mathbf{I} + \mathbf{B}\mathbf{K_1})| < 1 \\ &m = 1, ..., N \end{aligned} \tag{9}$$

holds, where the eigenvalues λ_m, $m \in (1, ..., n)$ of the above matrix are all in the unit circle.

It should be noted that the stability condition in (9) is global because each subsystem's controller gain $k_{i,1}$, $i \in (1, ..., n)$ computed by distributed algorithm illustrated in last section should satisfy the global stability condition.

4. SIMULATION

In this section, an irrigation system is used to illustrative the effectiveness of the proposed NDMPC based on Nash optimally algorithm for the large scale irrigation system. A schematic of this process is shown in Fig. 4, and the parameter values of this irrigation system is given in Table 1. It can be seen that the irrigation system is composed by one reservoir and five pools, and each pool can supply water to one or serval farms. The water level can be adjust by the gates (Each gate is controlled by local controller).

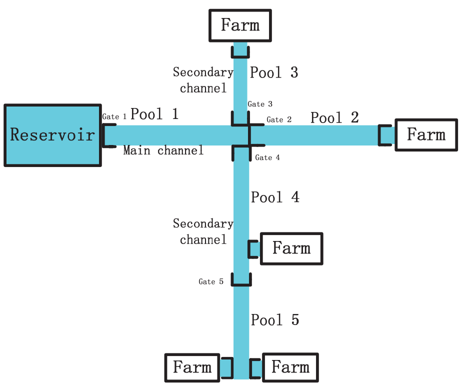

Fig.4. Schematic description of the irrigation system in simulation

Table 1: Parameter values of irrigation system

Pool	Surface area α	Set point of water level
1	210 m^2	5.5 m
2	160 m^2	5.3 m
3	170 m^2	4.5 m
4	180 m^2	5.0 m
5	180 m^2	5.0 m

Let $M = P = 5$, $Q_i = 400$, $R_i = 1$, $i \in (1, ..., 5)$. Then, the controllers are designed by Algorithm 1, which presented in section 3. The simulation results are shown in Fig. 5 and Fig. 6. Water level response of irrigation system in case of starting at the point which departure from the set points is shown in Fig. 5, and the corresponding control decision is shown in Fig. 6. It can seen from Fig. 5 and Fig. 6 that the water level would be adjust to the set points quickly by using the proposed algorithm when the irrigation system start at the point which departure from the set points. At $t = 150s$, there is a disturbance at pool 5, which may be the water releasing to the nearby farms. The water levels of pool 5, pool 4 and pool 1 are affected by the disturbance, because the upstream pools need to convey water down to the downstream pools to makeup the shortage of water . As shown in Fig. 5, it is more slower for the water levels of the upstream pools converges to the set points, but the offset of the water levels do not amplify but minish in the upstream pools, compared to the converge speed in downstream pools, and all the water levels of pools converges to the set points gradually.

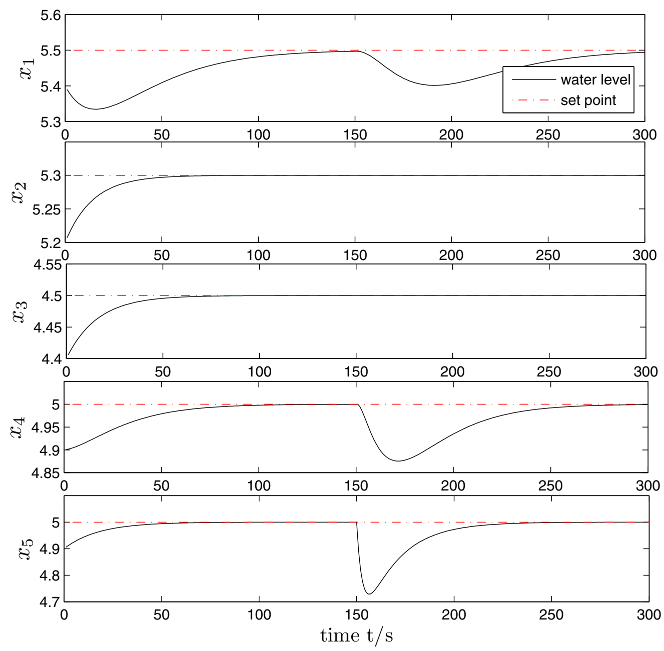

Fig.5. Water level response of irrigation system

5. CONCLUSION

In this paper, a NDMPC based on Nash optimality is proposed to manage large scale irrigation system. Firstly, a common irrigation system model is presented. Based on the irrigation system model, a NDMPC based on Nash optimality algorithm is presented to improve the system performance under the situation of strong coupling between

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pools, communication constrains and large disturbance from environment. Finally, a simulation example is given to demonstrate the effectiveness of the proposed algorithm. In addition, there are still some problems needing much further investigation, such as the irrigation system model should be describe more practically. More complex and practical case studies with time delay will be considered in the future.

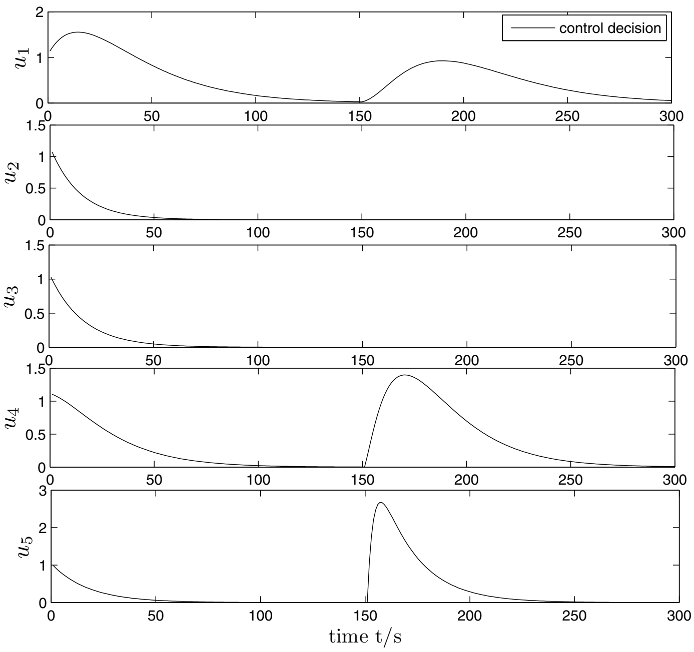

Fig.6. Control decision of the irrigation system

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