Model-based Fault-Tolerant Control of Uncertain Particulate Processes: Integrating Fault Detection, Estimation and Accommodation

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Abstract: This work presents a methodology for the integrated identification, estimation and accommodation of control actuator faults in particulate processes with discretely-sampled measurements and plant-model mismatch. Initially, a stabilizing state feedback controller is designed on the basis of a reduced-order model of the infinite-dimensional system, and the closed-loop stability region is characterized in terms of the model uncertainty, the fault magnitude, the sampling period and the control design parameters. When state measurements are unavailable, the reduced-order inter-sample model predictor generates state estimates which are updated at each sampling time. A moving-horizon optimization problem is then formulated and solved for on-line actuator fault detection, isolation and estimation using past state and input data. The resulting estimates are used to locate the operating point with respect to the closed-loop stability region, which in turn is used to carry out the fault accommodation logic via updating the pot-fault control model and/or adjusting the controller design parameters. The developed methodology is illustrated using a non-isothermal continuous crystallizer example.

Keywords: Fault identification, fault accommodation, sampled-data systems, reduced-order model, model-based control, particulate processes.

1. INTRODUCTION

Valuable products from the agricultural, chemical, food, mineral, and pharmaceutical industries are derived from particulate processes whose qualities are determined by the characteristics of the particle size distribution. Faulttolerant control of particulate processes is an important problem since control malfunctions have potentially negative effects on the particle size distribution which can lead to poor end product quality. This topic has received limited attention despite the significant body of research work on feedback control of particulate processes (e.g., see Semino and Ray (1995); Christofides (2002); Doyle et al. (2003); Hu et al. (2005); Larsen et al. (2006); Nagy (2009), for some results and references in this area).

The complex and uncertain dynamics, coupled with the infinite-dimensional nature of particulate process models, are some of the key challenges that complicate the design of model-based fault-tolerant control systems. While the literature on fault detection, isolation and accommodation is quite extensive (e.g., see Frank and Ding (1997); Blanke et al. (2003); Mhaskar et al. (2013); Raimondo et al. (2013); Paulson et al. (2014) for some results and references); results for particulate processes are limited at present (e.g., Giridhar and El-Farra (2009)).

Discrete measurement sampling is another important implementation issue that arises in the design of modelbased fault-tolerant control systems. In Napasindayao and El-Farra (2013), a stability-based fault detection scheme utilizing a time-varying residual alarm threshold was pro-

posed for detecting destabilizing faults in sampled-data particulate processes, and in Napasindayao and El-Farra (2014), a data-based fault identification scheme that allows the on-line detection of sudden and incipient faults while determining their location and magnitude was introduced. An advantage of this scheme is the fact that it can be used even after control system reconfiguration, which is an advantage over the detection strategy in Napasindayao and El-Farra (2013) where a new alarm threshold had to be calculated following every fault accommodation event which could cause delays in fault detection. The timely identification of faults location and magnitude allows for systematic scheduling of plant maintenance and equipment repair or replacement; however, this identification strategy was developed based on an accurate plant model. This assumption needs to be re-examined since model uncertainties are always present and can lead to inaccurate diagnosis of the fault or health status, and even closedloop instability, if not properly accounted for. In addition, the process was controlled using a sample-and-hold scheme because of the measurement sampling. This approach is simplistic and may lead to limited control capabilities especially for large sampling periods.

Motivated by these considerations, we develop in this work a model-based framework for fault-tolerant control of sampled-data particulate processes with model uncertainty and actuator faults. Model reduction techniques are initially applied to obtain an approximate reducedorder model which is used in designing a state feedback controller. The controller makes use of an inter-sample state estimator between the sampling times. The intersample state estimator is updated when sensor readings are received. Through stability analysis, an explicit characterization of the stability region is obtained as a function of the controller design parameters, the sampling period, the model uncertainty, and the actuator health status. This characterization is used for fault accommodation. Fault identification is carried out by solving a data-based moving horizon optimization problem in which the fault magnitude is estimated. The fault is then accommodated by either modifying the post-fault model in the intersample state estimator or the controller design parameter based on the stability analysis for all values within the estimation interval. Finally, the proposed fault-tolerant control framework is applied to a simulated model of a non-isothermal continuous crystallizer.

2. MOTIVATING EXAMPLE

In this section, we introduce a well-mixed non-isothermal continuous crystallizer as a motivating example that will be used throughout the paper to demonstrate the development and implementation of the proposed fault-tolerant control approach. The following reduced-order model of the crystallizer is adapted from Christofides (2002) by appropriately augmenting the population and component mass balances for the isothermal continuous crystallizer with the energy balance for the non-isothermal batch crystallizer. The model has the form:

$$\frac{d\mu_0}{dt} = \frac{-\mu_0}{\tau_r} + \left(1 - \frac{4}{3}\pi\mu_3\right)\bar{k}_2 e^{\frac{-k_3}{\left(\frac{c}{c_s} - 1\right)^2}} e^{\frac{-E_b}{RT}}$$

$$\frac{d\mu_v}{dt} = \frac{-\mu_v}{\tau_r} + v\mu_{v-1}\bar{k}_1(c-c_s)e^{\frac{-E_g}{RT}}, v = 1, 2, 3$$

$$\frac{dc}{dt} = \frac{c_0 - c - 4\pi\bar{k}_1 e^{\frac{-E_g}{RT}}\tau_r(c-c_s)\mu_2(\rho-c)}{\tau_r\left(1 - \frac{4}{3}\pi\mu_3\right)}$$

$$\frac{dT}{dt} = -\frac{\rho H_c}{\rho_s C_p}\frac{d\mu_3}{dt} - \frac{UA_c}{\rho_s C_p V}(T-T_c) + \frac{(T_0 - T)}{\tau_r}$$
(1)

where μ_v is the v-th moment of the crystal size distribution, c is the solute concentration in the crystallizer; T is the crystallizer temperature; τ_r is the residence time; ρ is the particle density; c_s is the concentration of the solute at saturation given by $c_s = -3\bar{T}^2 + 38\bar{T} + 964.9$, $\bar{T} =$ $\frac{T-273}{333-273}$, c_0 is the concentration of solute entering the crystallizer; \bar{k}_1 , \bar{k}_2 and \bar{k}_3 are constants; H_c is the heat of crystallization; ρ_s is the density of the contents of the crystallizer; C_p is the heat capacity; U is the heat transfer coefficient; A_c is the heat transfer area with the cooling medium; and T_c is the temperature of the cooling medium. For typical values of the process parameters, the process dynamics is characterized by an unstable steadystate surrounded by a stable periodic orbit. The open-loop oscillatory behavior exhibited by the crystallizer is due to the relative nonlinearity of the nucleation rate as compared to the growth rate (see Randolph (1980) for an early review of studies on crystal size distribution instabilities).

2.1 Control problem formulation

The control objective is to suppress this oscillatory behavior and stabilize the crystallizer at the following open-loop unstable steady-state: $x^s = [\mu_0^s \ \mu_1^s \ \mu_2^s \ \mu_3^s \ c^s \ T^s]^T =$

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 $[0.0047 \quad 0.0020 \quad 0.0017 \quad 0.0022 \quad 992.95 \quad 298.31]^T$, which corresponds to the desired crystal size distribution, by manipulating the solute feed concentration (c_0) and/or the residence time (τ_r) in the presence of actuator faults. Sampled measurements of the moments of the crystal size distribution $(\mu_v, \quad v = 0, 1, 2, 3)$, solute concentration, and temperature are collected discretely and sent to the controller where the control action is calculated and finally sent to the actuators.

2.2 Control actuator fault modeling

Linearizing the reduced-order system of 1 around the desired steady state, the linearized system takes the form: $\dot{x}(t) = Ax(t) + B^k \alpha^k u^k(t)$ (2)

where
$$x(t)$$
 is the vector of state variables; $u^k(t) = [u_1^k(t) \cdots u_m^k(t)]^T$ is the vector of manipulated inputs in
deviation variable form, m is the number of manipulated
inputs, k is a discrete variable denoting the active control
actuator configuration, and $\alpha^k = \text{diag}\{\alpha_1^k, \cdots, \alpha_m^k\}$ is
a diagonal fault parameter matrix that accounts for the
presence of actuator faults or malfunctions in the system.
Each diagonal element in the fault parameter matrix (α^k)
characterizes the local health status of the individual ac-
tuators. The entries of the fault matrix (α^k) take values
between 0 and 1, where 0 denotes total actuator failure,
and 1 denotes the fault-free state. In the absence of faults,
 $\alpha^k = I$ where I is the identity matrix. The state vector is
expressed as a deviation variable, $x(t) = \chi(t) - x_s$, where
 $\chi(t) = [\mu_0(t) \ \mu_1(t) \ \mu_2(t) \ \mu_3(t) \ c(t) \ T(t)]^T$; and A and
 B^k are constant matrices.

3. MODEL-BASED CONTROL SYSTEM DESIGN AND STABILITY ANALYSIS

3.1 Sampled-data controller design

To compensate for the unavailability of continuous measurements, an explicit inter-sample state estimator is used in the controller design. At each sampling time, the corresponding values of the measured states are instantaneously transmitted to the controller and are used to update the corresponding model states. The model-based state feedback controller is then implemented as follows:

$$u^{k}(t) = K\hat{x}(t), \ t \in [\tau_{j}, \tau_{j+1})$$

$$\dot{x}(t) = \hat{A}\hat{x}(t) + \hat{B}^{k}\hat{\alpha}^{k}u^{k}(t), \ t \in [\tau_{j}, \tau_{j+1})$$
(3)
$$\hat{x}(\tau_{j}) = x(\tau_{j}), \ j \in \{0, 1, \cdots\}$$

where K is the feedback gain, \hat{x} is the vector of estimated state variables used in generating the inter-sample control until the next state measurement is available, j is a sampling instance, and τ_j are the update times when values of the state are collected. $\hat{\alpha}^k = \text{diag}\{\hat{\alpha}_1^k, \cdots, \hat{\alpha}_m^k\}$ is a diagonal matrix whose diagonal elements represent estimates of the actuator faults which are used by the inter-sample model predictor to account for the effect of the faults. These elements are decision variables to be determined from the fault identification scheme which will be discussed in later sections. The plant matrices, A and B^k are approximated by the constant matrices \hat{A} and \hat{B}^k , where $\hat{A} = A + \Delta A$ and $\hat{B}^k = B^k + \Delta B^k$.

The controller gain (K) is chosen to ensure that the eigenvalues of $\widehat{A} + \widehat{B}^k \widehat{\alpha}^k K$ lie in the open left-half of the

complex plane. This choice ensures exponential stability of the origin of the closed-loop model, which implies that the closed-loop model state satisfies:

$$\|\widehat{x}(t)\| \le \gamma \|\widehat{x}(\tau_0)\| e^{-\varphi(t-\tau_0)} \tag{4}$$

for some constants $\gamma \geq 1$ and $\varphi > 0$.

3.2 Closed-loop stability analysis

To simplify the analysis, we consider the case when the sampling period is constant and equal for all the sensors, i.e., all the state measurements are available to the controller every Δ units of time, where Δ is the sampling period. Defining the model estimation error as $e(t) = \hat{x}(t) - x(t)$, the overall closed-loop system can be formulated as a discrete-continuous system of the form:

$$\dot{x}(t) = (\widehat{A} - \Delta A)(t) + (\widehat{B}^k - \Delta B^k)\alpha^k u^k(t)
\dot{x}(t) = \widehat{A}\widehat{x}(t) + \widehat{B}^k \widehat{\alpha}^k u^k(t)
e(\tau_i) = 0, \quad j = 0, 1, 2, \cdots$$
(5)

By analyzing the above discrete-continuous system, a sufficient condition for stability of the sampled-data closedloop system can be obtained in terms of the sampling period, the model uncertainty, the faults, and the controller design parameters. This condition is given in the following theorem.

Theorem 1. Consider the closed-loop system of (2) subject to the control and update law of (3). If Δ is chosen such that

$$\Gamma^{k}(\Delta) := \gamma \left(e^{-\varphi \Delta} + \frac{L_{A}}{\varphi + L_{B^{k}}} (e^{L_{A}\Delta} - e^{-\varphi \Delta}) \right) < 1 \quad (6)$$

then the sampled closed-loop states satisfy:

$$\|x(\tau_{j+1}^{-})\| < \|x(\tau_{j})\|, \ \forall j = 0, 1, 2, \cdots$$
(7)

where $L_A = \|\widehat{A} - A\|$ and $L_{B^k} = [\|\widehat{B}^k\|\|\widehat{\alpha}^k - \alpha^k\| + \|\Delta B^k\|\|\widehat{\alpha}^k\|]\|K\|$. Furthermore, $\lim_{i \to \infty} \|x(t)\| = 0$.

Remark 1. It can be seen from (6) and the definition of L_A and L_{B^k} that the closed-loop stability region defined by (6) is dependent on the degree of plant-model mismatch, the sampling period, the fault size and the controller design parameters. The stability condition can therefore be used to explicitly characterize the how these various factors influence the stability region. This characterization will be used as the basis for choosing an appropriate fault accommodation strategy.

4. DATA-DRIVEN ESTIMATION AND IDENTIFICATION OF ACTUATOR FAULTS

4.1 Discrete-time model formulation

To facilitate the fault identification step which relies on discrete process data, the continuous-time plant and model are first converted into discrete-time form to allow comparing the discrete state estimates to the historical state and input data. The discrete-time plant and model take the form:

$$x[j+1] = A_d x[j] + B_d^k \alpha^k u^k[j]$$

$$\hat{x}[j+1] = \hat{A}_d \hat{x}[j] + \hat{B}_d^k \hat{\alpha}^k u^k[j]$$
(8)

subject to the update law:

$$\widehat{x}[j] = x[j], \ j \in \{0, 1, \cdots\}$$
(9)

where $x[j] = x(\tau_j)$ is the vector of discrete process states, $\hat{x}[j] = \hat{x}(\tau_j)$ is the vector of discrete state estimates, and

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 $u^{k}[j] = u^{k}(\tau_{j})$ the vector of discrete input data. The update period $\Delta = \tau_{j+1} - \tau_{j}$ is the time interval between discrete consecutive measurements, j is the update instance, and A_{d} , B_{d}^{k} , \widehat{A}_{d} and \widehat{B}_{d}^{k} are discrete versions of the constant matrices A, B^{k} , \widehat{A} , and \widehat{B}^{k} ; respectively.

4.2 Moving horizon fault estimation

Fault identification involves estimating the value of the fault parameter matrix α^k using past state measurements and inputs. This is done by solving the following moving-horizon optimization problem (inspired by the formulation in Samar et al. (2006)):

$$\min_{\alpha^*} J(\zeta_j, \alpha^*) \tag{10}$$

s.t.
$$0 \le \alpha_{1,2,\cdots,m}^* \le 1$$

where the cost function is given by:

$$J(\zeta_j, \alpha^*) = \sum_{p=j}^{j-N_I+1} \left(\left\| x[p+1] - \widehat{A}_d x[p] - \widehat{B}_d^k \alpha^* u^k[p] \right\|^2 \right)$$
(11)

and $\zeta_j = \{(x[j-p], u^k[j-p]) | p = 1, 2, \cdots, N_I\}$ denotes the past N_I historical data of the state measurements and the manipulated inputs for each *j*-th sampling instance. The idea is to compute the fault estimates t hat would minimize the discrepancy between the actual and predicted state values over a certain horizon of length N. Note that a large value for N_I may result in discontinuities in the values of α^* , particularly right after a fault has occurred, since the pool of data used in the calculations will involve conflicting data taken both before and after the fault. This may delay the accurate identification of the fault. This parameter should therefore be carefully selected. Note also that the fault identification scheme makes use of the constant matrices, \hat{A}_d and \hat{B}_d^k , from the inter-sample model predictor since the plant dynamics are not fully known by the system in actual applications.

Due to the discrepancy between the inter-sample predictor model and the process (i.e., the fact that $A \neq \hat{A}$ and $B^k \neq \hat{B}^k$ which leads to $A_d \neq \hat{A}_d$ and $B^k_d \alpha^k \neq \hat{B}^k_d \hat{\alpha}^k$), the optimal estimate of the fault, α^* , obtained from the optimization problem of (10) will not be exactly the same as the actual fault parameter α^k . However, given bounds on the size of the model uncertainty, an estimation confidence interval for α^k_l dependent on α^*_l may be obtained. This interval is denoted by $\Psi_l(\alpha^*_l)$ and helps provide an estimate of the size (or range) of the fault and can thus be used for fault detection. Specifically, for a given sampling period that satisfies the stability condition of (6), a fault in the *l*-th actuator can be detected at T_d when the upper bound of $\Psi_l(\alpha^*_l)$ is less than 1, since we can easily conclude that $\alpha^k_l < 1$ in that situation.

5. STRATEGIES FOR FAULT ACCOMMODATION

Following the detection of a fault in the operating control configuration, we need to determine whether corrective action (e.g., updating $\hat{\alpha}^k$ using α^* or using a new feedback gain K^*) is required to preserve closed-loop stability. When the partial fault is not significant enough to impair the stability properties of the process, switching to a new control configuration may be unnecessary. Considering this, we develop in this section a stability-based fault accommodation logic which is formulated in Algorithm

1. The key idea is to maintain the current control configuration if all the elements of the estimation interval $\Psi_l(\alpha_l^*)$; $l \in \{1, \dots, m\}$, satisfy the stability condition of (6); otherwise, the system should switch to a new control configuration which can guarantee the stability of the closed-loop system.

$Algorithm \ 1.$

- 1. choose Δ that satisfies (6) and set $\alpha_l^k = \widehat{\alpha}_l^k = 1$, j = 0
- 2. solve (10)-(11) for α^* and estimate $\Psi_l(\alpha_l^*)$ for each $\alpha_l^*, l \in \{1, \cdots, m\}$
- 3. if for any $\vartheta_l \in \Psi_l(\alpha_l^*)$, (6) is violated with $\alpha_l^k = \vartheta_l$
- 4. **if** for all $\omega_l \in \Psi_l(\alpha_l^*)$, α_l^* satisfies (6) with $\alpha_l^k = \omega_l$ and $\widehat{\alpha}_l^k = \alpha_l^*$
- 5. **update** $\hat{\alpha}_l^k$ using α_l^* at the next transmission time and GOTO step 12
- 6. else if any K^* satisfies (6)
- 7. **update** K using K^* at the next transmission time and GOTO step 12
- 8. **else**
- 9. **replace** *l*-th actuator with a new actuator that satisfies (6), **set** $\alpha_l^k = \widehat{\alpha}_l^k = 1$ at the next transmission time and GOTO step 12
- 10. end if
- 11. else
- 12. j = j + 1 and GOTO step 2
- 13. end if

Besides considering the stability requirement to compensate for the destabilizing effect of an actuator fault, it is possible to incorporate performance considerations in the accommodation logic to determine the optimal solution among all stabilizing backup control configurations. Also note that Algorithm 1 can be applied in the case of multiple and simultaneous faults. In these cases, the solution of the optimization problem of (10)-(11) also determines which of the actuators is faulty. The faults may be accommodated following an approach similar to the one described in Algorithm 1, by updating all $\hat{\alpha}_l^*$ associated with the faulty actuators or using a new feedback gain.

6. SIMULATION STUDY

The non-isothermal continuous crystallizer example introduced in Section 2 is used in this section to illustrate the implementation of the proposed fault-tolerant control scheme. The closed-loop stability regions are obtained using the condition $\Gamma^k(\Delta) < 1$ which is derived from the closed-loop stability analysis of the discrete-continuous system in (6). These stability regions are obtained as explicit functions of the controller gain (K), the sampling period (Δ), the fault parameter (α^k), the plant-model mismatch ($\Delta A, \Delta B$), and the control configuration selection (B^k, \hat{B}^k).

Each of the diagonal elements in the fault matrix (α) characterizes the local health status of the individual actuators. In the initial control configuration (k = 1), two actuators are utilized for control: α_1 represents the health status of the actuator related to the first manipulated variable (u_1^1) used to vary the inlet concentration (c_0), and α_2 is for the other actuator used to adjust the second manipulated variable (u_2^1), the residence time (τ_r). The

regions of stability are plotted as a function of the health status of the first actuator (α_1) against the fault model parameter estimate $(\hat{\alpha}_1)$ as shown in Fig.1 for the case when a perfect model is used (plot (a)) and the case when model uncertainty is taken into account (plot (b)). The uncertainty is considered to be in \bar{k}_3 , which is an experimentally determined constant that influences the growth rate of particles in the crystallizer (see 1).

The blue area enclosed by the unit contour line in each plot shows the region where the closed-loop process is guaranteed to be stable. These two contour plots are useful when there is a single fault in the actuator controlling the inlet concentration since these were generated by setting the fault parameters of other actuators equal to 1, thereby signifying their fault-free status. Similar plots may be generated for other conditions. Such plots are useful in predicting the behavior of the process and in determining the appropriate fault-tolerant response once a fault is identified. A partial malfunction in any of the actuators could possibly occur such that the operating point is shifted somewhere within the stability region. Such faults are not detrimental to process stability and may not harm product quality and; therefore, do not necessarily warrant immediate fault accommodation or control reconfiguration. Based on this knowledge, the plant supervisor is then able to strategically prioritize which specific control loop or plant equipment requires maintenance or replacement through this stability-based closed-loop analysis. In cases where there are more variables to consider (e.g., a larger number of manipulated variables), instead of a two-dimensional contour plot, a look-up table with values of $\Gamma^k(\Delta)$ for varying magnitudes of the process parameters can be generated off-line and then used to judge if an identified fault requires urgent attention.

The contour plot of the region of stability for different systems with and without plant-model mismatch show that uncertainties can significantly limit the range of parameters under which the process is still stabilizable (see Fig.1). When there is a perfect model, the system is more tolerant to differences in values of α_1 and $\hat{\alpha}_1$ and is still closed-loop stable under severe malfunctions. Both regions of stability take the form of a diagonal figure that is symmetric around the $\alpha_1 = \hat{\alpha}_1$ axis. This is reasonable since, in the best case scenario, the fault model parameter $\hat{\alpha}_1$ should be equal to the actual fault parameter α_1 .

For the two fault scenarios considered below, the controller gain (K) is calculated by specifying the location of the poles of $\widehat{A} + \widehat{B}^k \widehat{\alpha}^k K$ at $[-1 \ -2 \ -3 \ -4 \ -5 \ -6]$. Fault identification is carried out by solving for the fault estimation matrix (α^*) in the optimization problem in (10)-(11). This is done by comparing the past 20 data points $(N_I = 20)$ of the state measurements and manipulated input to values generated by the discrete-time model in (8). Actuator faults in the manipulated variable responsible for controlling the inlet solute concentration (c_0) are investigated and the simulation is carried out under a sampling period of $\Delta = 0.1$ h. All faults are introduced after 1 h of operation. In both cases, the fault identification scheme is shown to be effective in quantifying and almost instantaneously locating faults—limited primarily by the measurement sampling. However, jumps in the calculated



Fig. 1. Plots (a)-(b): Region of stability is larger with a perfect model (a) compared to one with model uncertainty (b) using the feed concentration (c_0) and residence time (τ_r) as manipulated variables. Contour plots of $\Gamma^k(\Delta)$ plotted against different values of the fault parameter (α_1^1) and fault model parameter ($\hat{\alpha}_1^1$).

values of the fault estimation parameters are occasionally observed right after a fault. This is attributed to the sudden disruptions in the data points used in the databased identification method which includes values of the state and the manipulated variable before and after the fault occurs. This is why the optimization horizon (N_I) has to be properly selected and the selection of the appropriate fault accommodation strategy should be deferred until the fault identification scheme settles to a final value for the fault estimation parameter.

The first case involves a malfunction wherein the actuator controlling the feed concentration (c_0) , the first manipulated variable (u_1^1) , becomes 90% effective $(\alpha_1 = 0.9)$, while the other actuator used in varying the residence time (τ_r) remains fault-free $(\alpha_2 = 1)$ (see Fig.2(a)). Prior to the fault, the fault parameters (α_1, α_2) and fault model parameters $(\hat{\alpha}_1, \hat{\alpha}_2)$ for all the actuators are equal to 1. The fault is almost immediately identified and is reflected by changes in the calculated values of the fault estimation parameter (α_1^*) which eventually settles to a final value of 0.82. The offset between the fault magnitude (α_1) and the optimized fault estimate (α_1^*) is due to the model uncertainty. Since there is plant-model mismatch, the estimated value of the fault is unreliable but yields some information about the range of possible values of the fault parameter. Based on the uncertainty bounds, the range of possible values for the fault is found to be $\Psi_1 = [0.95, 1]$ (see Fig.2(b)). An examination of the stability region plot shows that this range of values for α_1 is completely within the region of stability if the fault model parameter estimate $\hat{\alpha}_1$ is maintained at its initial value of $\hat{\alpha}_1 = 1$. Hence, the fault will not result in system instability, and plant operations may resume without having to modify the fault model parameter $\hat{\alpha}_1$. This is verified by the dynamics of the total particle size (μ_1) and inlet solute concentration (c_0) which reveal that the fault does not significantly disrupt process performance and the states eventually settle to their steady state values (see Figs.2(c)-(d)).

In the second scenario, a fault causes a 40% drop in the effectiveness of u_1^1 , the actuator modifying the feed concentration (c_0) ; this time leading to instability. The fault parameter matrix then shifts from $\alpha = \text{diag}\{\alpha_1, \alpha_2\} = \text{diag}\{1, 1\}$ to $\alpha = \text{diag}\{0.4, 1\}$ (see Fig.3(a)). The fault identification scheme is able to estimate the fault at $\alpha_1^* = 0.3675$, and the range of possible fault parameter values in

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Fig. 2. Plots (a)-(d): Fault identification after a partial fault ($\alpha_1 = 0.9$) at t = 1 h. Plot (a): Dynamics of fault parameter (α_1) and fault estimation parameter (α_1^*). Plot (b): Region of stability with the estimation interval $\Psi_1 = [0.95, 1]$ for $\hat{\alpha}_1 = 1$ (red line). Plots (c)-(d): Dynamics of the state (μ_1) (c) and the faulty actuator controlling the manipulated variable u_1^1 , the feed concentration (c_0) (d).

this case is found to be $\alpha_1 \in \Psi(\alpha_1^*) = [0.4, 0.475]$ which lies completely outside the stability region (see Fig.3(b)).

From the contour plot, one can observe that no matter how $\hat{\alpha}_1$ is adjusted to try to accommodate the fault, a part of the possible fault range will always lie outside the stability region (i.e., there are values of α_1 that fall outside the stability region for all values of $\hat{\alpha}_1$). There is, therefore, no guarantee of closed-loop stability for this fault accommodation option.



Fig. 3. Plots (a)-(b): Fault identification after a partial fault ($\alpha_1 = 0.4$) at t = 1h. Plot (a): Dynamics of fault parameter (α_1) and fault estimation parameter (α_1^*). Plot (b): Region of stability with the estimation interval $\Psi_1 = [0.4, 0.475]$ for $\hat{\alpha}_1 = 1$ (red vertical line).

Since there is no stabilizing $\hat{\alpha}_1$ value for $\alpha_1 = 0.4$, the next alternative is to search for a stabilizing feedback gain K^* value when $\alpha_1 = 0.4$ and $\hat{\alpha}_1 = 1$ (see Fig.4(a)). The stability region is then plotted as a function of the fault parameter (α_1) and the control design parameter. Recall that the feedback gain K is selected using pole placement such that the poles of the closed-loop model $\hat{A} + \hat{B}^k \hat{\alpha} K$ are at $p = [\lambda \ p_2 \ p_3 \ p_4 \ p_5] = [-1 \ -2 \ -3 \ -4 \ -5 \ -6].$ The first pole value (λ) is chosen as the control design parameter and used to adjust K for fault accommodation. The stability region generated reveals that there is no stabilizing feedback gain (K^*) when $\alpha_1 = 0.4$ and $\hat{\alpha}_1 = 1$ (the dark green zone is the stability region). The next option is reverting to a different control configuration that does not use the faulty actuator responsible for controlling u_1^1 , the feed concentration (c_0). The residence time (τ_r), the second manipulated variable in the original control configuration (u_2^1) , becomes the sole manipulated variable (u_1^2) in the second control configuration. This causes a change in the stability properties of the closed-loop system as reflected in the new stability region shown in Fig.4(b). Through this reconfiguration-based fault accommodation, the controller is able to maintain closed-loop stability after a potentially destabilizing fault (see Fig.5).

It should be noted that the regions of stability are not only useful in determining the appropriate control action once a fault has occurred, but may also provide insight in selecting the best control design parameters for faulttolerance. The stability region plotted against values of the fault parameter (α_1) and the controller design parameter (λ) reveal that the stability region is less robust to faults for large values of (λ) when the fault model parameter ($\hat{\alpha}_1$) is equal to 1. Thus, K was initialized using $\lambda = -1$ (see Fig.4(a)) which leads to the largest range of tolerable faults. If the pole is pushed further to the left in the complex plane, the range of tolerable faults shrinks and even small faults can become destabilizing.



Fig. 4. Plots (a)-(b): Regions of stability used in selecting the best fault accommodation strategy after a partial fault ($\alpha_1 = 0.4$) at t = 1h. Plot (a): Stability region for different values of the fault parameter (α_1^1) and the controller design parameter (p_1) using the feed concentration (c_0) and residence time (τ_r) as the manipulated variables ($\hat{\alpha}_1^1 = 1$). Plot (b): Stability region plotted against the fault parameter (α_1^2) and the fault model parameter ($\hat{\alpha}_1^2$) using the residence time (τ_r) as the only manipulated variable (u_1^2).

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- Fig. 5. Plots (a)-(b): Dynamics of the state (μ_1) (a) and the fallback manipulated variable u_1^2 varying residence time (τ_r) (b) shows that fault accommodation re-establishes stability after a potentially destabilizing fault.
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